



Tutorials for Verification
Exercise sheet 8

Exercise 1: NBA might be smaller than DBA

Prove: there exists an alphabet such that for all $n \geq 1$ there is an n -state NBA A so that there is a DBA recognizing $L_\omega(A)$ and so that any DBA recognizing $L_\omega(A)$ has $\Omega(2^n)$ states.

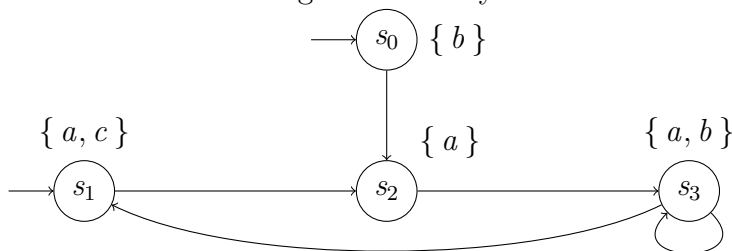
Exercise 2: Checking ω -regular properties

Let $AP = \{ req_1, crit_1 \}$. Consider the non-starvation property, which can be expressed in LTL as $P_{starve} = \text{Words}(\Box(req_1 \Rightarrow \Diamond crit_1))$.

- (a) Construct a Büchi-Automaton \mathcal{A} that recognizes the complement of P_{starve} .
- (b) Build the reachable part of the cross product $TS_{pet} \otimes \mathcal{A}$ where TS_{pet} is the transition system of Petersen's mutual exclusion. Check if there is a reachable cycle containing a final state. Does $TS_{pet} \models P_{starve}$?
- (c) Do the same for TS_{sem} the transition system for the semaphore based mutual exclusion.

Exercise 3: States satisfying LTL-formulae

Consider the following transition system over the set of atomic propositions $\{ a, b, c \}$:



Give for each of the following LTL-formulae the set of states for which the formula is satisfied.

- | | | |
|---------------------------|---------------------------|----------------------------------|
| (a) $a \wedge \bigcirc b$ | (b) $\bigcirc c$ | (c) $\bigcirc \bigcirc c$ |
| (d) $a \cup b$ | (e) $b \cup a$ | (f) $b \cup \Box a$ |
| (g) $a \cup \Box b$ | (h) $\neg(a \cup \Box b)$ | (i) $(\Diamond c) \cup (\Box a)$ |
| (j) $\Diamond \Box a$ | (k) $\Box \Diamond b$ | (l) $\Box \Diamond c$ |

Exercise 4*: Complements need exponential space in worst case

Prove or refute:

- (a) There is an alphabet such that for infinitely many n , there is an n -state NFA (nondeterministic finite automaton) A such that any NFA accepting $\Sigma^* \setminus L(A)$ has $\Omega(2^n)$ states.
- (b) There is an alphabet such that for infinitely many n , there is an n -state NBA (nondeterministic Büchi automaton) A such that any NBA accepting $\Sigma^\omega \setminus L_\omega(A)$ has $\Omega(2^n)$ states.

In positive cases, can you prove a larger lower bound than 2^n ?