Finite Asynchronous Automata

Julian Jarecki

Department of Computer Science,
SWT, Freiburg, Germany
http://swt.informatik.uni-freiburg.de

18. Juni 2012

Content

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Overview

- introduced by Wiesław Zielonka [Zie87]

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- restriction to a finite statespace

Recap: Petri Nets

$$L_n = \{wc \mid w \in \{a, b\}^* \ \wedge \ |w|_a = |w|_b = n\}$$

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- $L_1 = \{abc, bac\}$
- $L_2 = \{aabbc, ababc, baabc, babac, bbaac\}$
- $L_3 = \{aaabbbc, aababbc, \ldots, bbbaaac\}$

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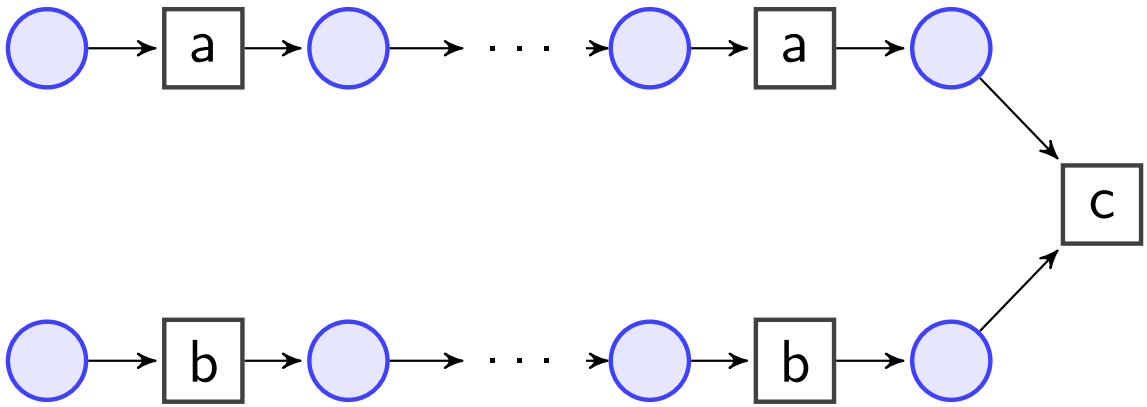

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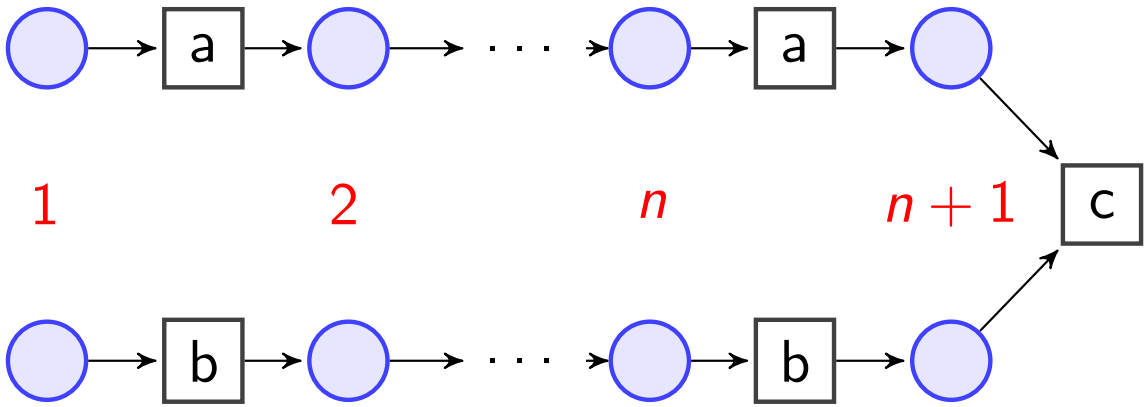

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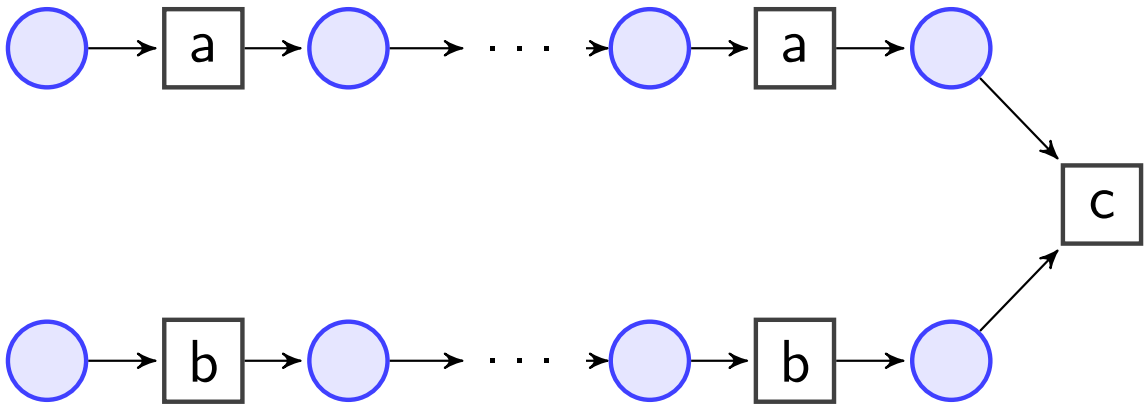

Exactly one final marking:

$$p \mapsto 0$$

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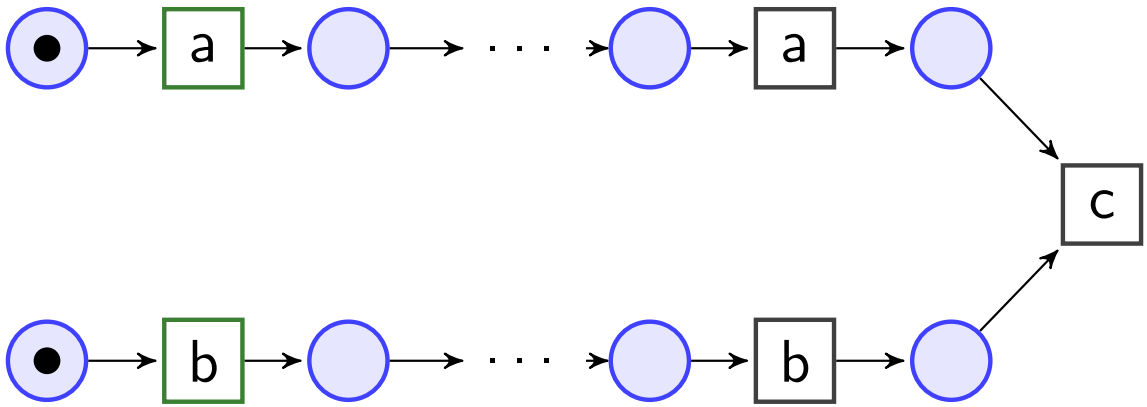

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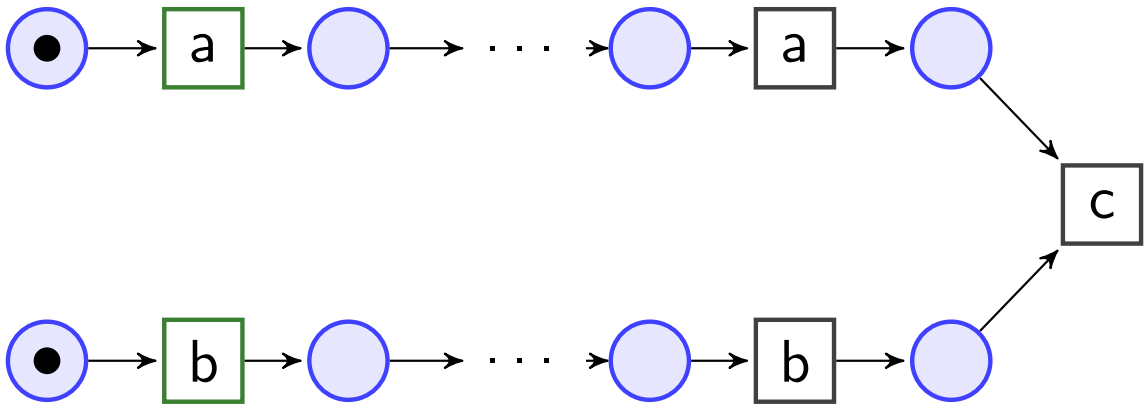

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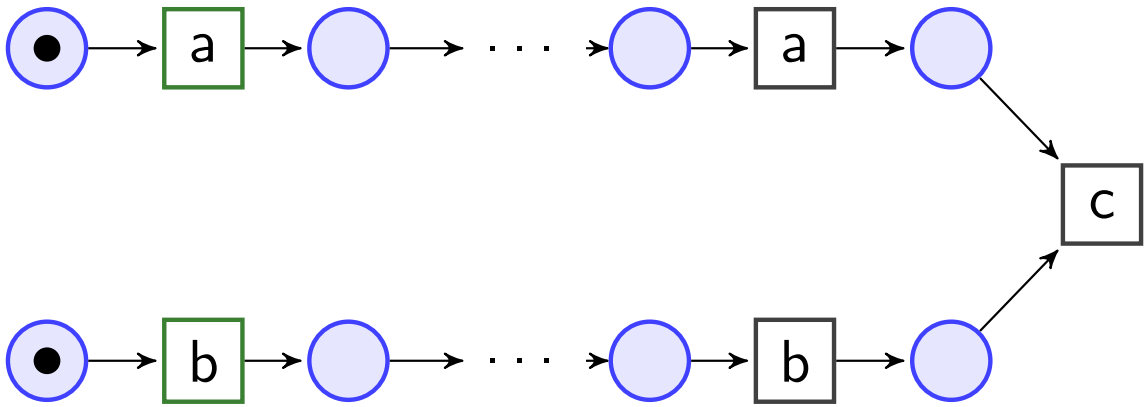

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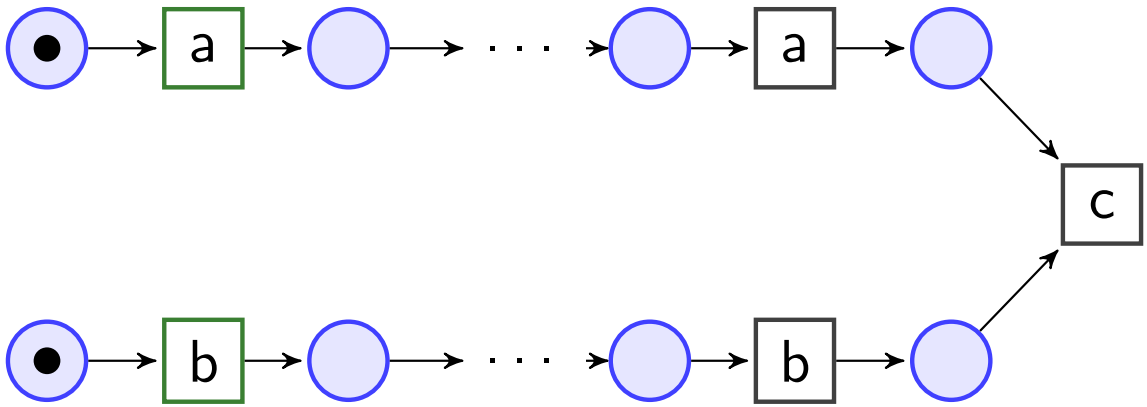

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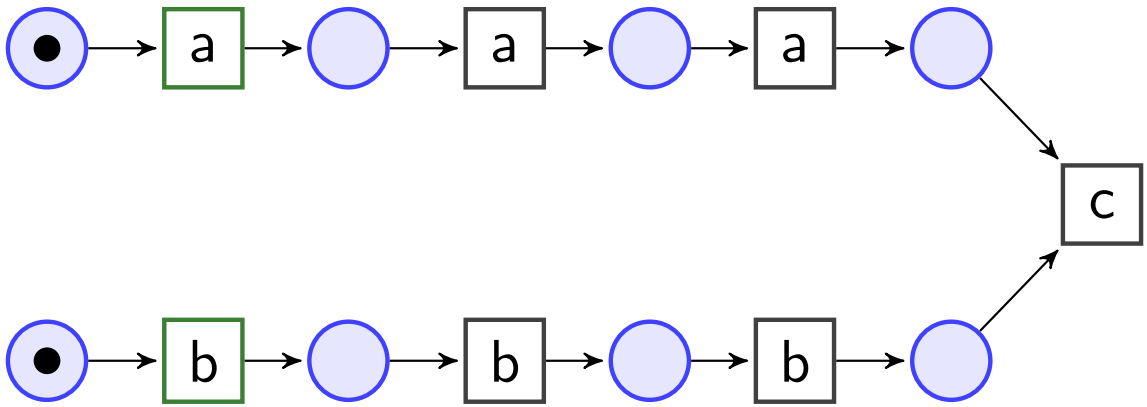

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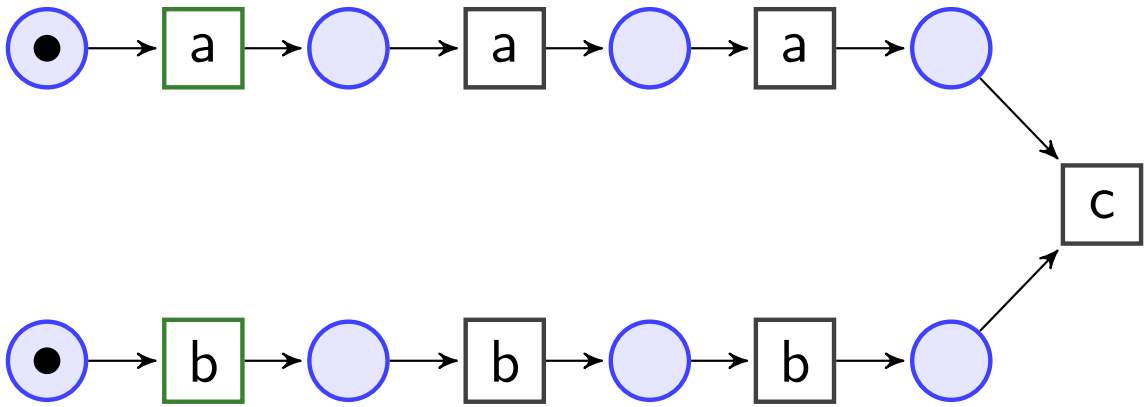

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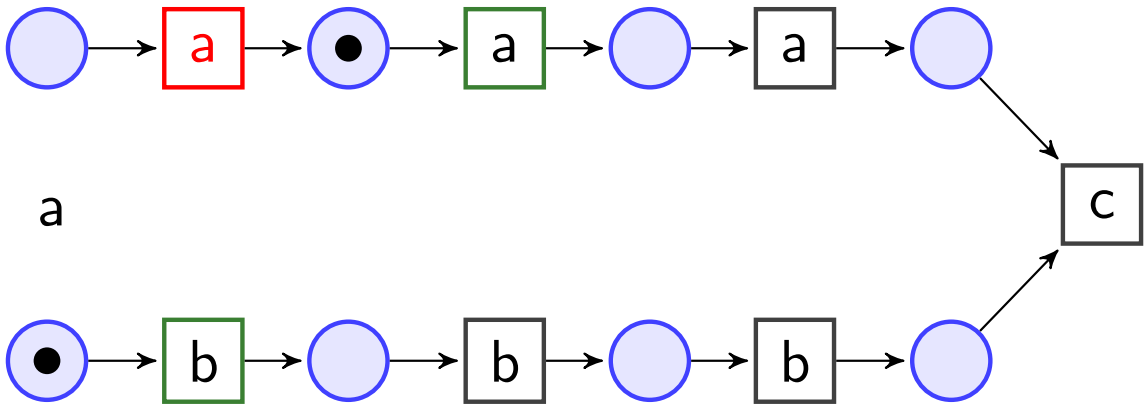

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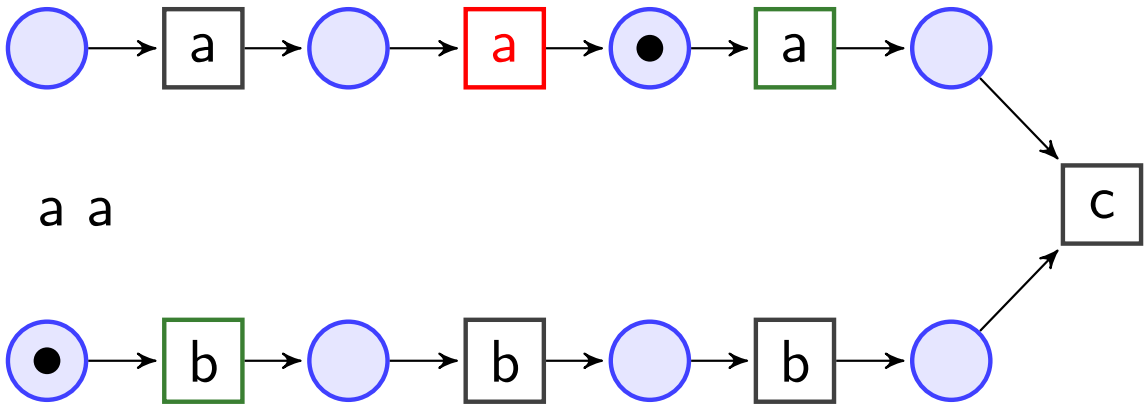

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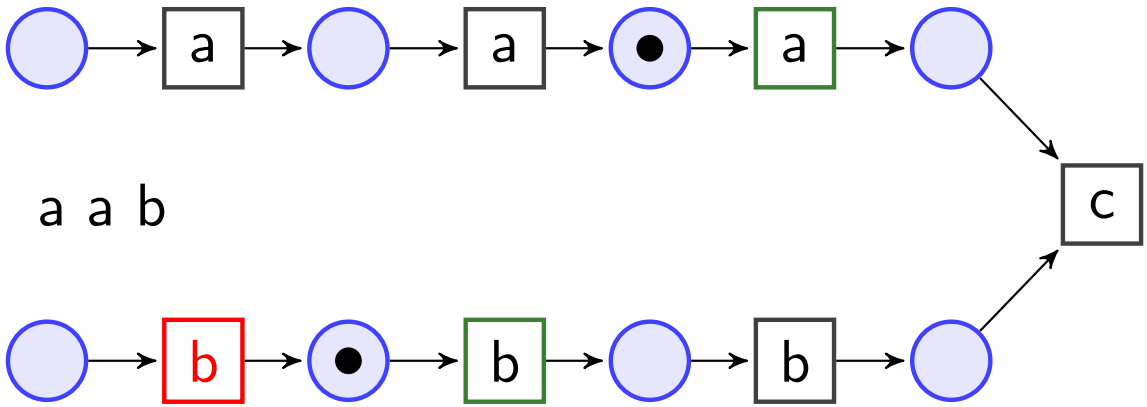

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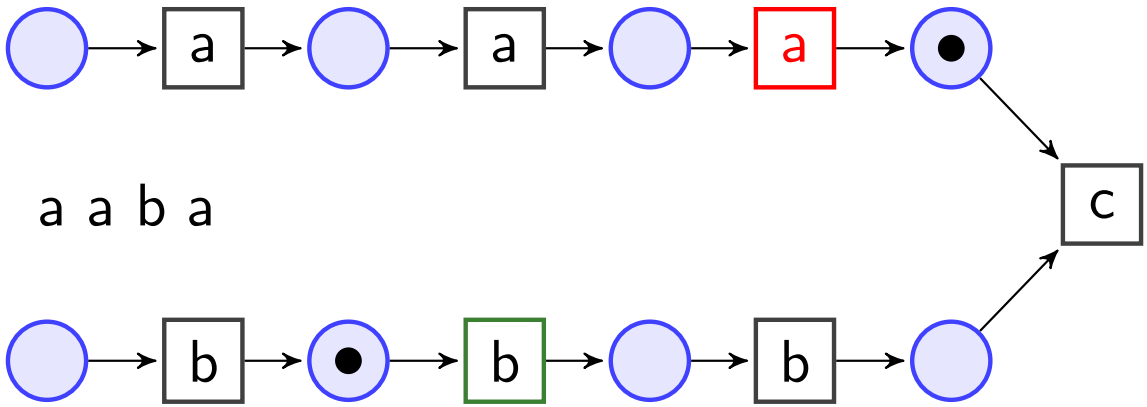

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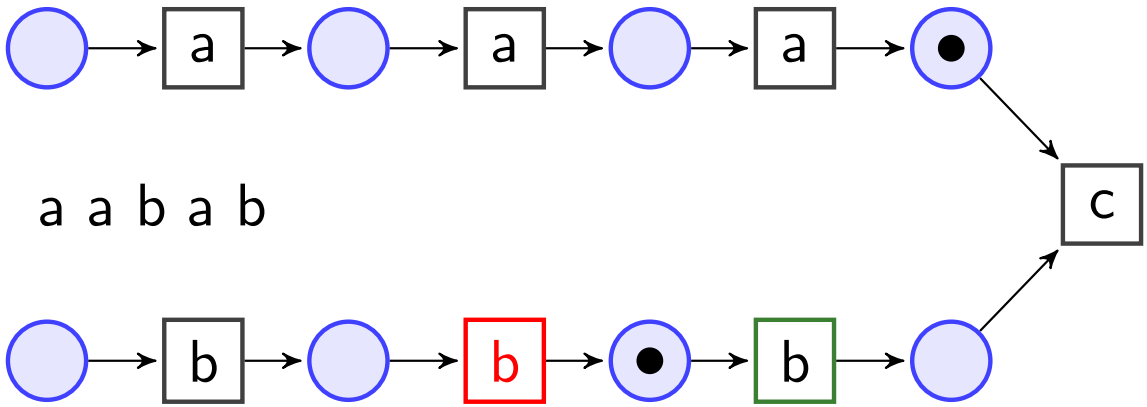

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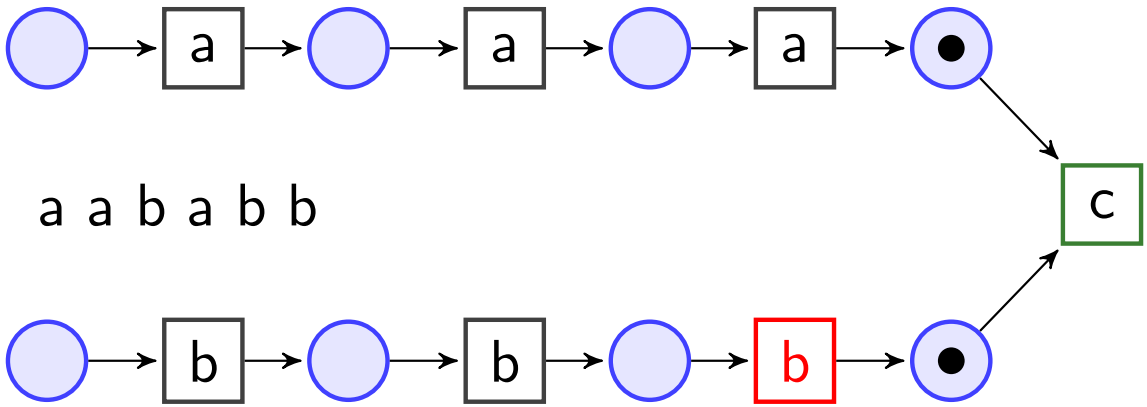

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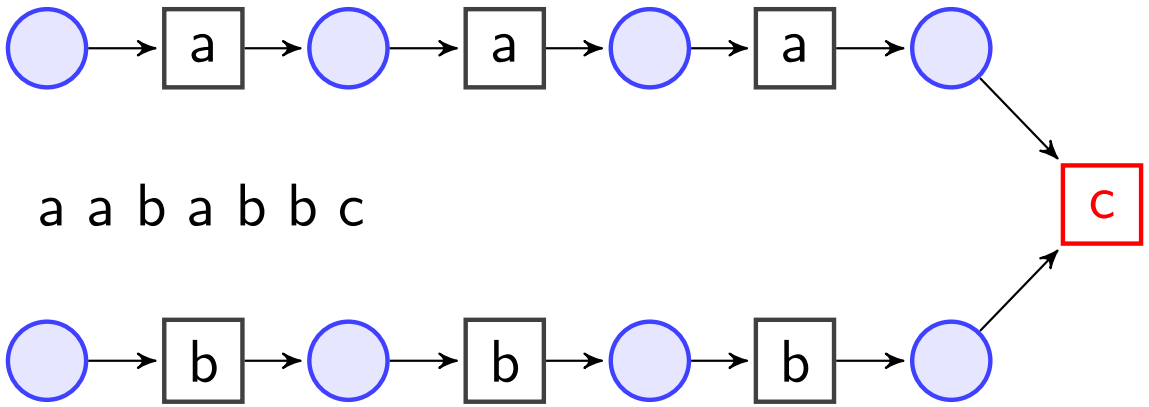

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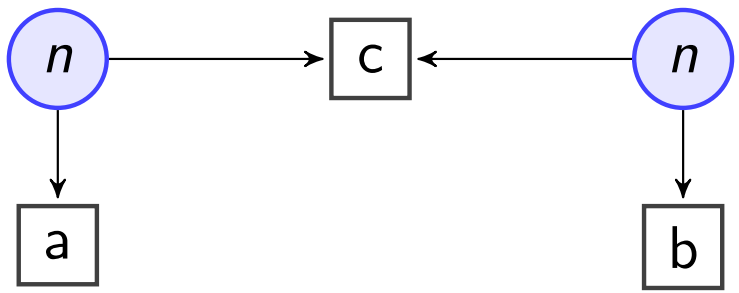

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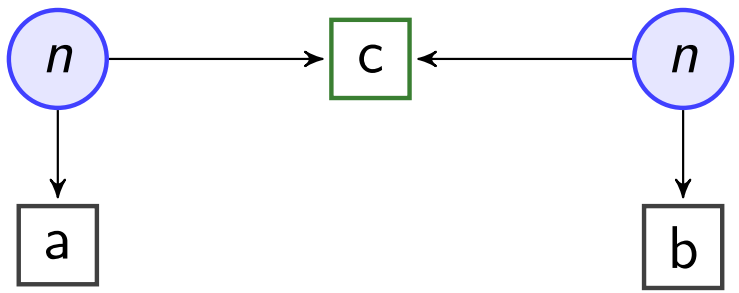

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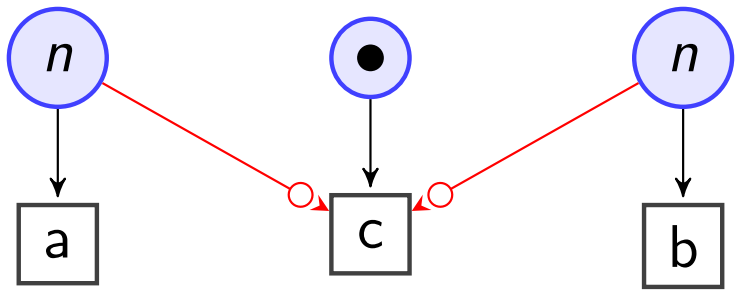

An inhibitor arc imposes the precondition that the transition may only fire when the place is empty; this allows arbitrary computations on numbers of tokens to be expressed, which makes the formalism Turing complete.[1]

[1] http://en.wikipedia.org/wiki/Petri_net

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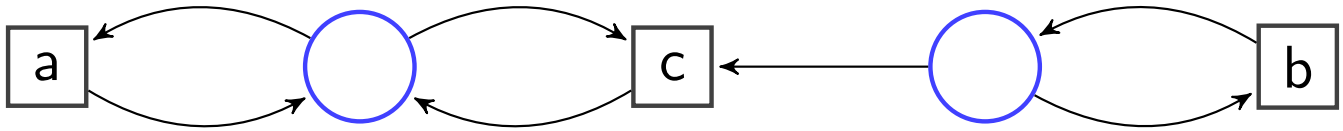
A Finite Asynchronous Automaton accepting L_n



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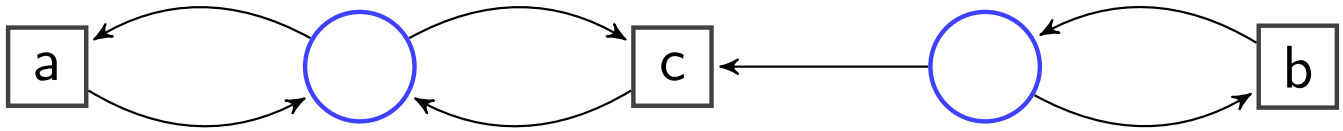

A *Signature*:

$$\sigma = (\Sigma, R, \mathcal{E})$$

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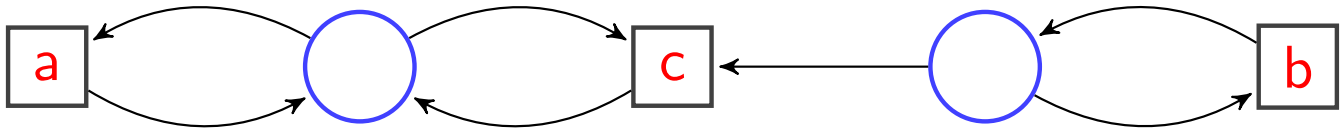

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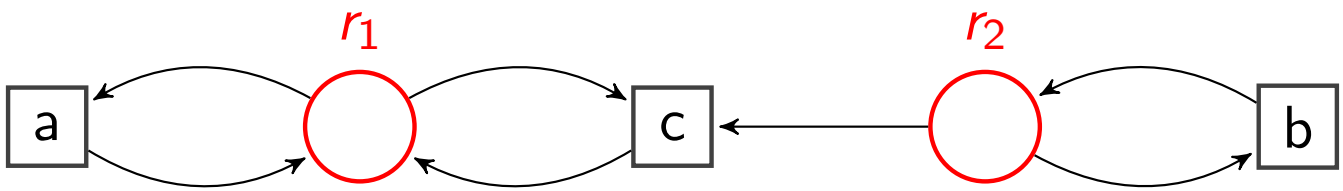

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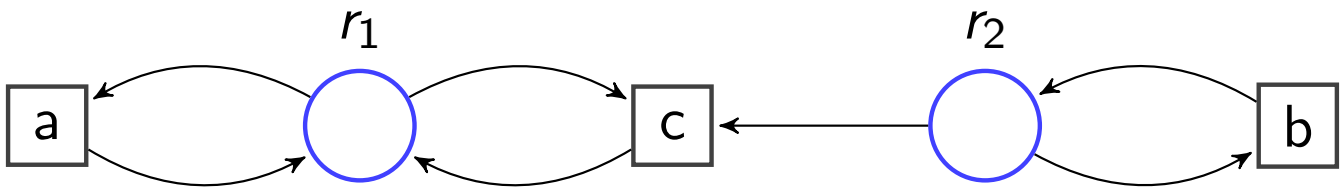

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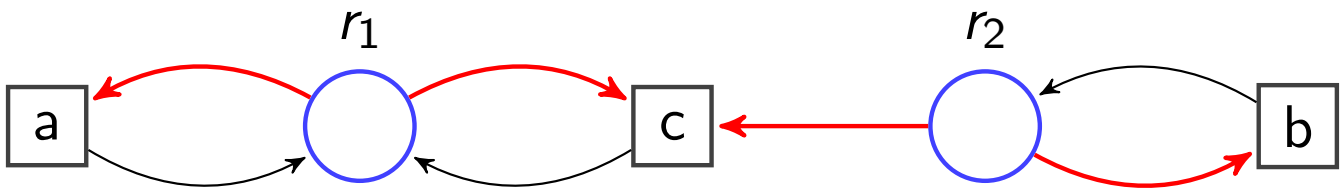

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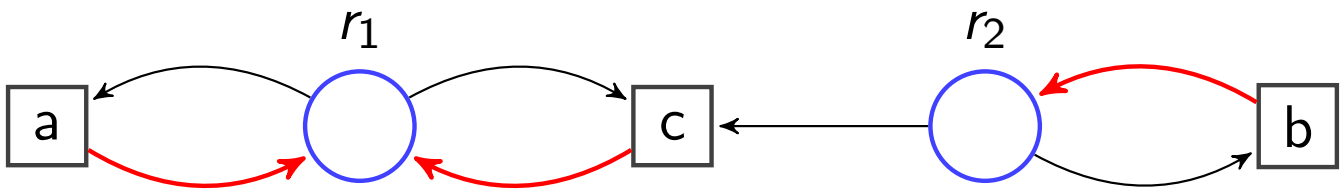

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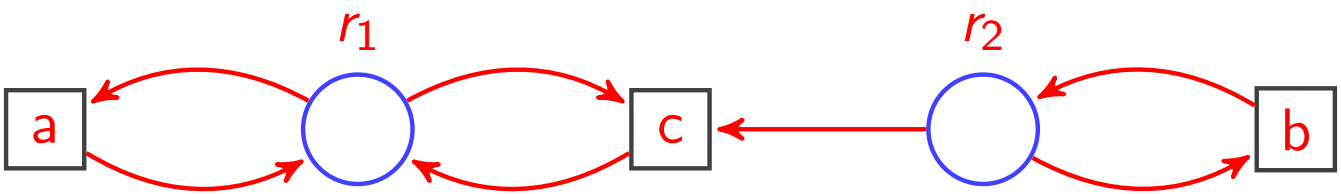

Finite Asynchronous Transition System (fat-system):

$$\tau = (\underbrace{\Sigma, R, \mathcal{E}}_{\sigma}, X, \Delta)$$

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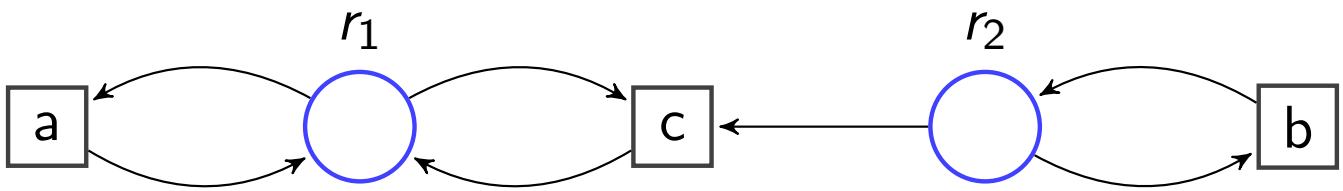

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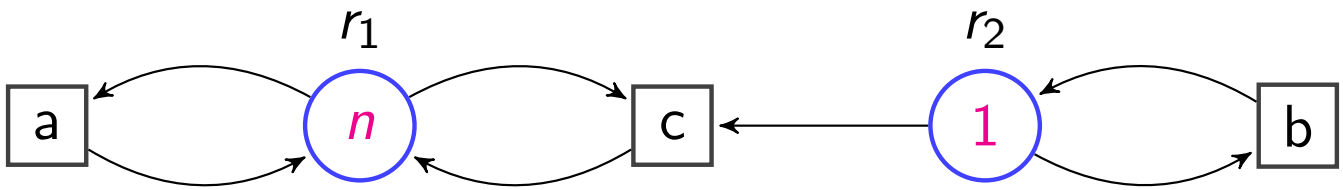

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Global states:

$$S = X^R$$

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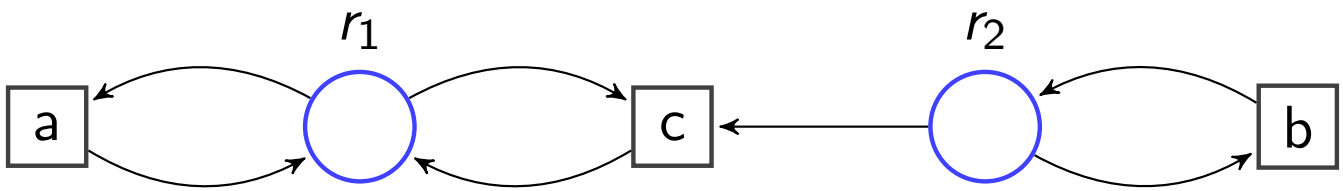

Finite Asynchronous Transition System (fat-system):

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- $\Delta = \{\delta_a \mid a \in \Sigma\}$ is a family of transition Relations

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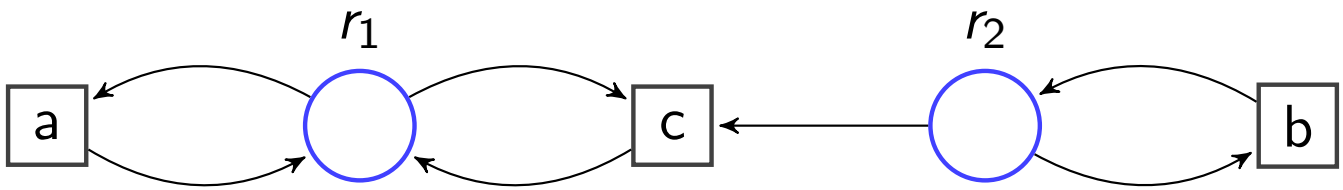

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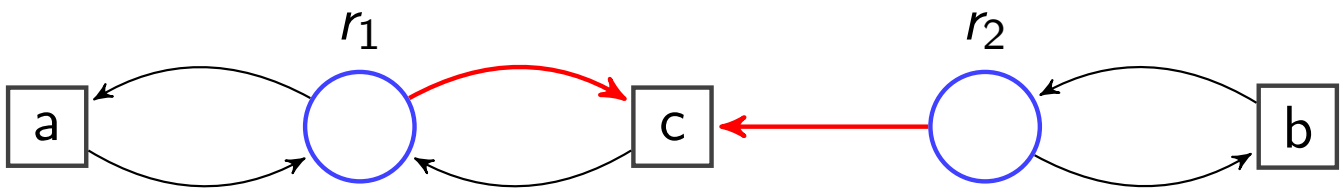

δ_c

r_1	r_2	r_1
0	0	s

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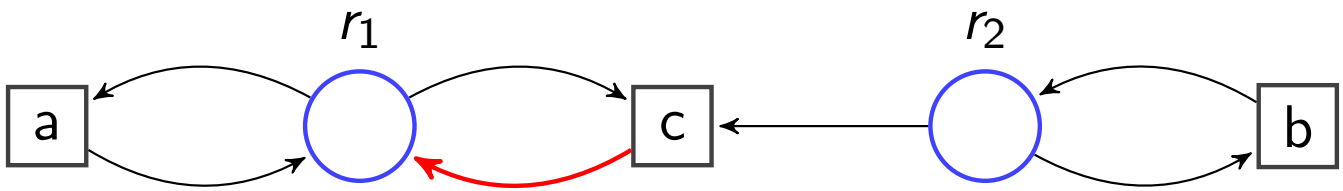

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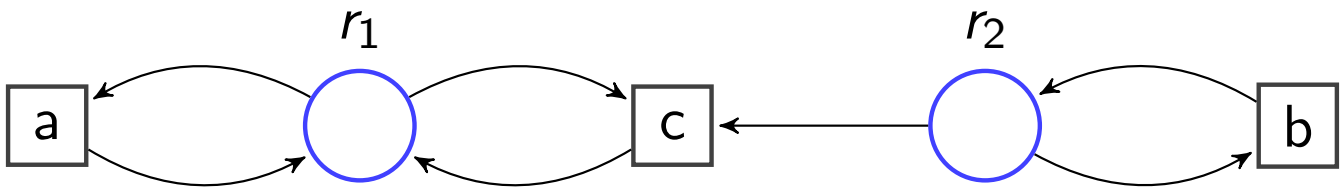

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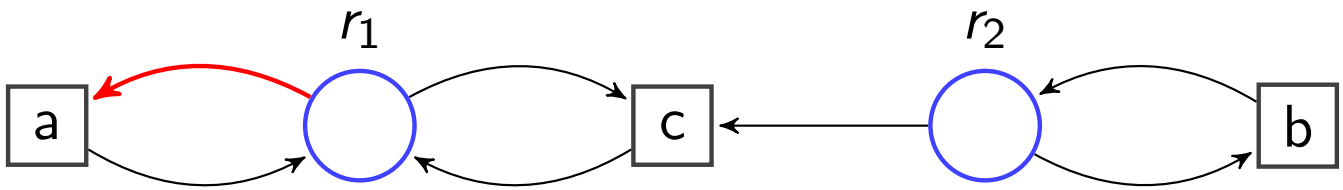

δ_a

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i	$i-1$

δ_c

r_1	r_2	r_1
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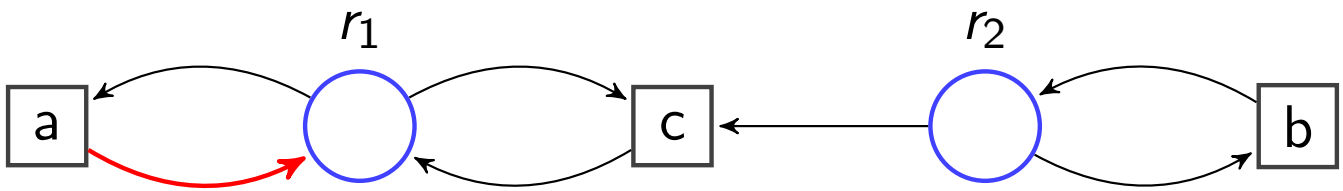

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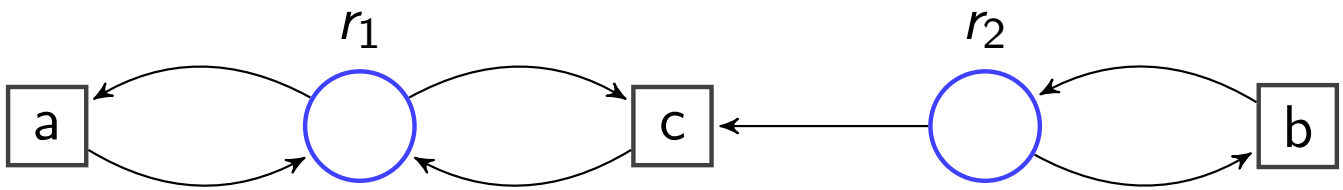

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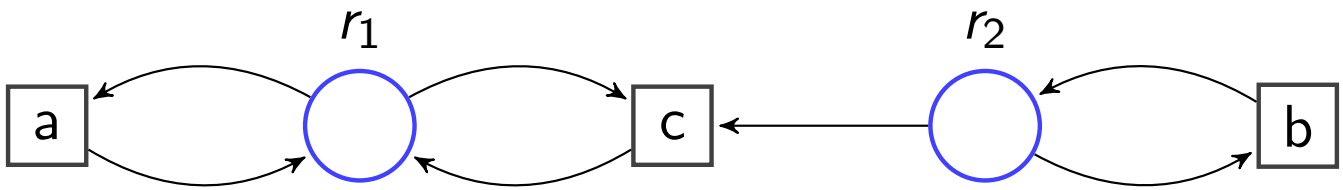

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δ_c

r_1	r_2	r_1
0	0	s

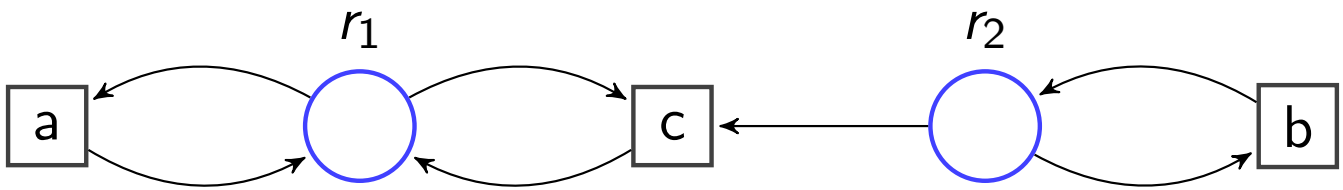
δ_b

r_2	r_2
i	$i-1$

Local transitions:

$$\delta_a \subseteq X^{\mathcal{E}a} \times X^{a\mathcal{E}}$$

A Finite Asynchronous Automaton accepting L_n



δ_a

r_1	r_1
i	$i-1$

δ_c

r_1	r_2	r_1
0	0	s

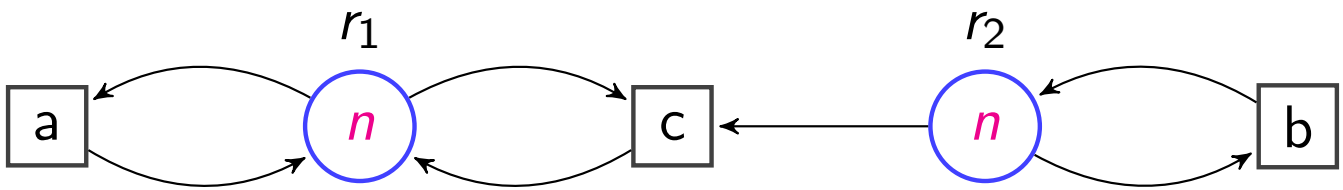
δ_b

r_2	r_2
i	$i-1$

Finite Asynchronous Automaton (FAA):

$$\mathcal{A} = (\underbrace{\Sigma, R, \mathcal{E}, X, \Delta}_{\text{fat-system}}, I, F)$$

A Finite Asynchronous Automaton accepting L_n



δ_a

r_1	r_1
i	$i-1$

δ_c

r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

Finite Asynchronous Automaton (FAA):

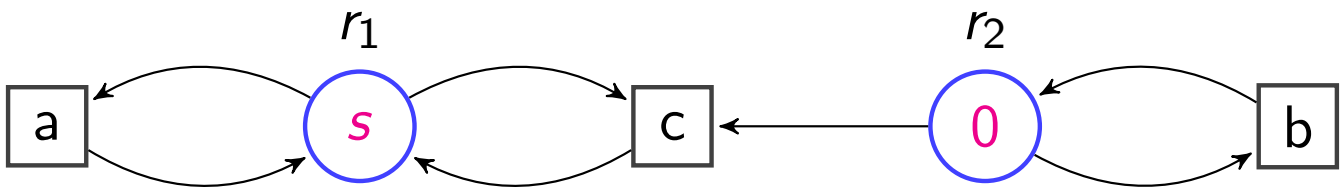
$$\mathcal{A} = (\Sigma, R, \mathcal{E}, X, \Delta, I, F)$$

$$I = \{f_I\}$$

$$f_I(r_1) = n$$

$$f_I(r_2) = n$$

A Finite Asynchronous Automaton accepting L_n



δ_a

r_1	r_1
i	$i-1$

δ_c

r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

Finite Asynchronous Automaton (FAA):

$$\mathcal{A} = (\Sigma, R, \mathcal{E}, X, \Delta, I, F)$$

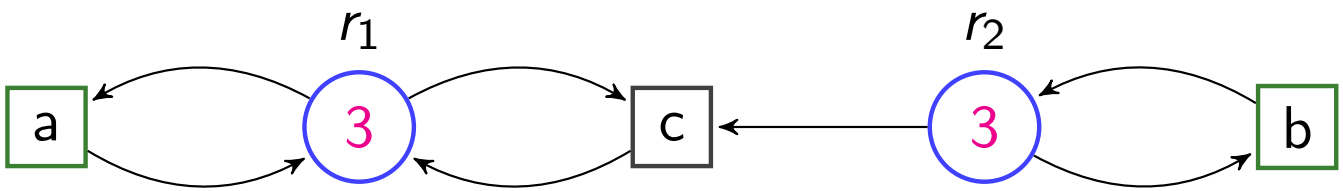
$$F = \{f_F\}$$

$$f_F(r_1) = s$$

$$f_F(r_2) = 0$$

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

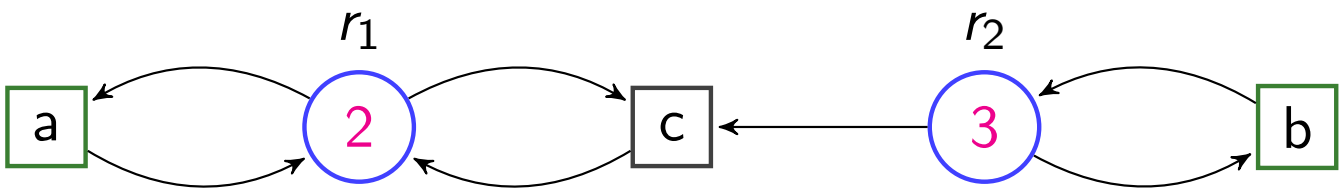
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

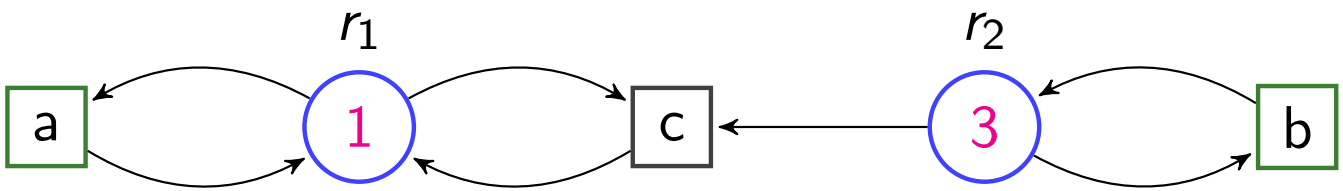
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

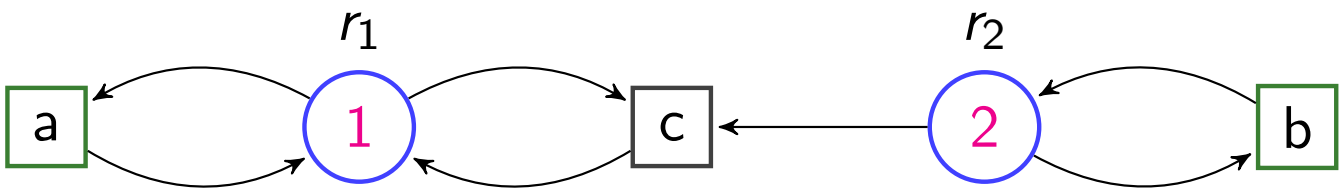
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

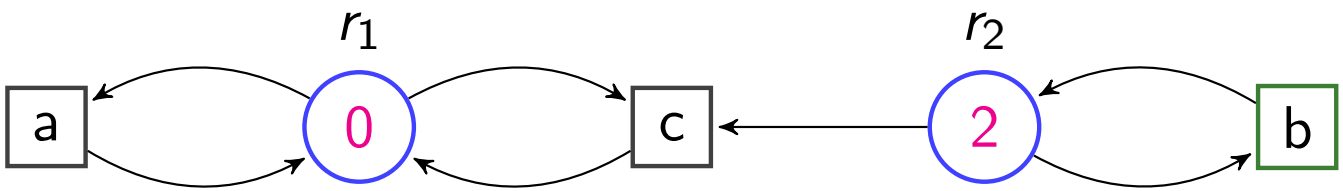
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

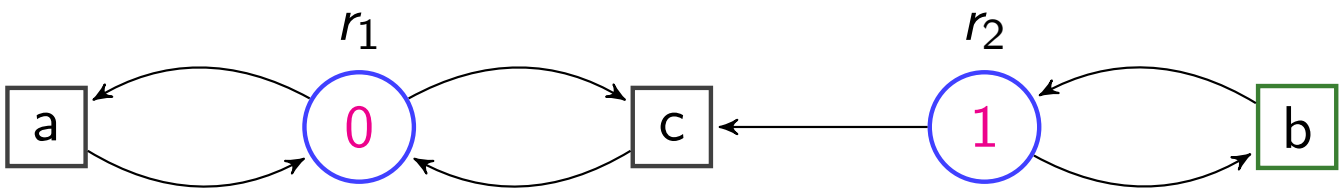
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

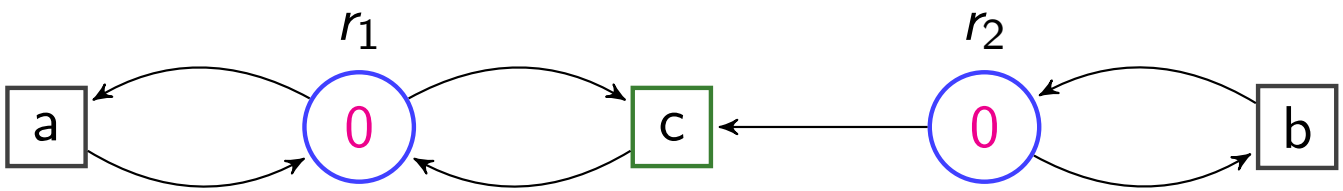
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

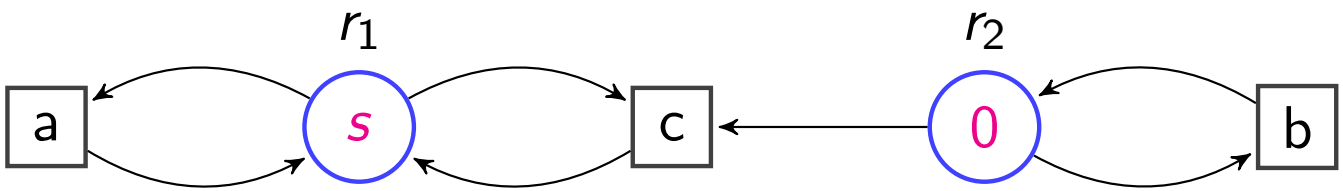
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

A Finite Asynchronous Automaton accepting L_3

Example Run for $aababbc \in L_3$



δ_a

r_1	r_1
3	2
2	1
1	0

δ_c

r_1	r_2	r_1
0	0	s

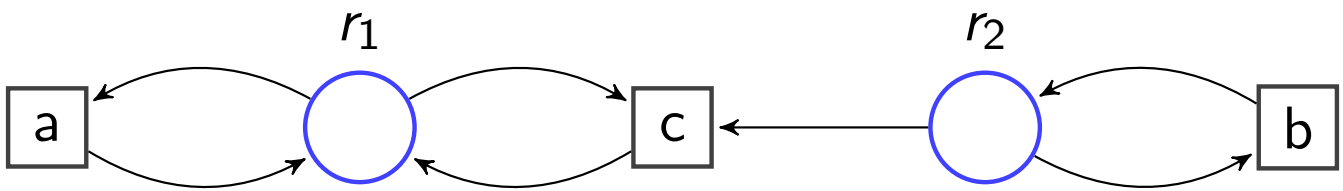
δ_b

r_2	r_2
3	2
2	1
1	0

a a b a b b c

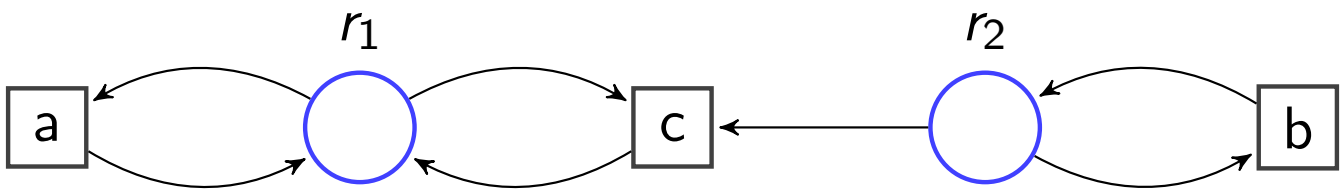
Independence

A closer look at the signature



Independence

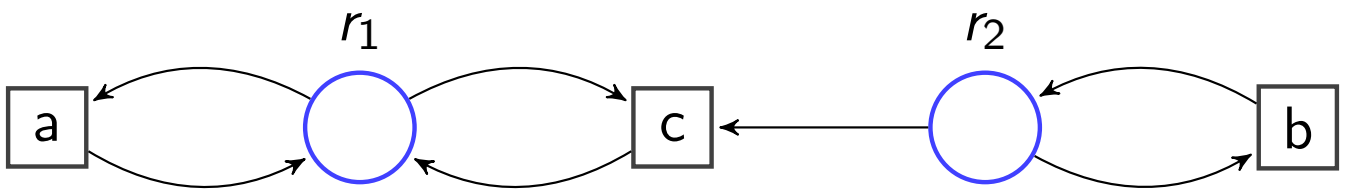
A closer look at the signature



When are two Actions $a, b \in \Sigma$ **independent** ?

Independence

A closer look at the signature

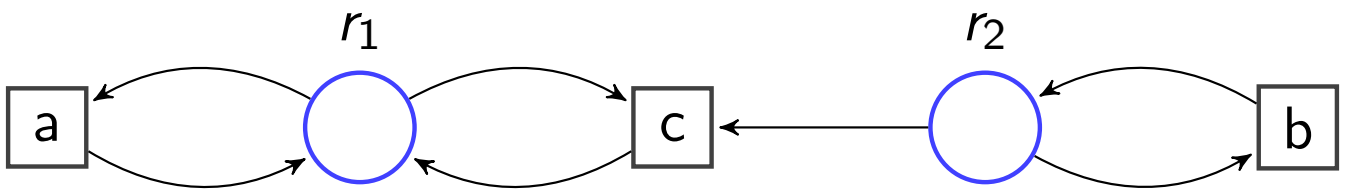


When are two Actions $a, b \in \Sigma$ **independent** ?

What does independent mean?

Independence

A closer look at the signature

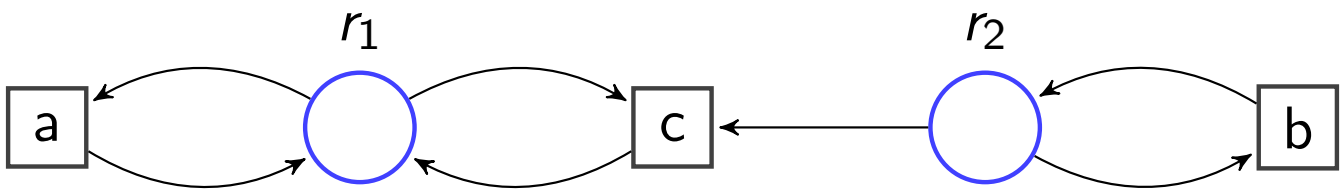


When are two Actions $a, b \in \Sigma$ **independent** ?

In a signature σ two Actions a, b are called independent,
if σ's structure doesn't allow a fat-system to distinguish ab and ba.

Independence

A closer look at the signature

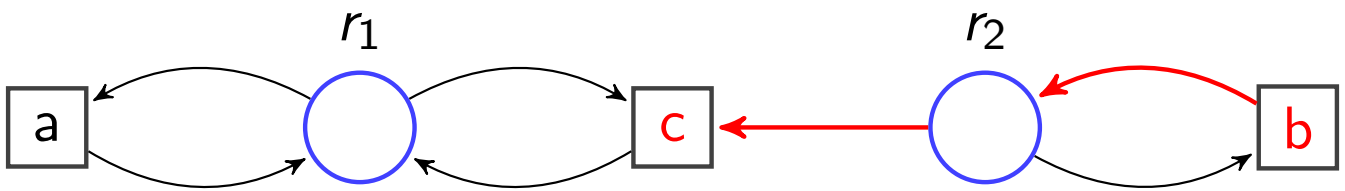


When are two Actions $a, b \in \Sigma$ **independent** ?

In a signature σ two Actions a, b are called independent,
if for all fat-systems with σ a and b can be executed in any order
without altering the result of the computation.

Independence

A closer look at the signature

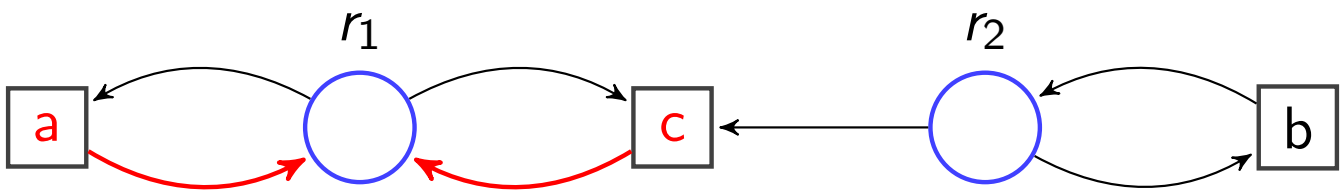


$C_\sigma = \mathcal{E}^2 \cap \Sigma^2$

$a \to \bigcirc \to b$

Independence

A closer look at the signature



$$C_\sigma = \mathcal{E}^2 \cap \Sigma^2$$

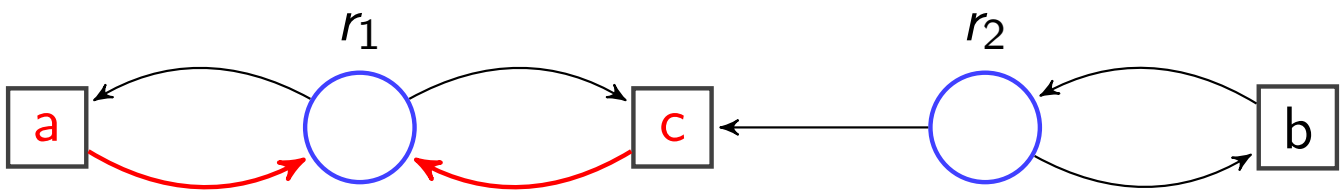
$$W_\sigma = \mathcal{E} \times \mathcal{E}^{-1} \cap \Sigma^2$$

$$a \to \bigcirc \to b$$

$$a \to \bigcirc \leftarrow b$$

Independence

A closer look at the signature



$$C_\sigma = \mathcal{E}^2 \cap \Sigma^2$$

$$W_\sigma = \mathcal{E} \times \mathcal{E}^{-1} \cap \Sigma^2$$

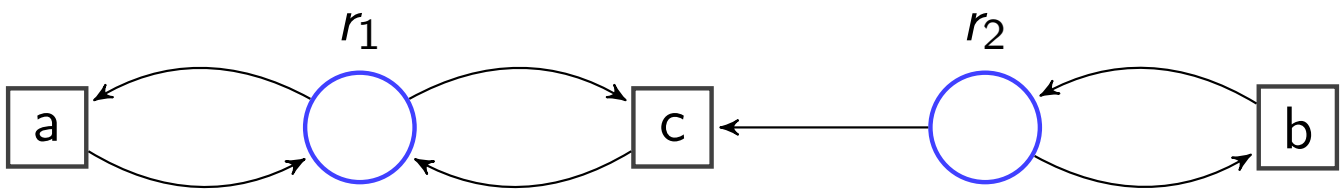
$$a \to \bigcirc \to b$$

$$a \to \bigcirc \leftarrow b$$

Whiteboard Example

Independence

A closer look at the signature



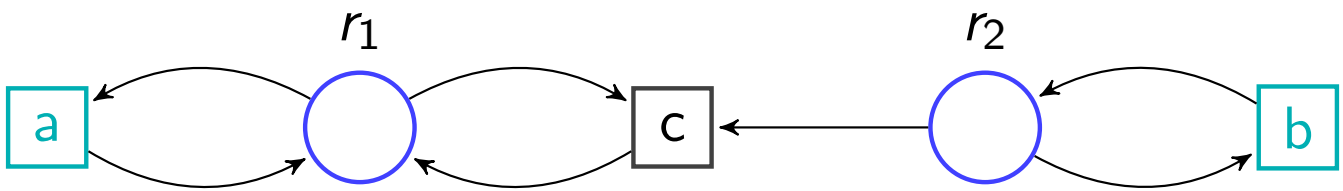
$$C_\sigma = \mathcal{E}^2 \cap \Sigma^2 \qquad a \to \bigcirc \to b$$

$$W_\sigma = \mathcal{E} \times \mathcal{E}^{-1} \cap \Sigma^2 \qquad a \to \bigcirc \leftarrow b$$

$$D_\sigma = C_\sigma \cup C_\sigma^{-1} \cup W_\sigma$$

Independence

A closer look at the signature



$$C_\sigma = \mathcal{E}^2 \cap \Sigma^2$$

$$W_\sigma = \mathcal{E} \times \mathcal{E}^{-1} \cap \Sigma^2$$

$$D_\sigma = C_\sigma \cup C_\sigma^{-1} \cup W_\sigma$$

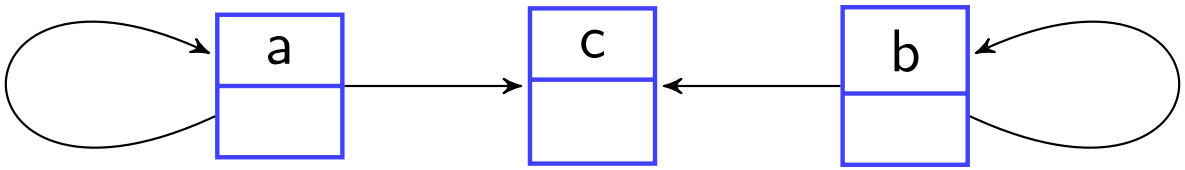
$$I_\sigma = \Sigma^2 \setminus D_\sigma$$

$a \to \bigcirc \to b$

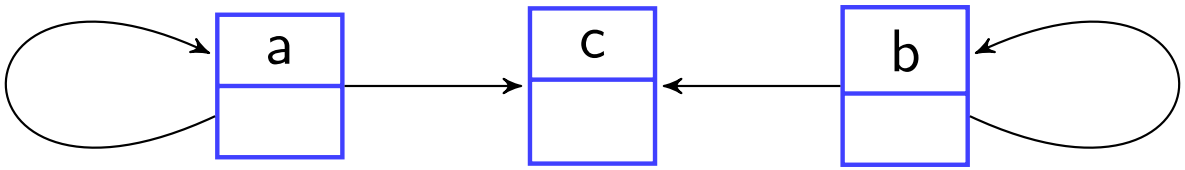
$a \to \bigcirc \leftarrow b$

Content

A particularly simple signature



A particularly simple signature

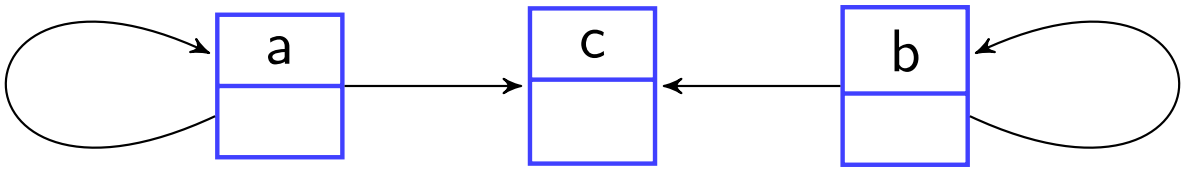


signature of a fat-system:

$$\sigma = (\Sigma, R, \mathcal{E})$$

Where $\mathcal{E} \subseteq R \times \Sigma \cup \Sigma \times R$

A particularly simple signature

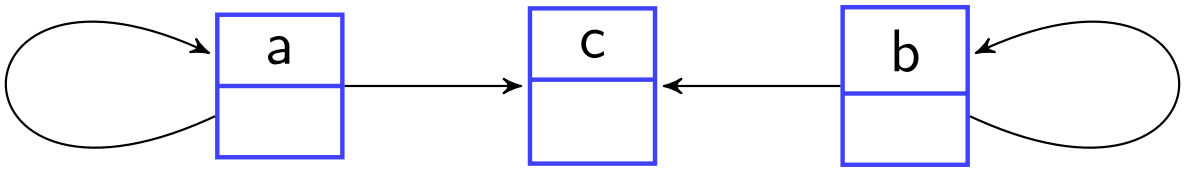


New Type of signature:

$$\sigma = (\Sigma, C)$$

Where $C \subseteq \Sigma^2$

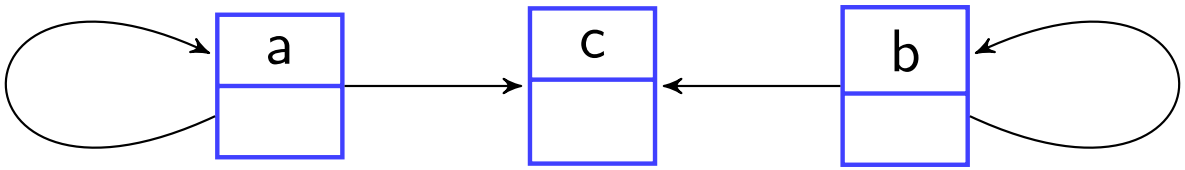
A particularly simple signature



Finite Asynchronous Cellular Transition System (fact-system):

$$\tau = (\Sigma, C, X, \Delta)$$

A particularly simple signature



Finite Asynchronous Cellular Transition System (fact-system):

$$\tau = (\Sigma, C, X, \Delta)$$

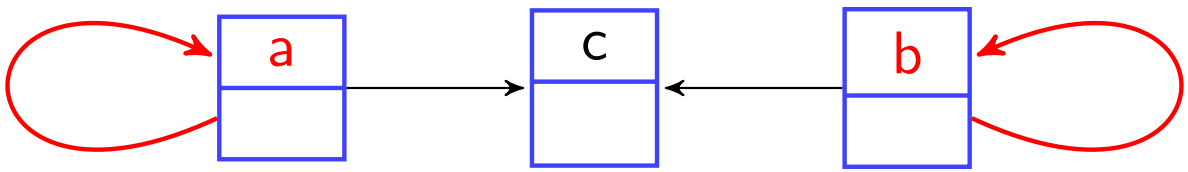
Where

$$\Delta = \{\delta_a \mid a \in \Sigma\}$$

And for $a \in \Sigma$

$$\delta_a \subseteq X^{Ca} \times X$$

A particularly simple signature



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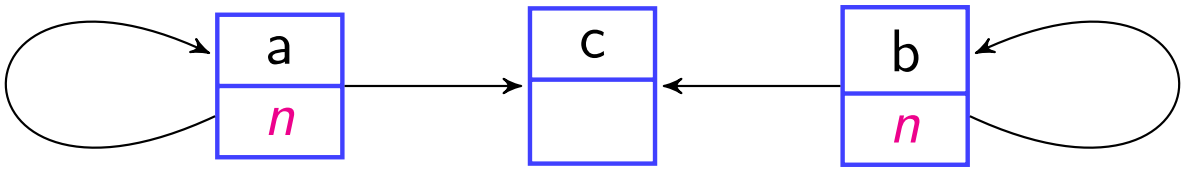

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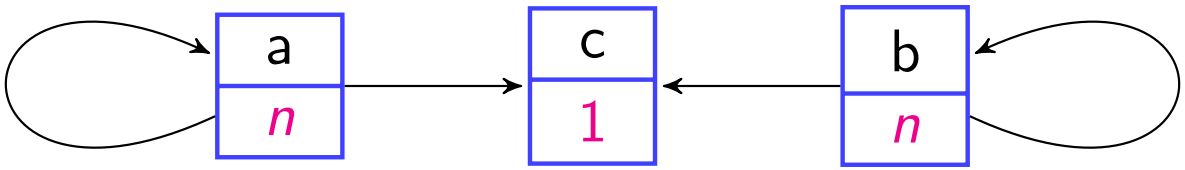

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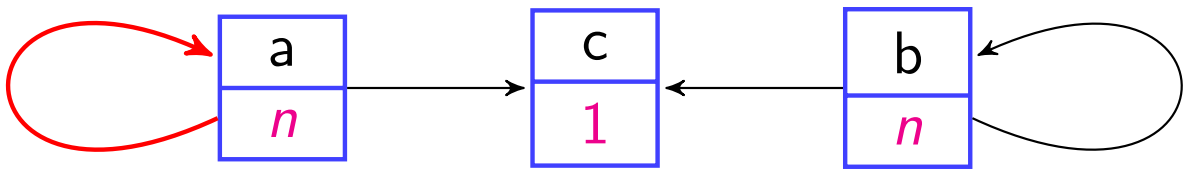


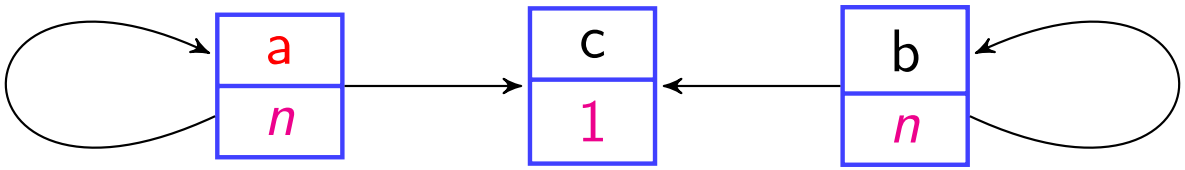
δ_a

a	a
i	$i-1$

δ_b

b	b
i	$i-1$

A particularly simple signature



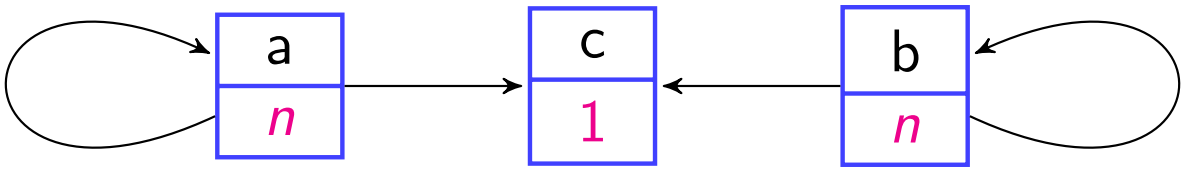
δ_a

a	a
i	$i-1$

δ_b

b	b
i	$i-1$

A particularly simple signature



δ_a

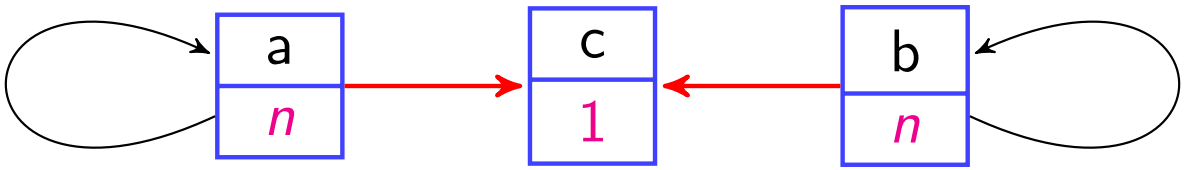
a	a
i	$i-1$

δ_b

b	b
i	$i-1$

$0 < i \leq n$

A particularly simple signature



δ_a

a	a
i	$i-1$

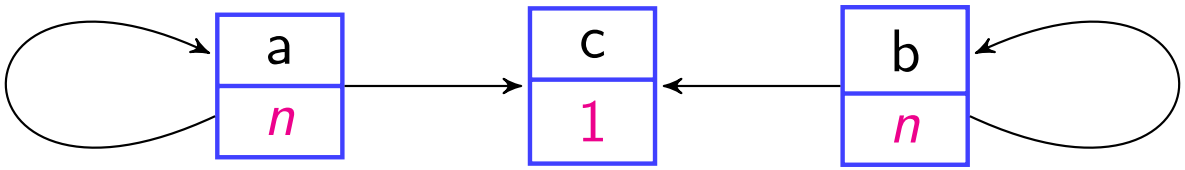
δ_c

a	b	c
0	0	0

δ_b

b	b
i	$i-1$

A particularly simple signature



δ_a

a	a
i	$i-1$

δ_c

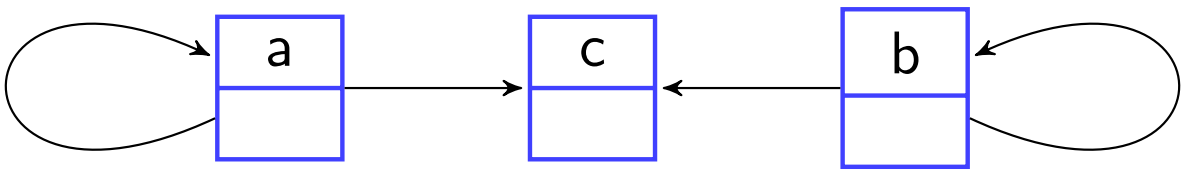
a	b	c
0	0	0

δ_b

b	b
i	$i-1$

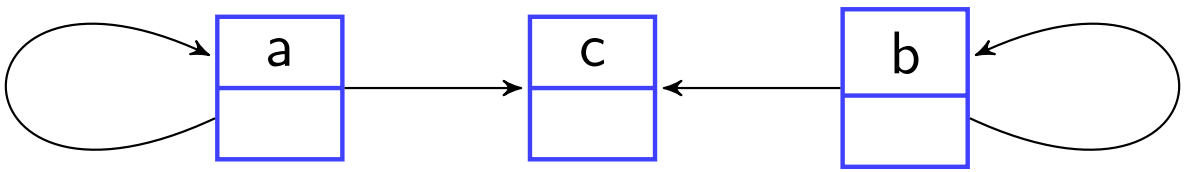
Independence

no writing conflicts



Independence

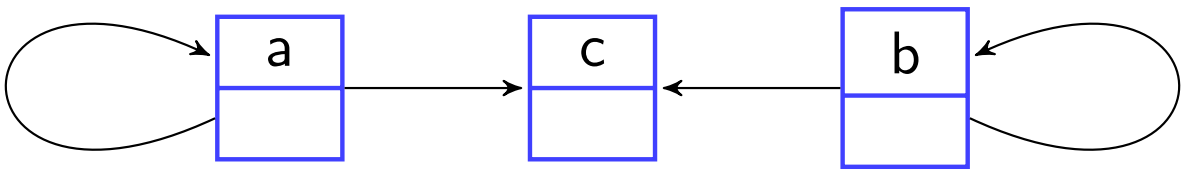
no writing conflicts



When are two Actions $a, b \in \Sigma$ **dependent** ?

Independence

no writing conflicts



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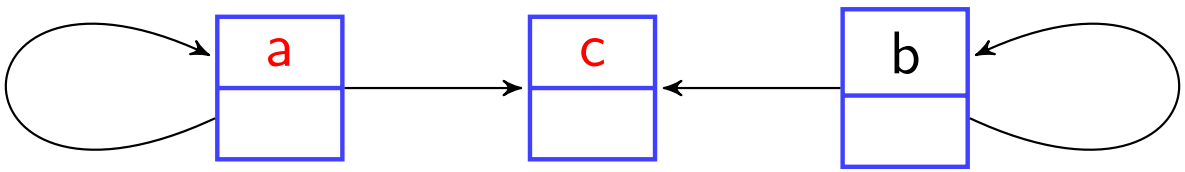
Independence

no writing conflicts



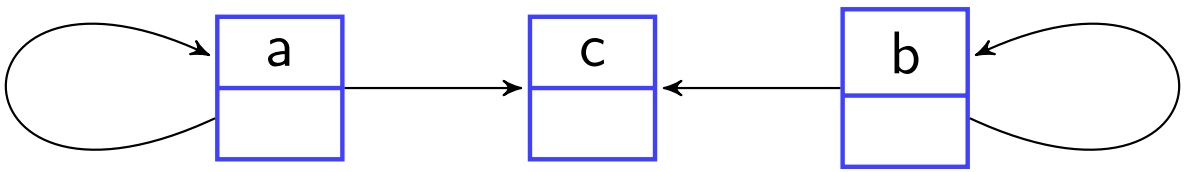
$$\sigma = (\Sigma, C)$$

$C_\sigma = C$

$(a \to \bigcirc \to b)$

Independence

no writing conflicts



$$\sigma = (\Sigma, C)$$

$C_\sigma = C$

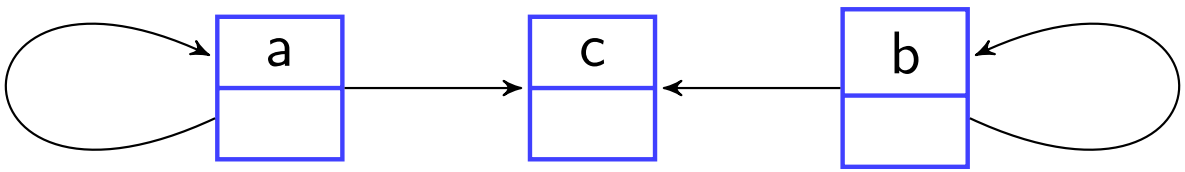
$W_\sigma = id_\Sigma$

$(a \to \bigcirc \to b)$

$(a \to \bigcirc \leftarrow b)$

Independence

no writing conflicts



$$\sigma = (\Sigma, C)$$

$C_\sigma = C$

$W_\sigma = id_\Sigma$

$D_\sigma = C_\sigma \cup C_\sigma^{-1} \cup W_\sigma$

$(a \to \bigcirc \to b)$

$(a \to \bigcirc \leftarrow b)$

Independence

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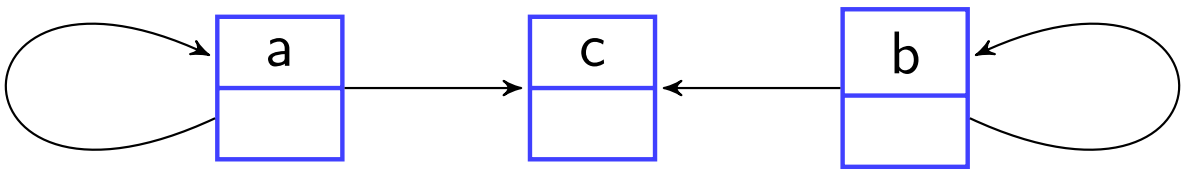

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Independence

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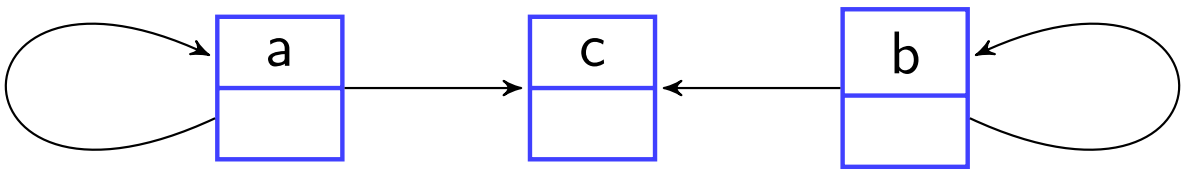

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$$I_\sigma = \Sigma^2 \setminus D_\sigma \qquad \{(a, b) \mid a, b \text{ not adjacent}\}$$

Simulations

Are fact-systems regular?

Simulations

NDT $\rightarrow$ fact

Proof.

Let $\tau = (\Sigma, Q, \delta)$ be a regular finite transition system.
WLOG let $Q = \{q_0, \ldots, q_{k-1}\}$.

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- $\Delta = \bigcup_{a \in \Sigma} \{\delta_a\}$

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let $a \in \Sigma, \beta \in X^\Sigma$ and $x \in X$.

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$(\beta, x) \in \delta_a \Leftrightarrow \exists s, s' :$

$(q_s, a, q_{s'}) \in \delta$

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$$\wedge \sum_{b \in \Sigma} \beta(b) \mod k = s$$

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$$\wedge s - \beta(a) + x \mod k = s'$$

τ_C simulates τ. $\square$

Simulations

fat $\rightarrow$ fact

Can a finite asynchronous cellular automaton (faca) simulate a faa?

Simulations

fat $\rightarrow$ fact

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– both accept regular languages.

Simulations

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What would be an interesting simulation?

Simulations

fat $\rightarrow$ fact

Can a finite asynchronous cellular automaton (faca) simulate a faa?

– both accept regular languages.

What would be an interesting simulation?

– One that maintains independence.

Simulations

fat $\rightarrow$ fact

How can we assure that all independence is maintained?

Simulations

fat $\rightarrow$ fact

How can we assure that all independence is maintained?

Try not to interfere with the signature:

Fat signature
$\sigma = (\Sigma, R, \mathcal{E})$

Simulations

fat $\rightarrow$ fact

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Try not to interfere with the signature:

Fat signature
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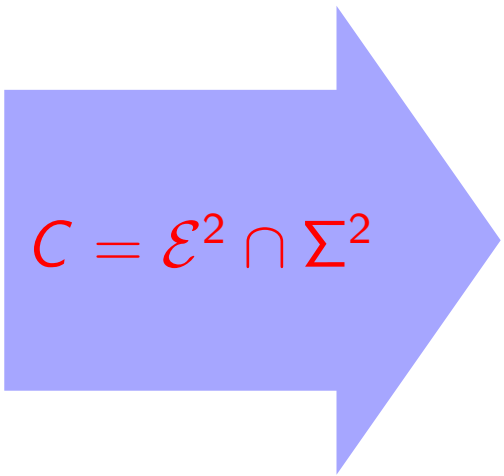

Simulations

fat $\rightarrow$ fact

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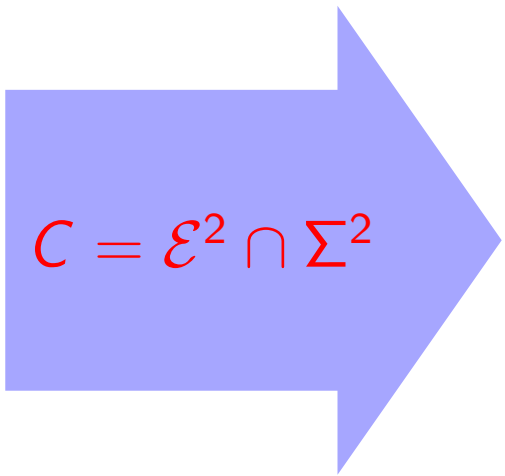


Fact signature
$\sigma' = (\Sigma, C)$

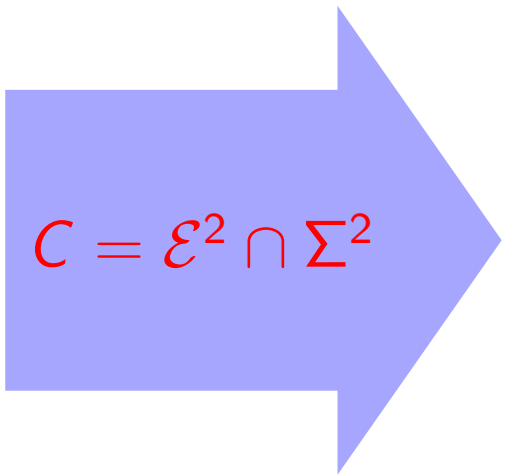
Simulations

fat $\rightarrow$ fact

How can we assure that all independence is maintained?

Try not to interfere with the signature:

Fat signature
$\sigma = (\Sigma, R, \mathcal{E})$



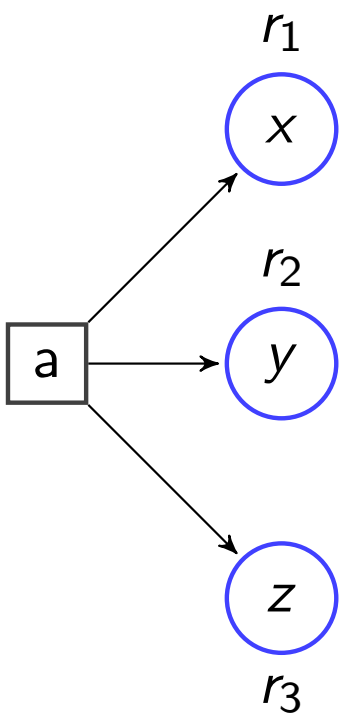
Fact signature
$\sigma' = (\Sigma, C)$

σ induces C

Simulations

fat $\rightarrow$ fact

If there are no writing-conflicts



Simulations

fat $\rightarrow$ fact

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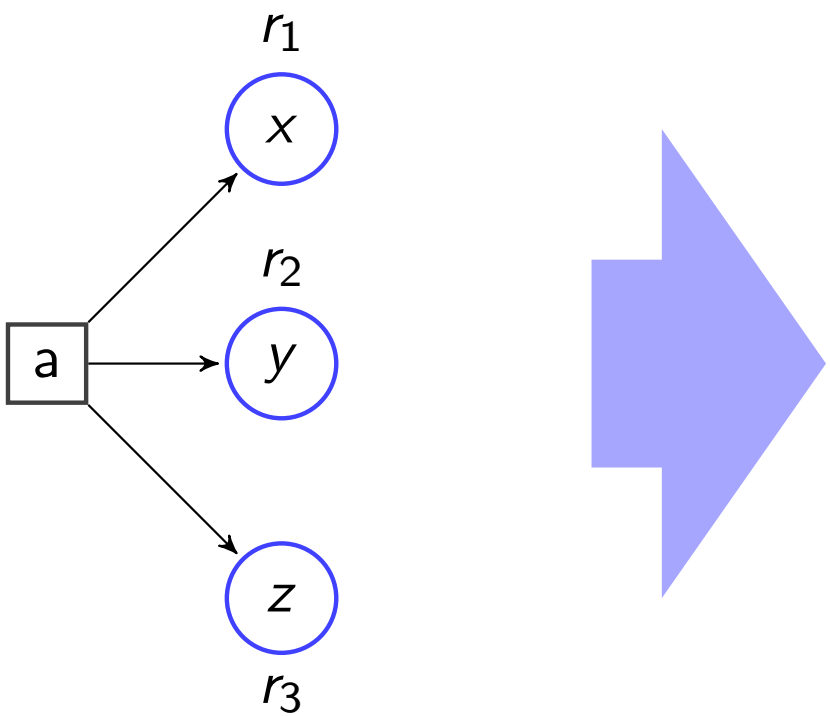

Simulations

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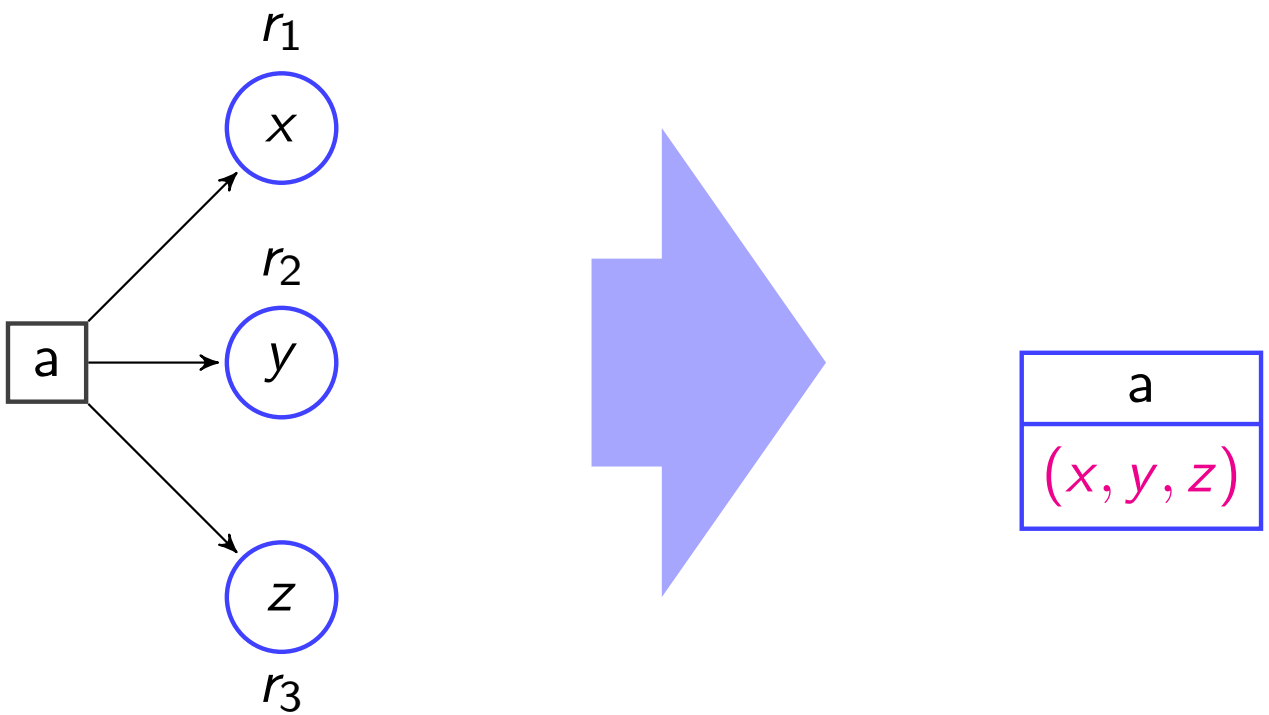

Simulations

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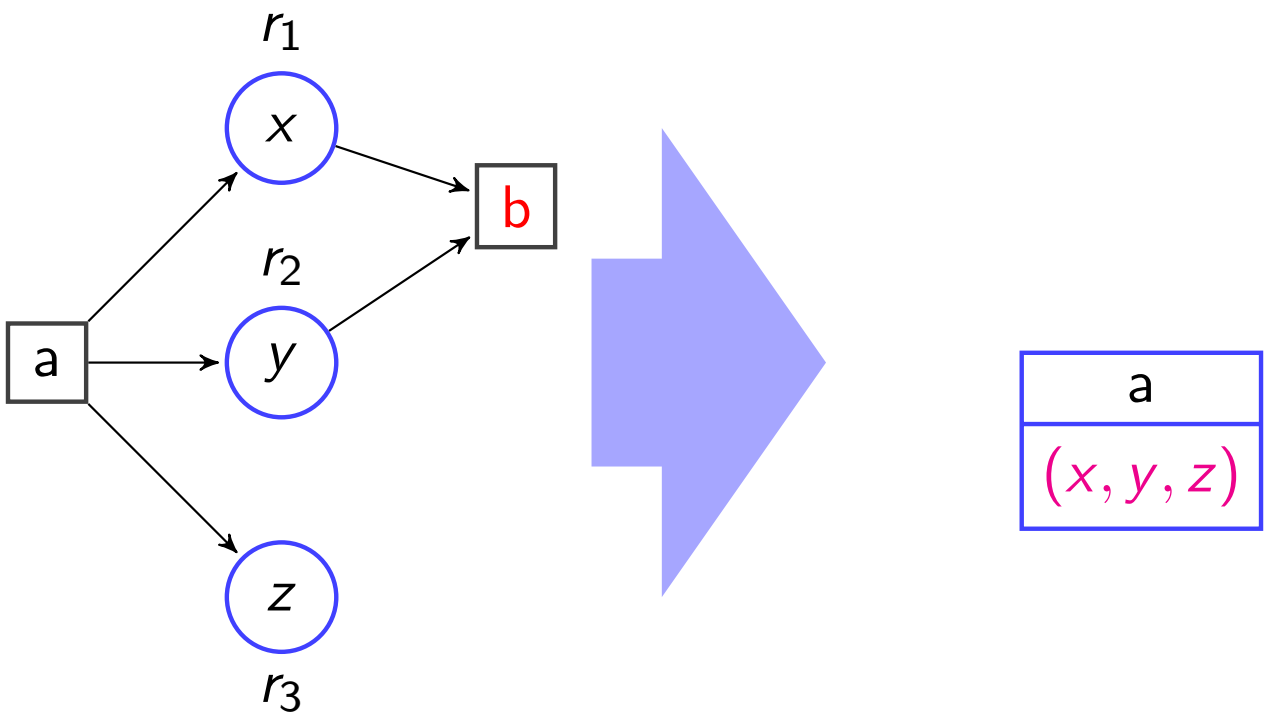

Simulations

fat $\rightarrow$ fact

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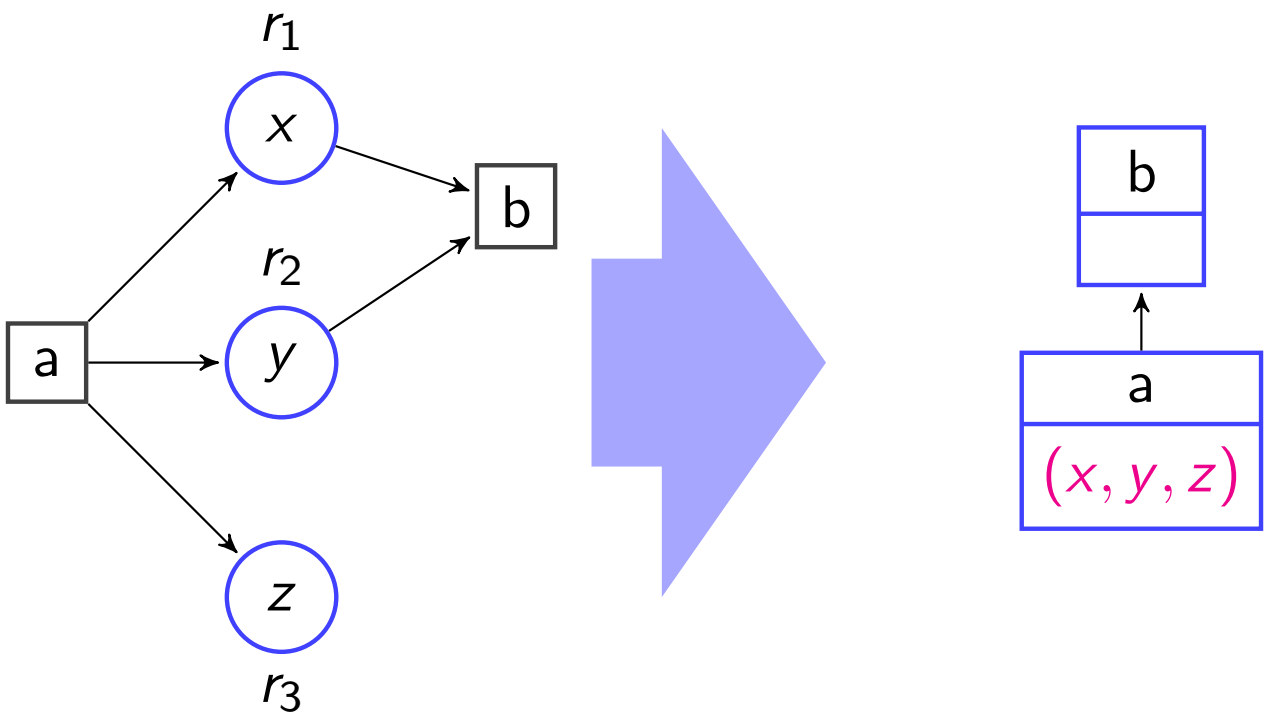


Simulations

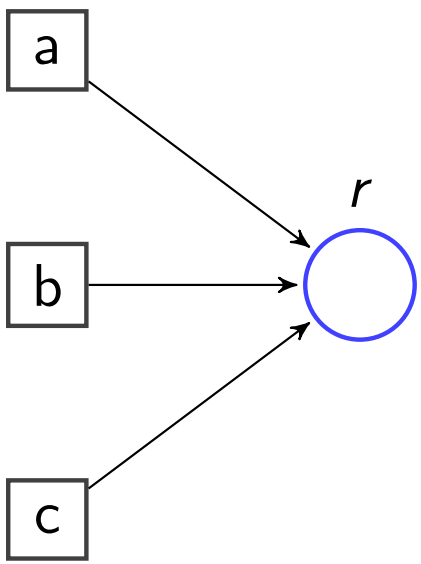
fat $\rightarrow$ fact

Writing-conflict

Simulations

fat $\rightarrow$ fact

Writing-conflict



Simulations

fat $\rightarrow$ fact

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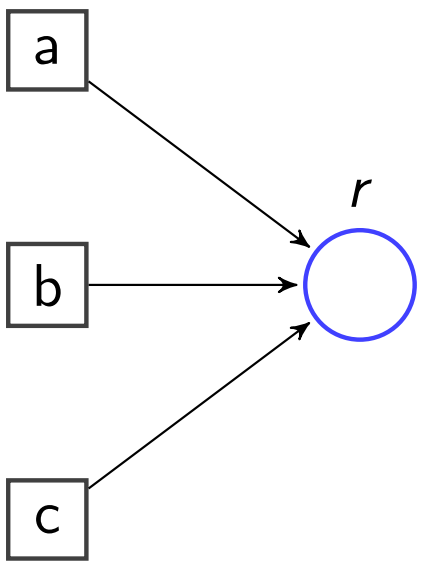
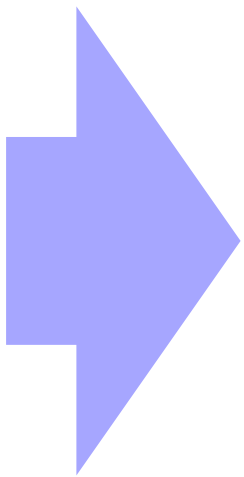
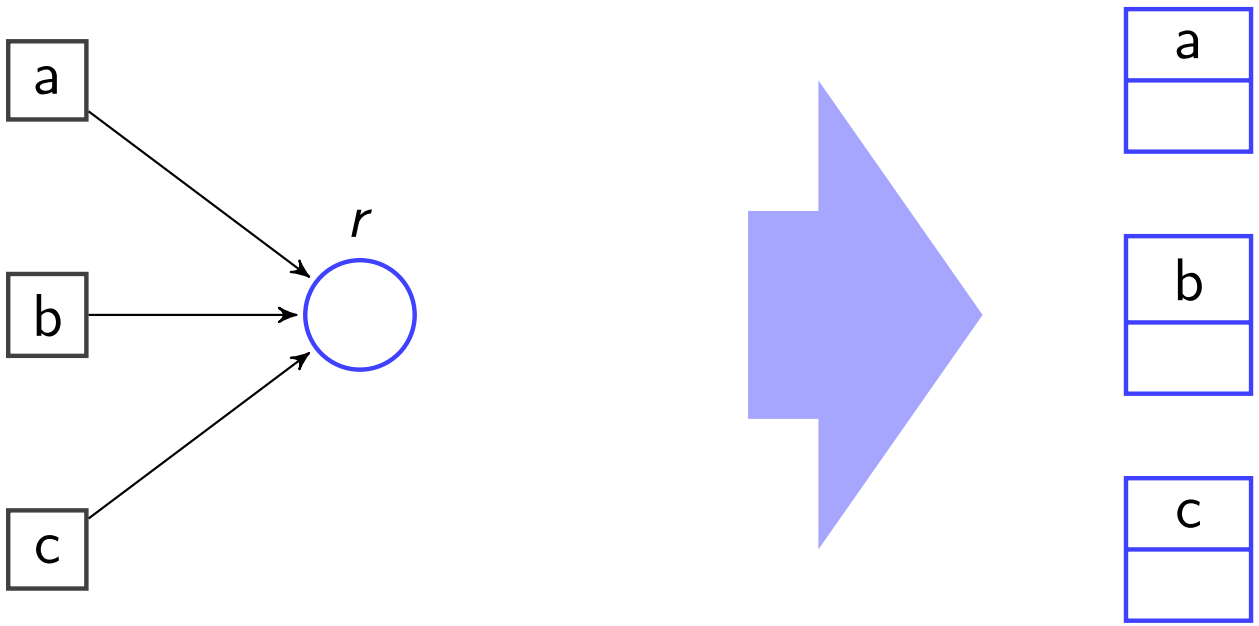




Simulations

fat $\rightarrow$ fact

Writing-conflict



Simulations

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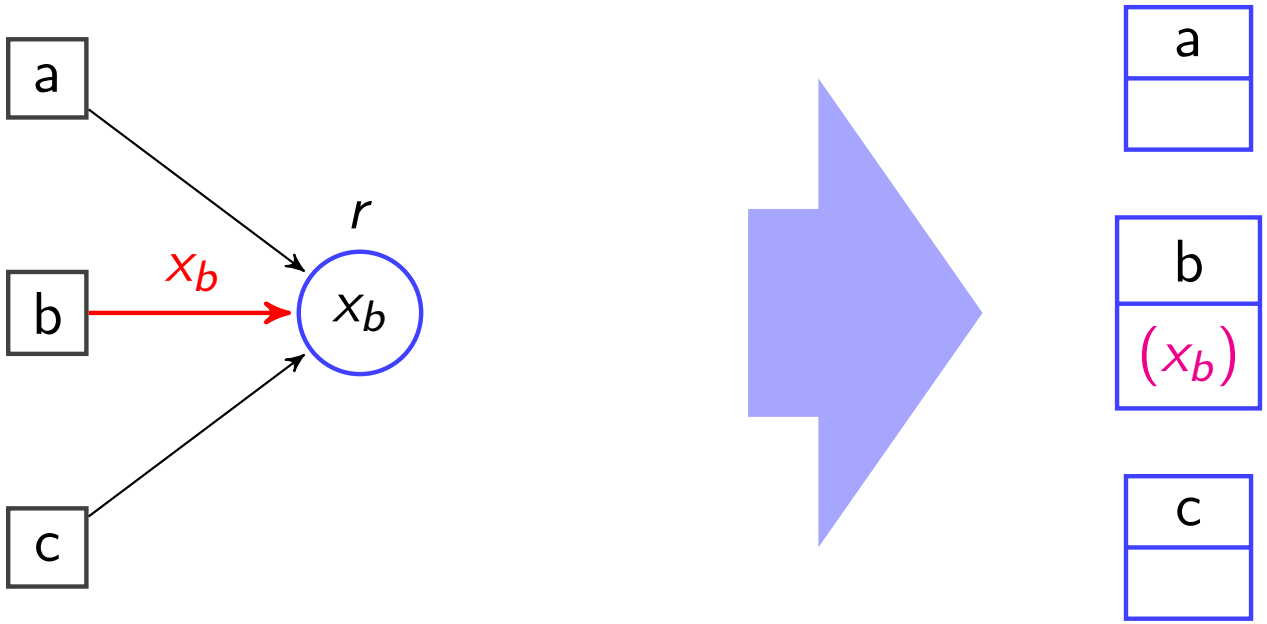

Simulations

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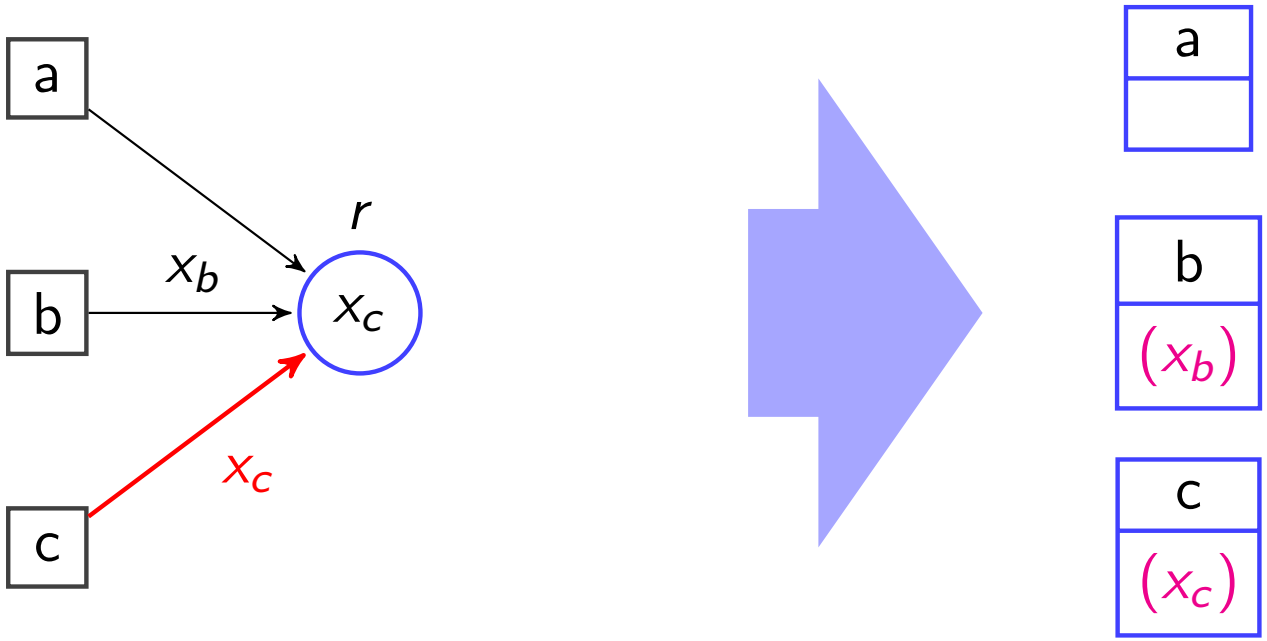
Writing-conflict



Simulations

fat $\rightarrow$ fact

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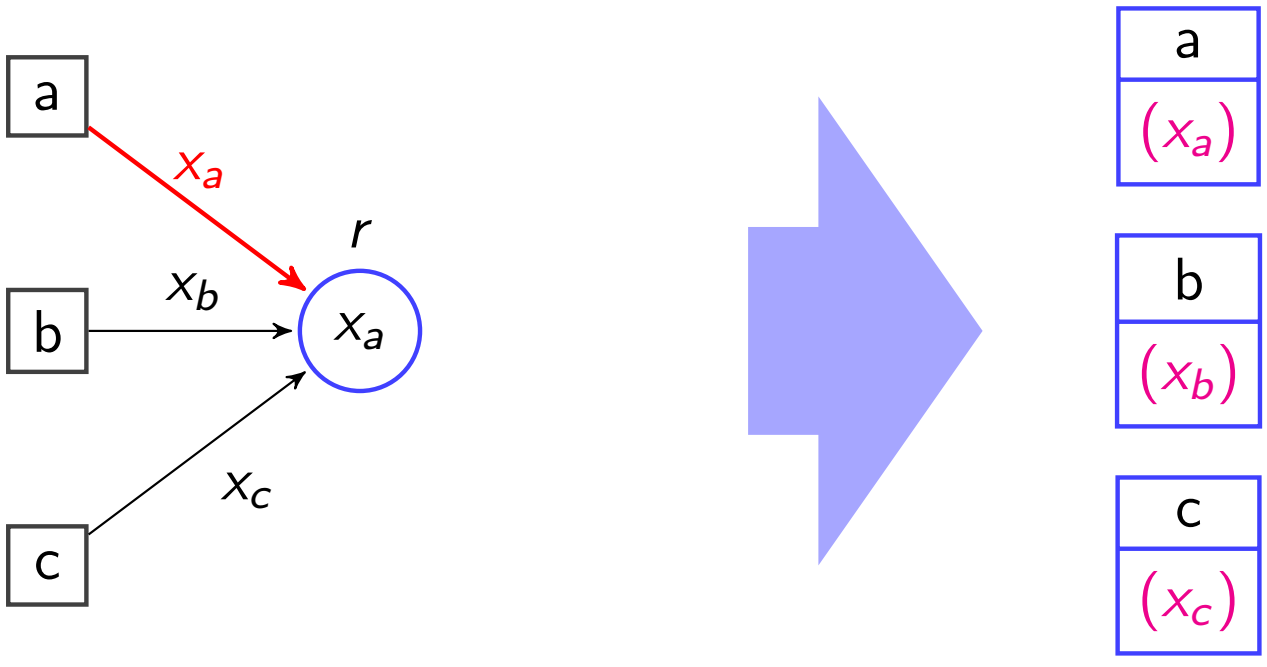
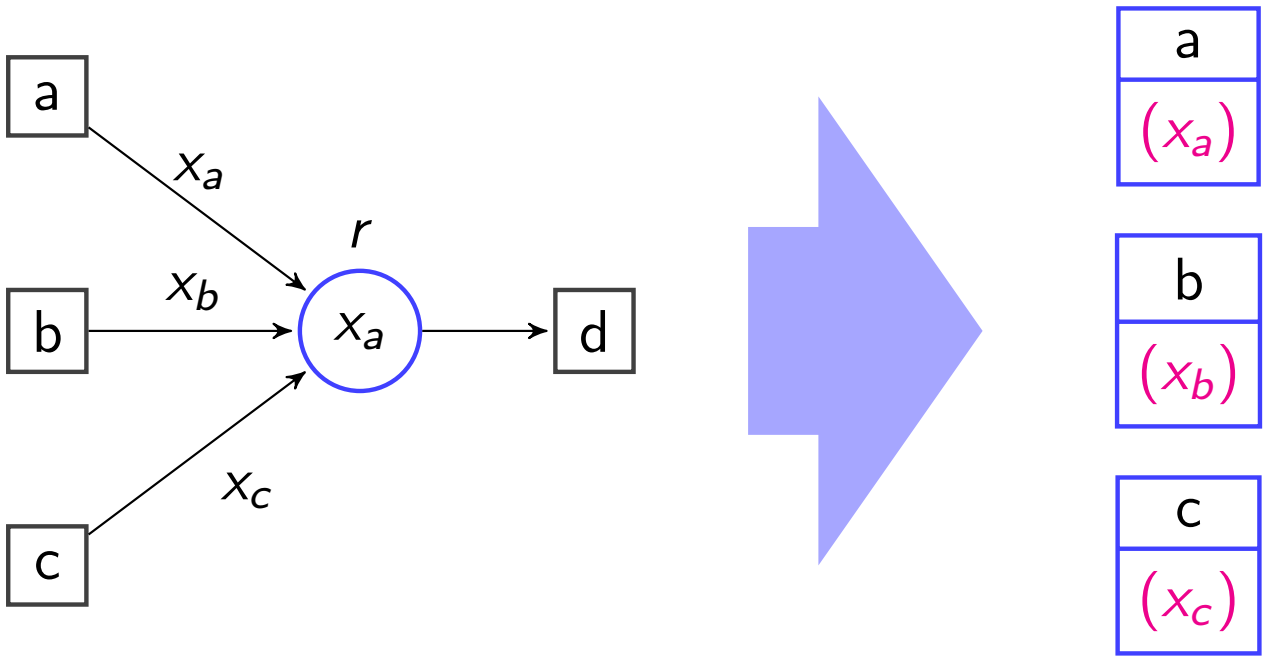


Simulations

fat $\rightarrow$ fact

Writing-conflict



Simulations

fat $\rightarrow$ fact

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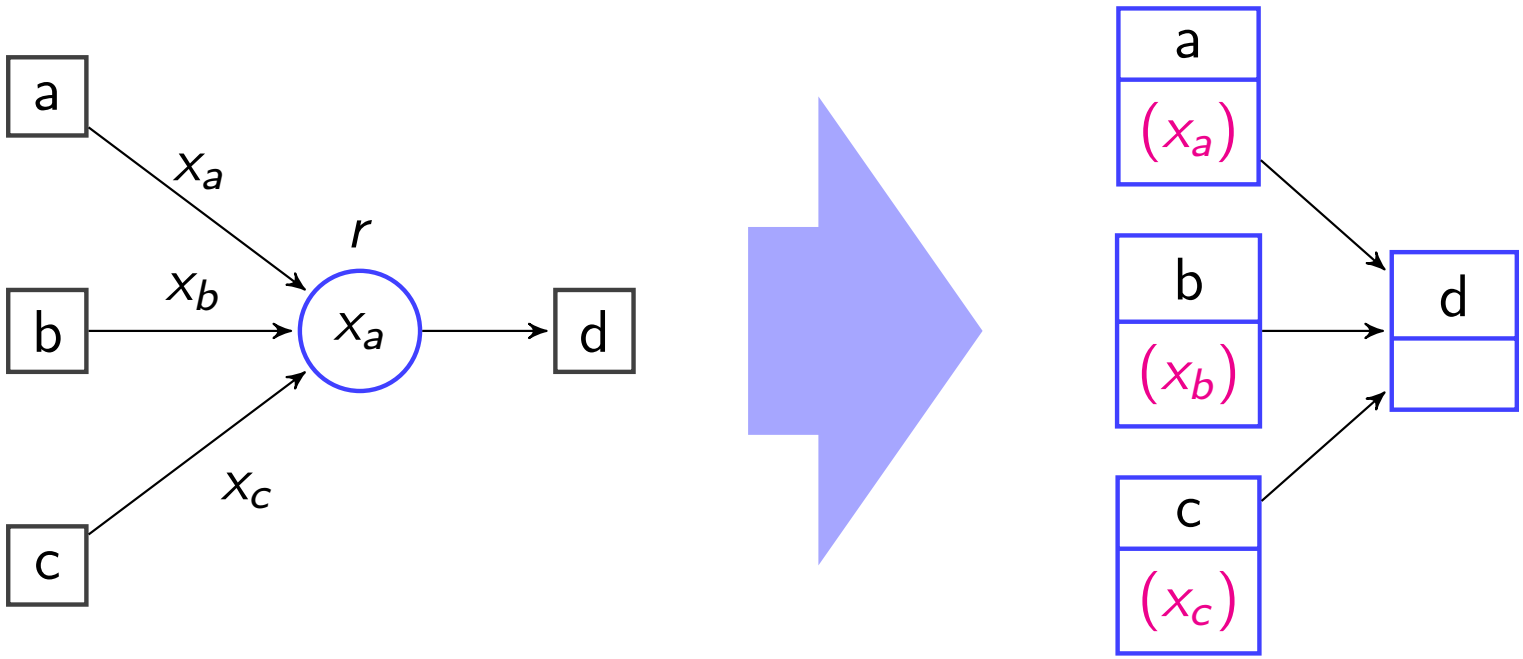

Simulations

fat $\rightarrow$ fact

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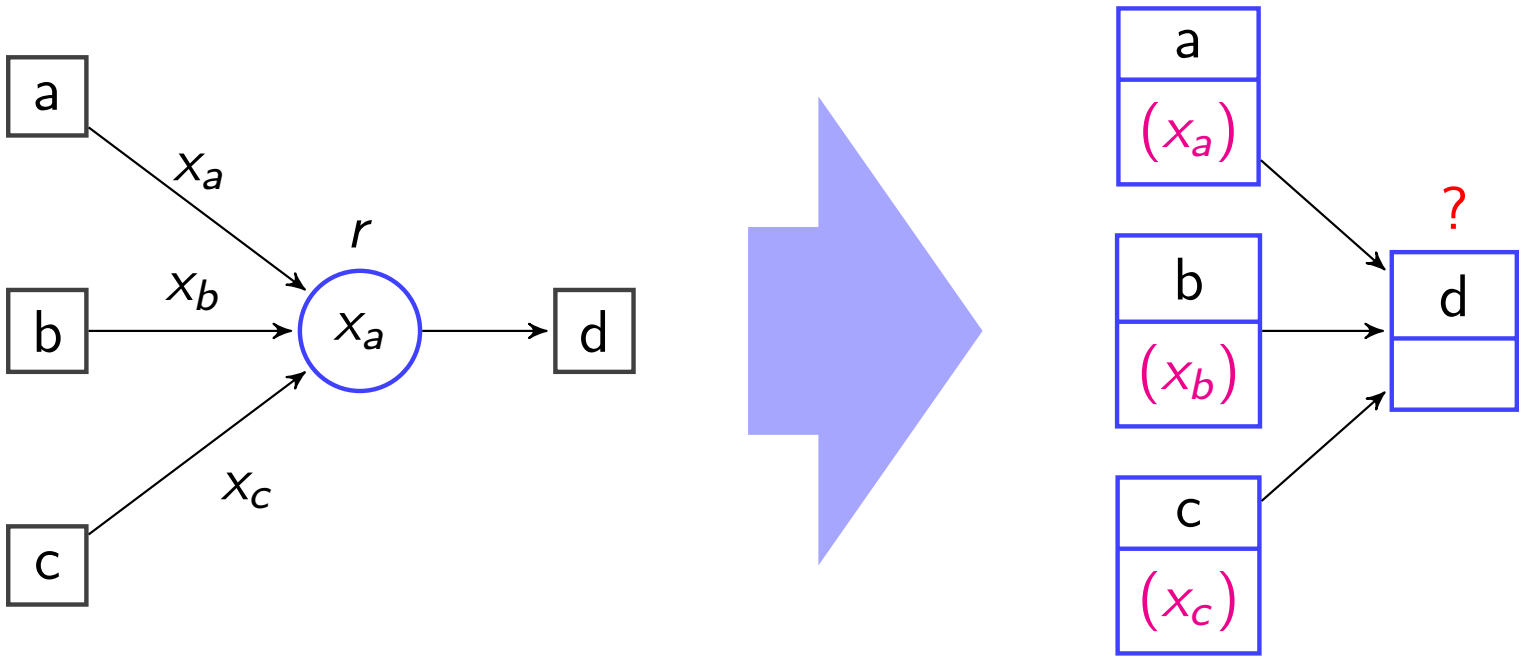
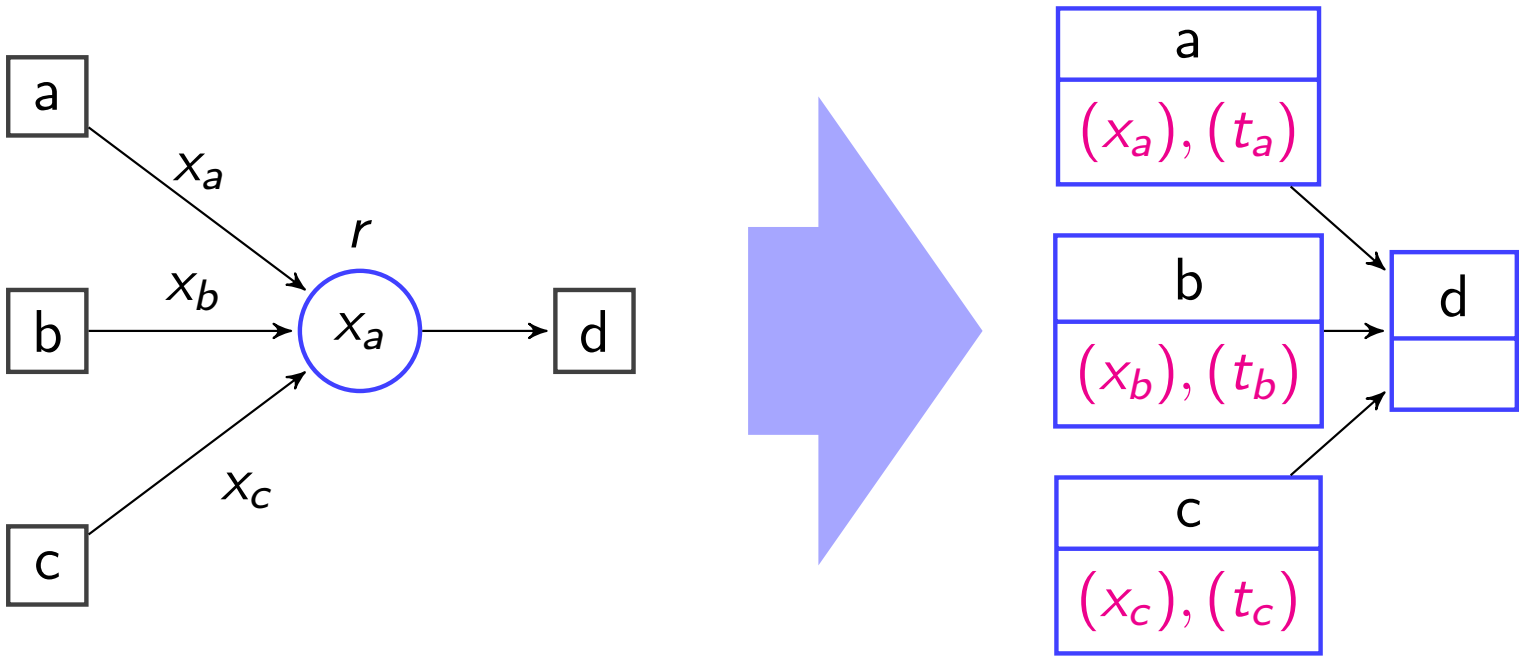


Simulations

fat $\rightarrow$ fact

Add a time-stamp



Simulations

fat $\rightarrow$ fact

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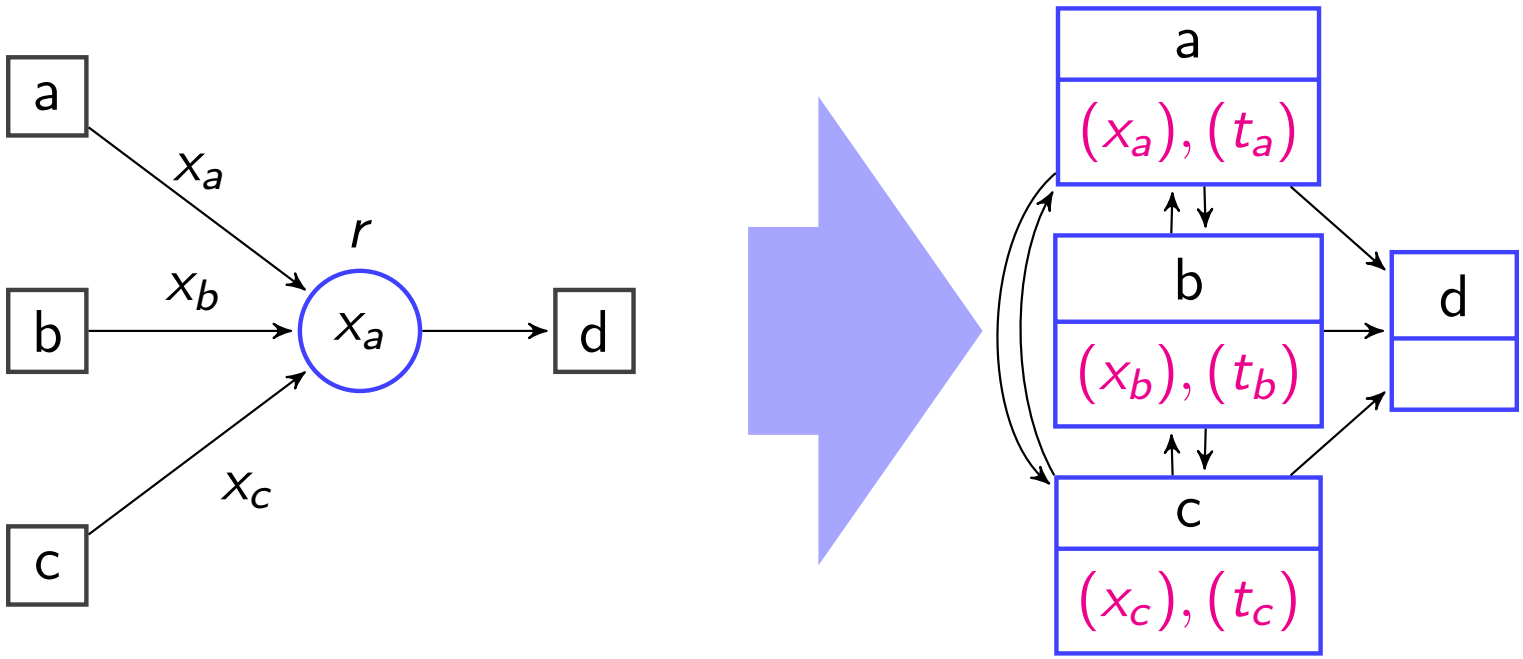

Simulations

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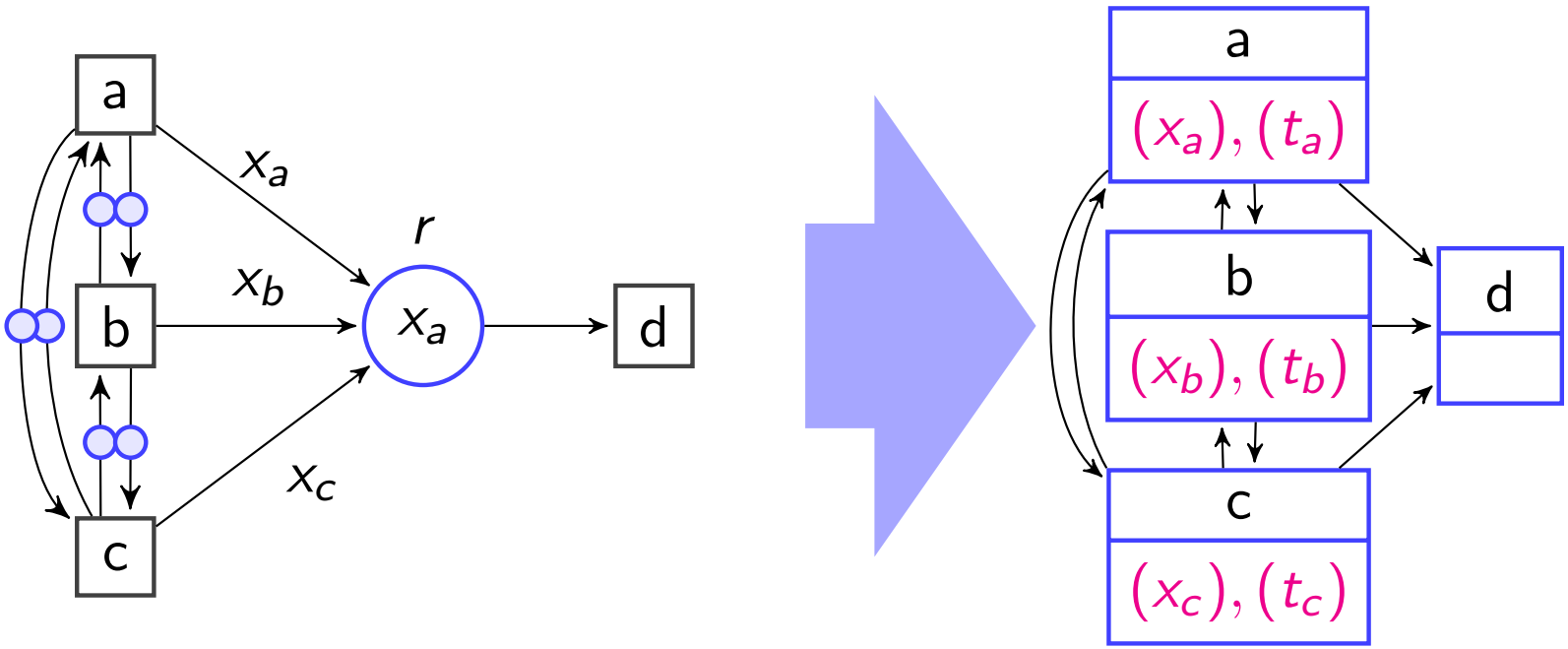

Simulations

fat $\rightarrow$ fact

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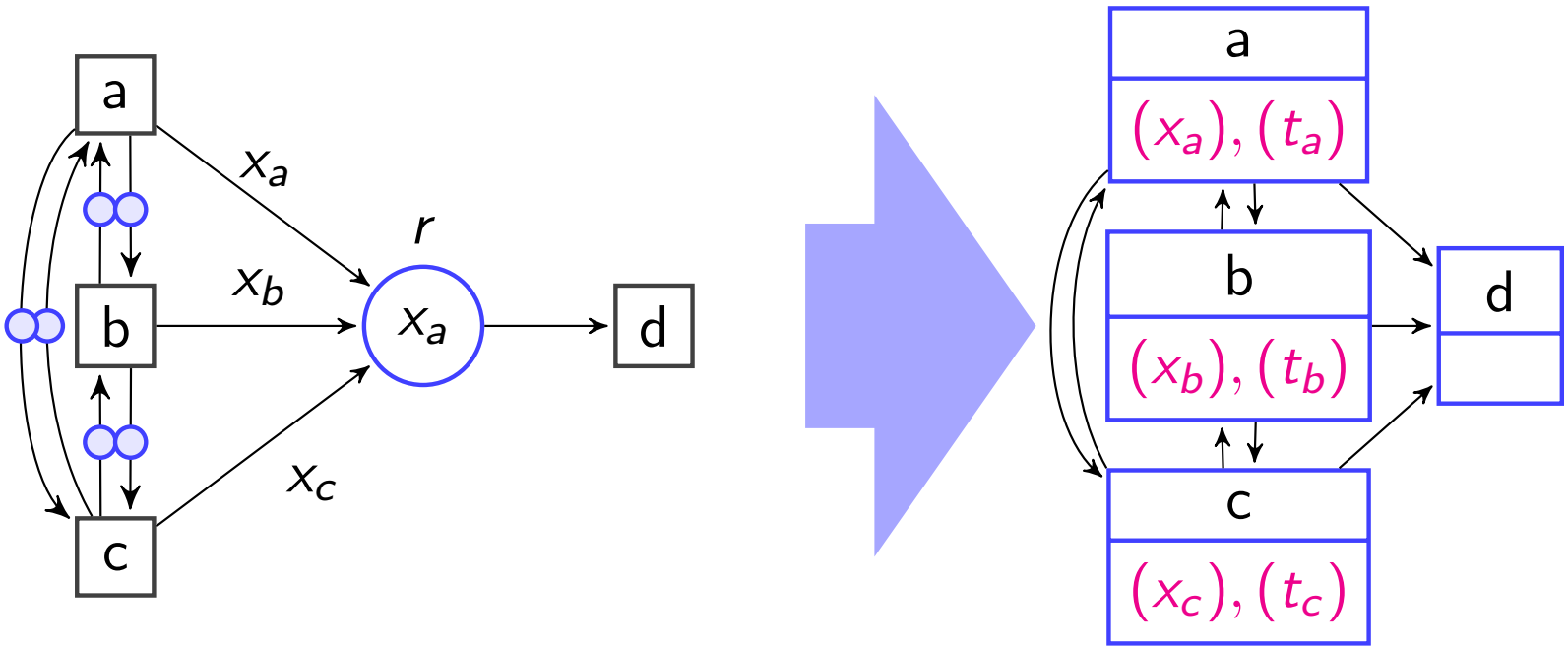
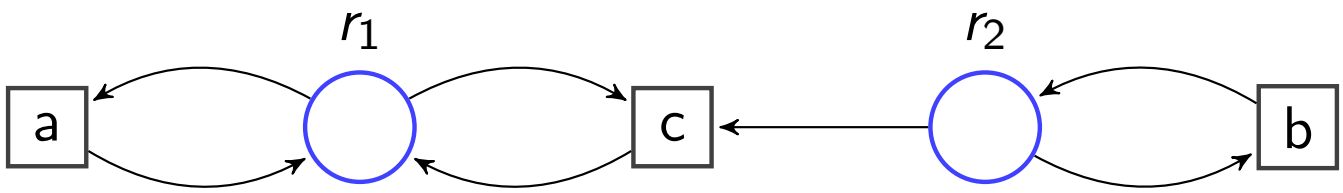


This writing conflict is ω-redundant.

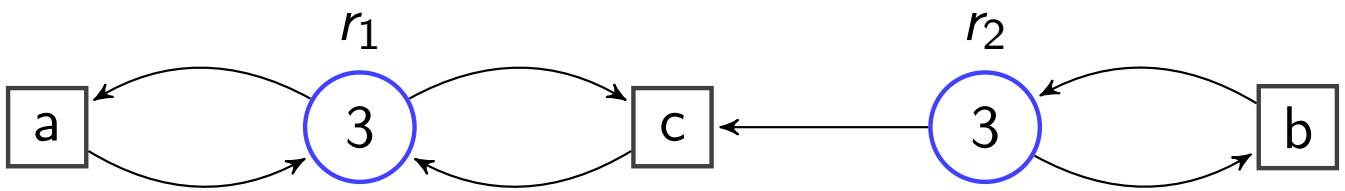
Simulations

fat $\rightarrow$ fact: Example



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

r_1	r_2	r_1
0	0	s

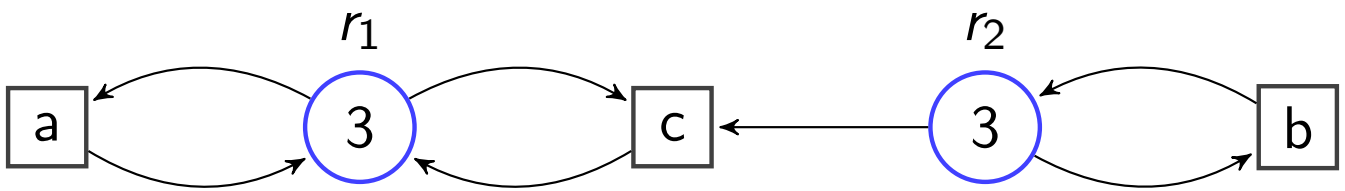
δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

Simulations

fat $\rightarrow$ fact: Example



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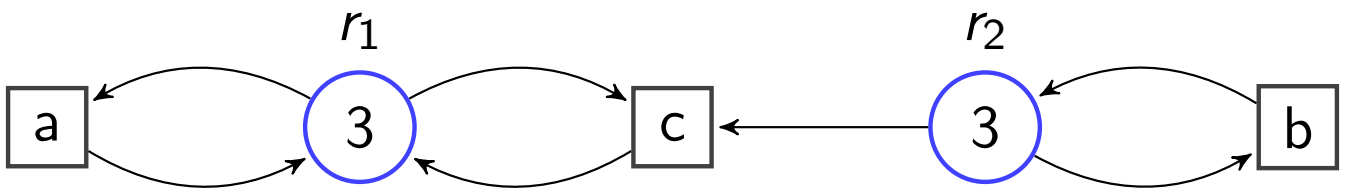
Simulations

fat $\rightarrow$ fact: Example



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δ_c

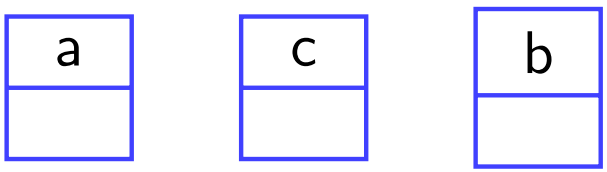
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

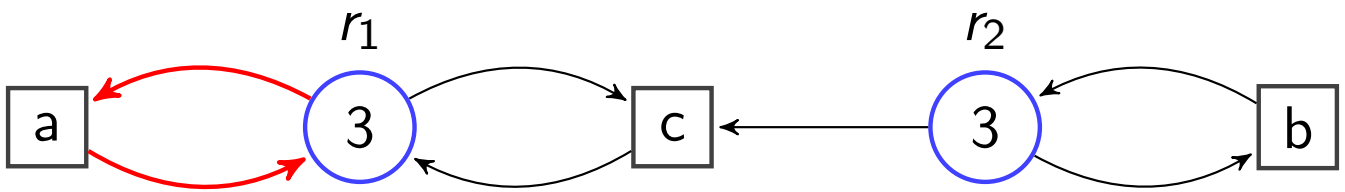
$0 < i \leq 3$

$$\Sigma = \Sigma_{fat}$$



Simulations

fat $\rightarrow$ fact: Example



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r_1	r_1
i	$i-1$

δ_c

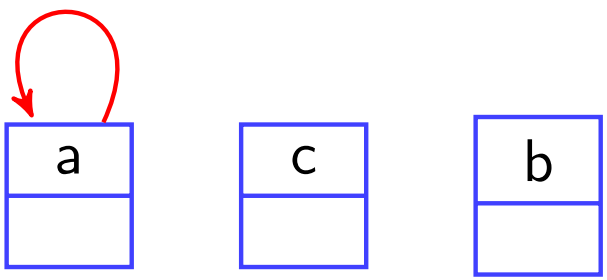
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

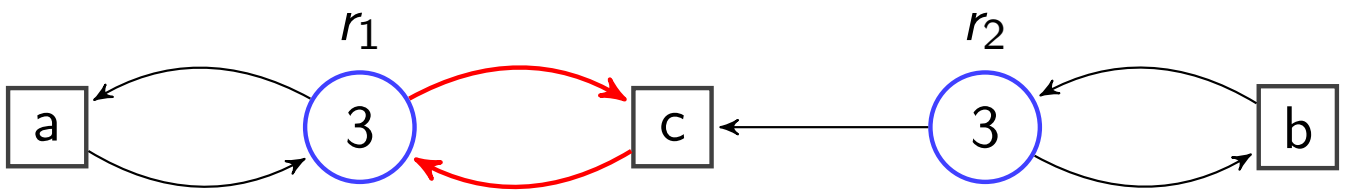
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

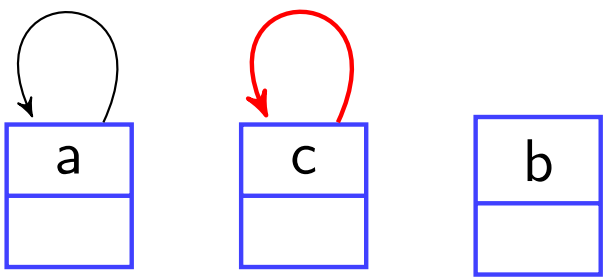
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

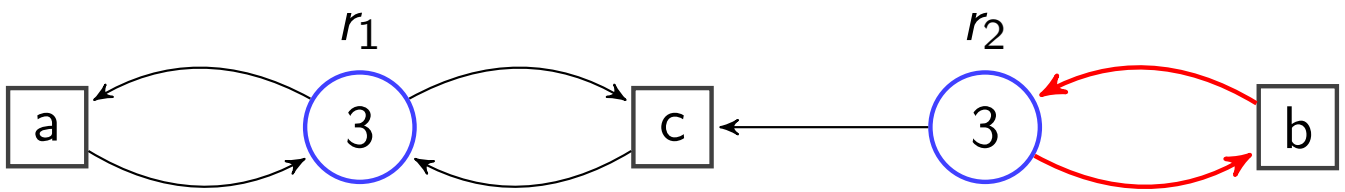
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



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δ_c

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0	0	s

δ_b

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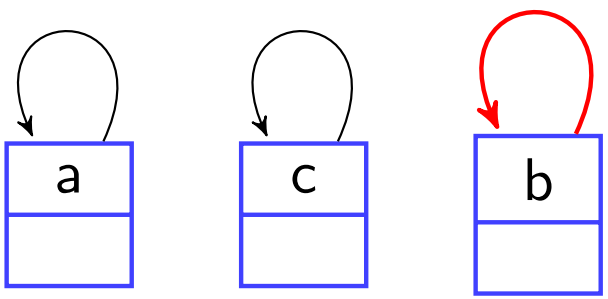
i	$i-1$

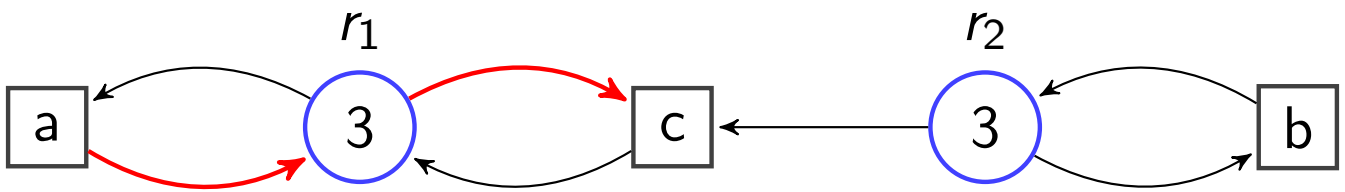
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

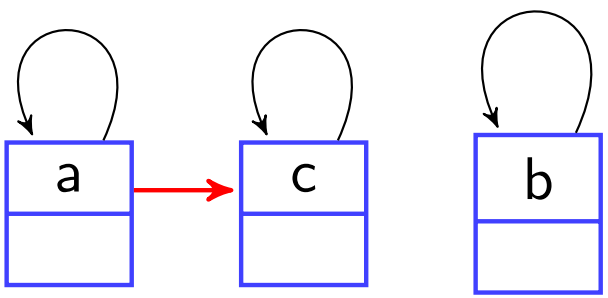
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

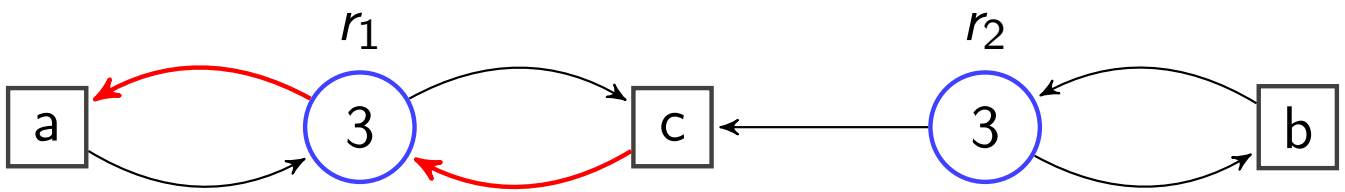
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

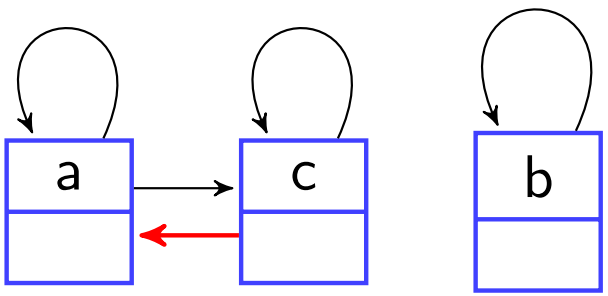
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

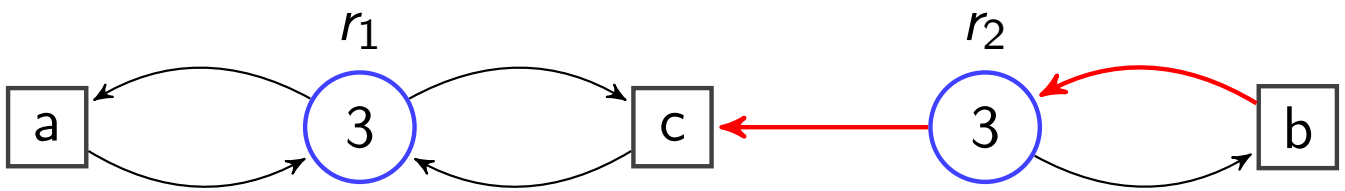
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

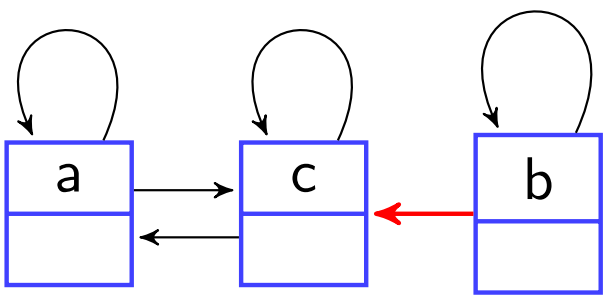
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

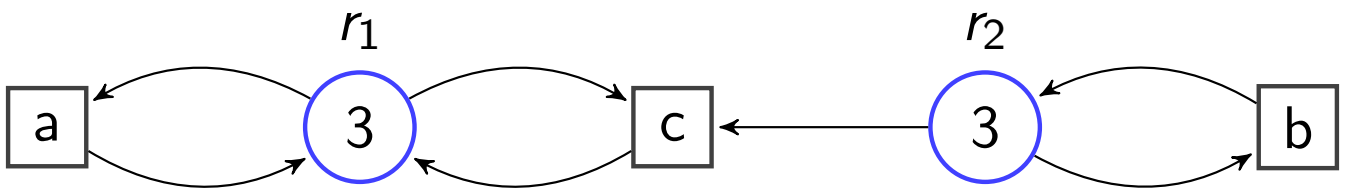
$0 < i \leq 3$

$$C = \mathcal{E}^2_{fat} \cap \Sigma^2$$



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

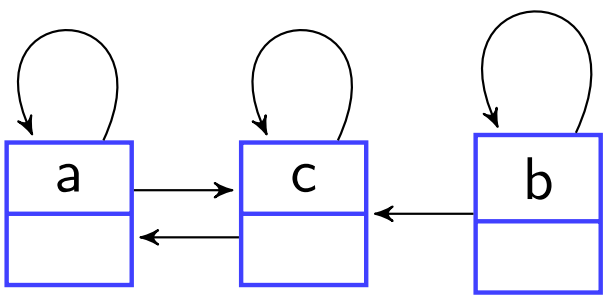
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

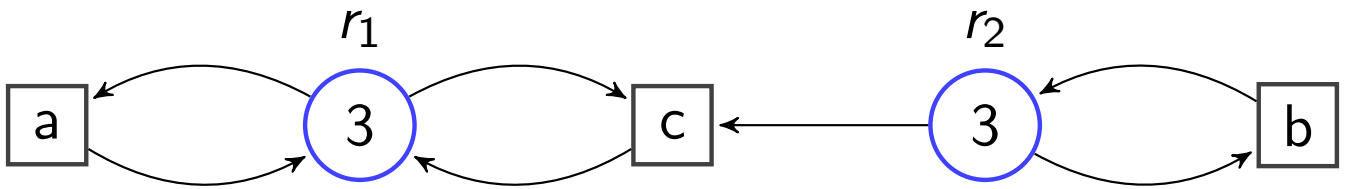
$$X = \bigcup_{a \in \Sigma} X_a$$



$X = \bigcup_{a \in \Sigma} X_a$

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

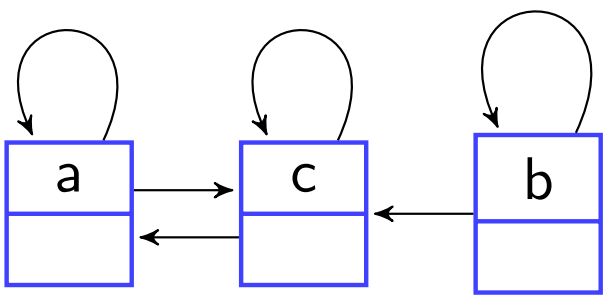
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

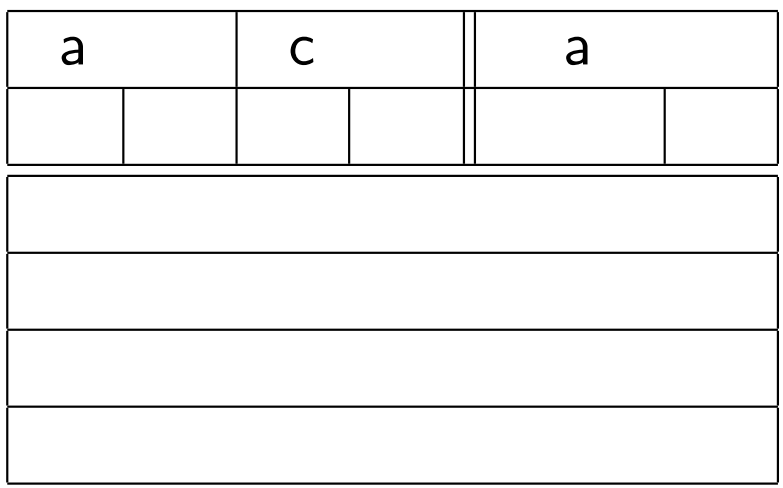
$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



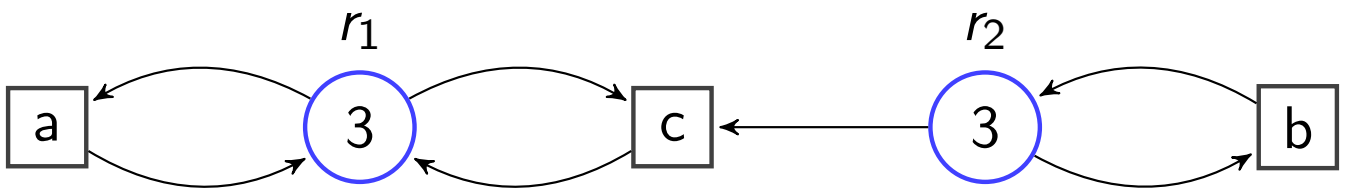
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

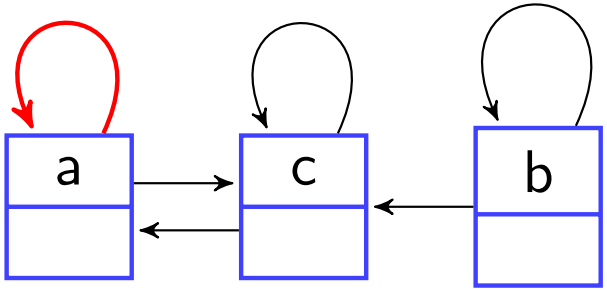
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

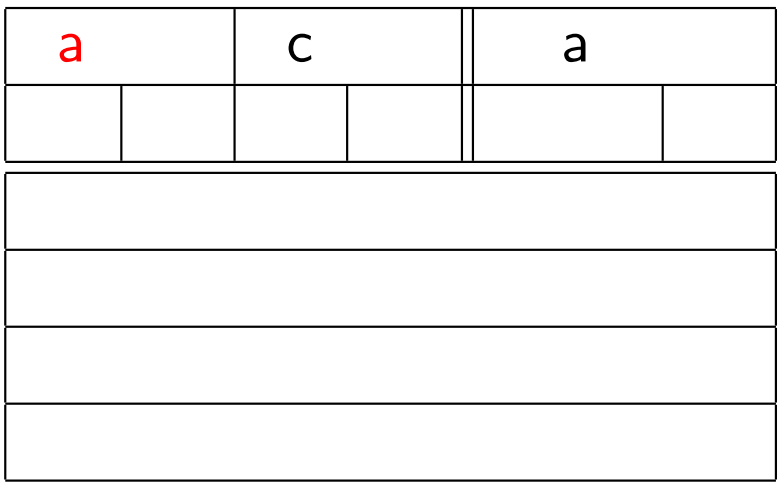
$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



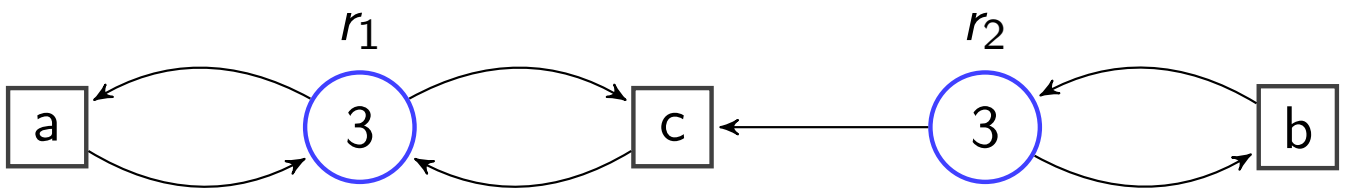
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

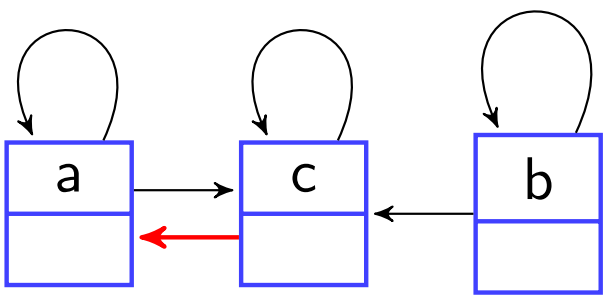
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

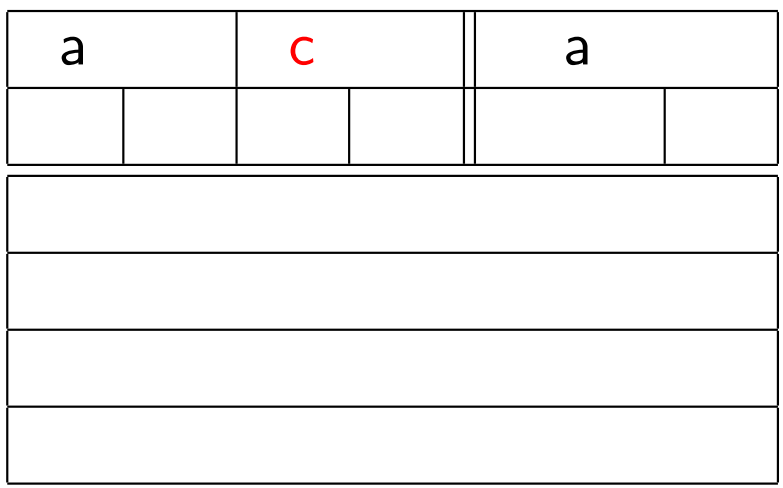
$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



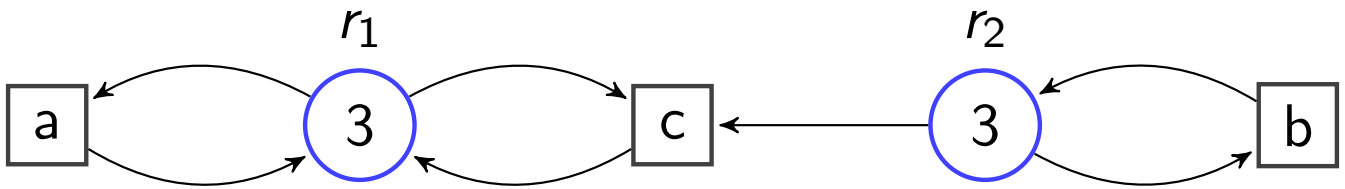
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

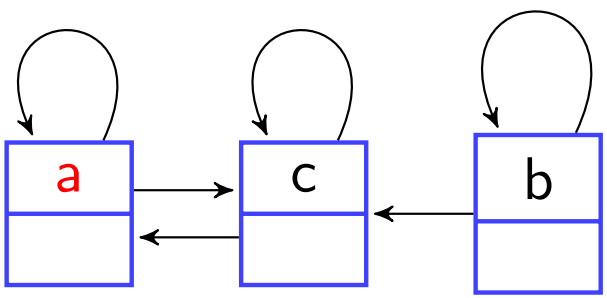
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

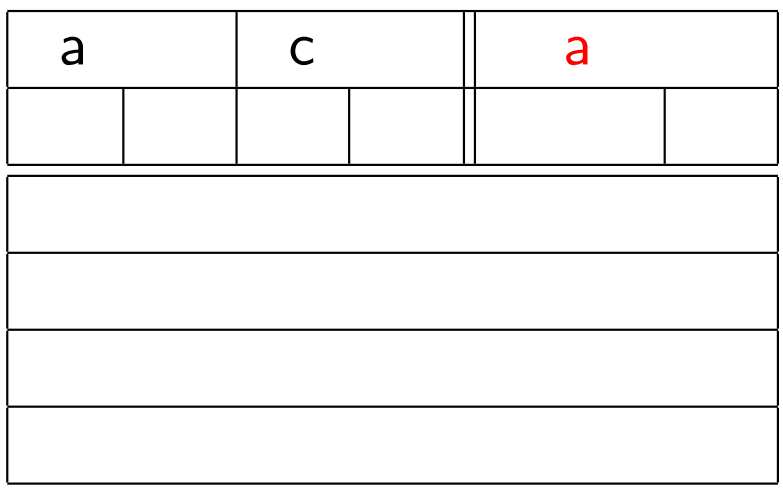
$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



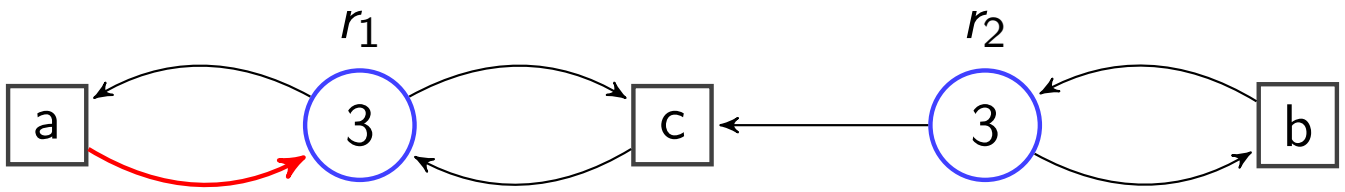
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:



Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

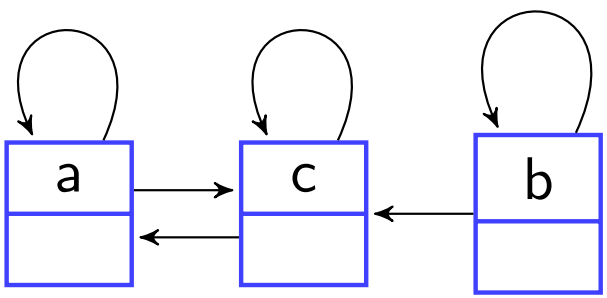
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



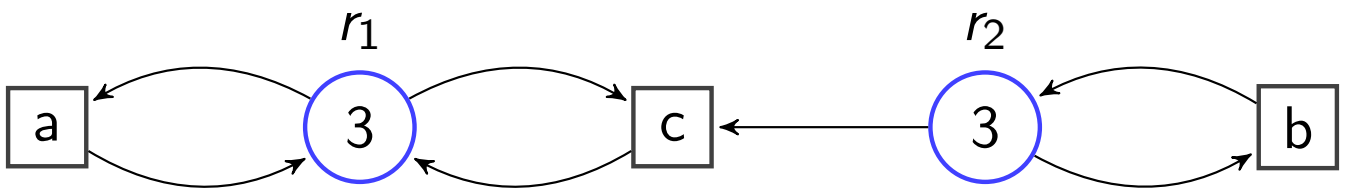
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a		c			a	
r_1					r_1	

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

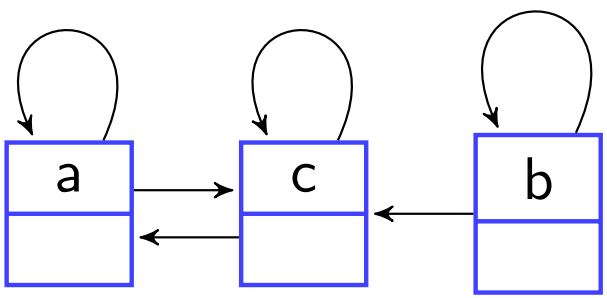
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



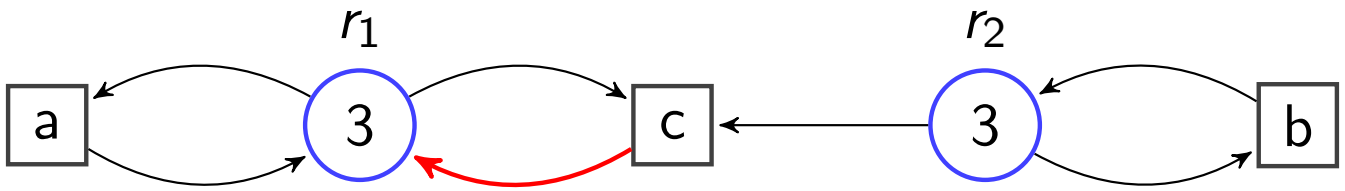
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a		c		a	
r_1	t_a			r_1	t_a

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

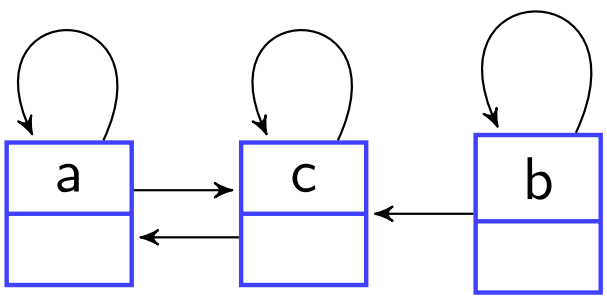
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



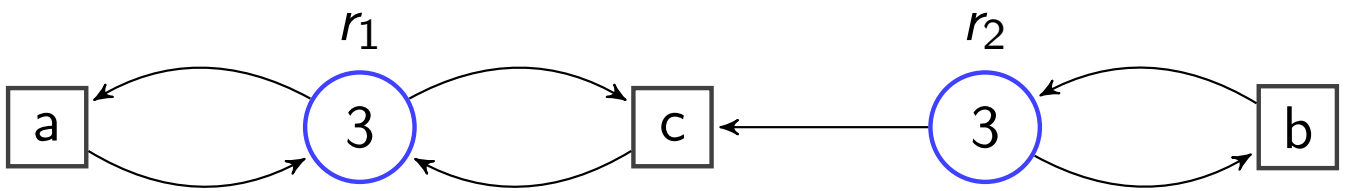
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a		c		a	
r_1	t_a	r_1		r_1	t_a

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

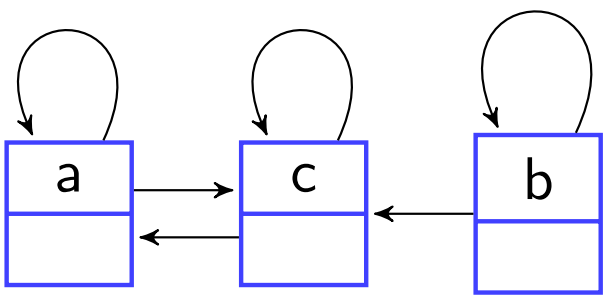
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



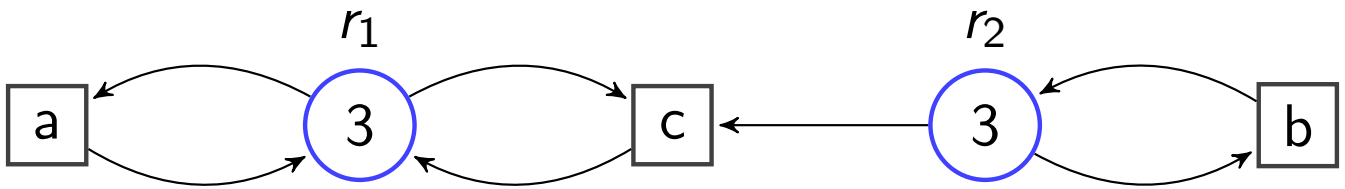
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a		c		a	
r_1	t_a	r_1	t_c	r_1	t_a

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

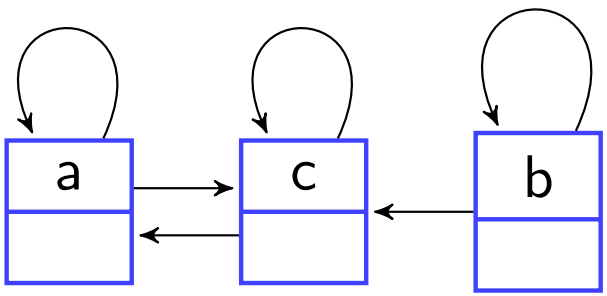
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



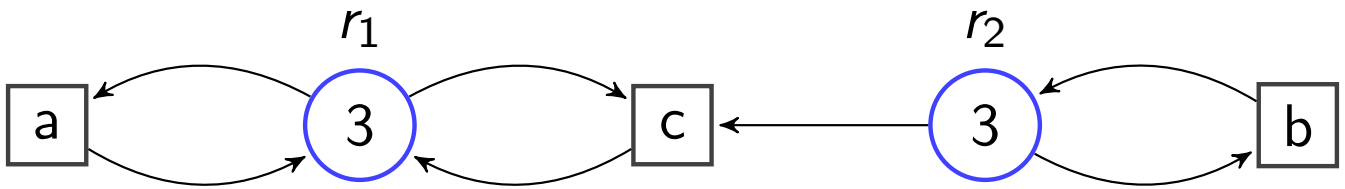
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a (0)		c		a (0)	
r_1	t_a	r_1	t_c	r_1	t_a

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

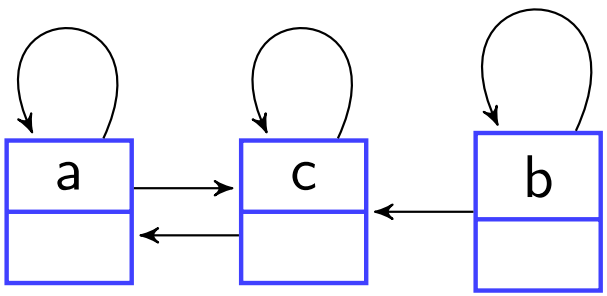
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



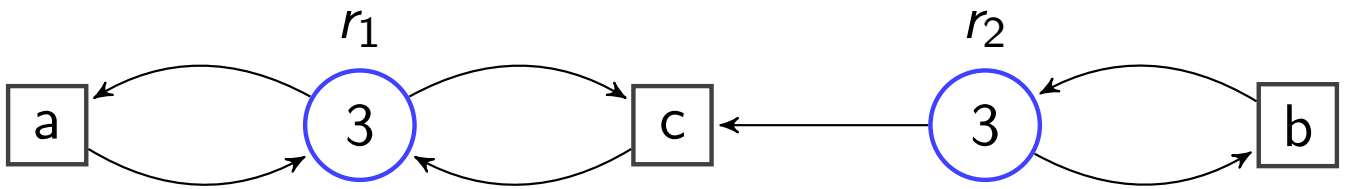
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a (0)		c (1)		a (0)	
r_1	t_a	r_1	t_c	r_1	t_a

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

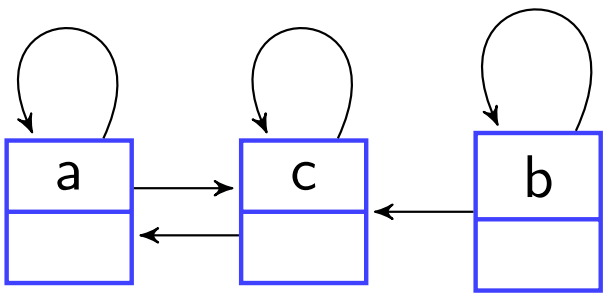
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



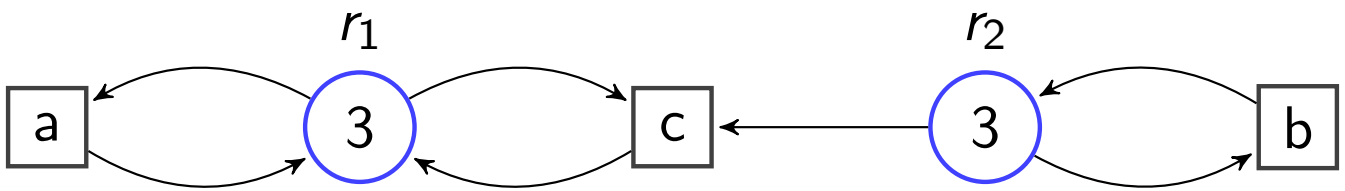
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a (0)		c (1)		a (0)	
r_1	t_a	r_1	t_c	r_1	t_a
i	0	*	0	$i-1$	0

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

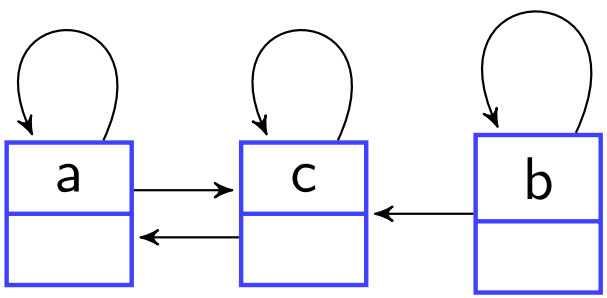
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



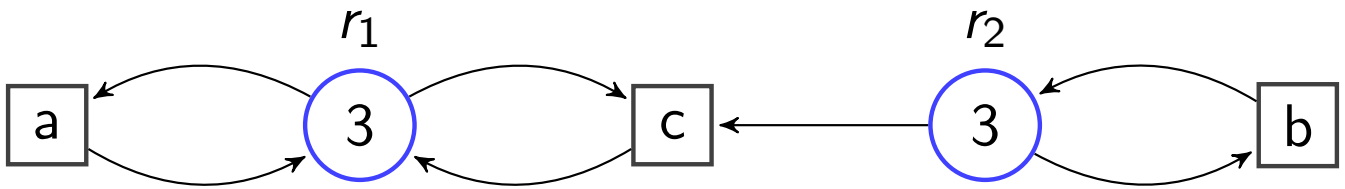
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a (0)		c (1)		a (0)	
r_1	t_a	r_1	t_c	r_1	t_a
i	0	*	0	$i-1$	0

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

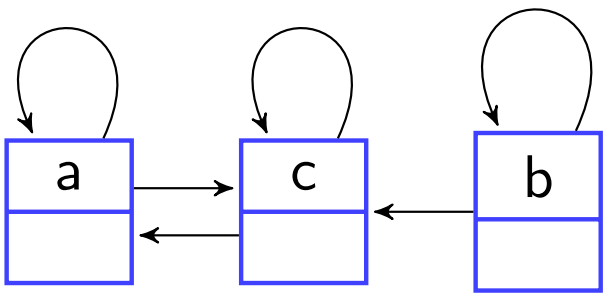
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



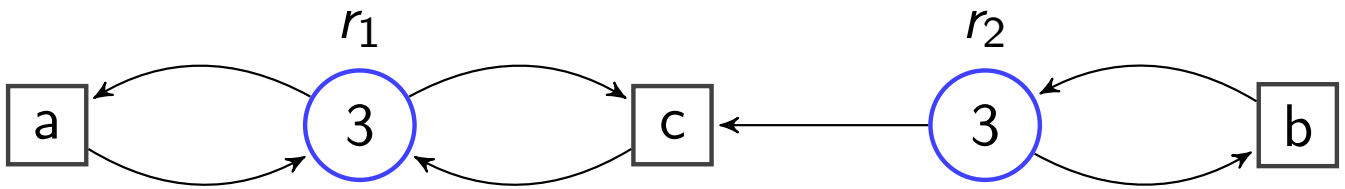
$$X = \bigcup_{a \in \Sigma} X_a$$

δ_a:

a (0)		c (1)		a (0)	
r_1	t_a	r_1	t_c	r_1	t_a
i	0	*	0	$i-1$	0
i	1	*	1	$i-1$	1

Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

δ_c

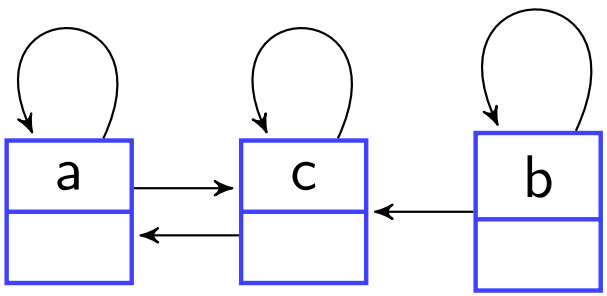
r_1	r_2	r_1
0	0	s

δ_b

r_2	r_2
i	$i-1$

$0 < i \leq 3$

$$\delta_a \subseteq \prod_{\alpha \in Ca} X_\alpha \times X_a$$



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r_1	t_a	r_1	t_c	r_1	t_a
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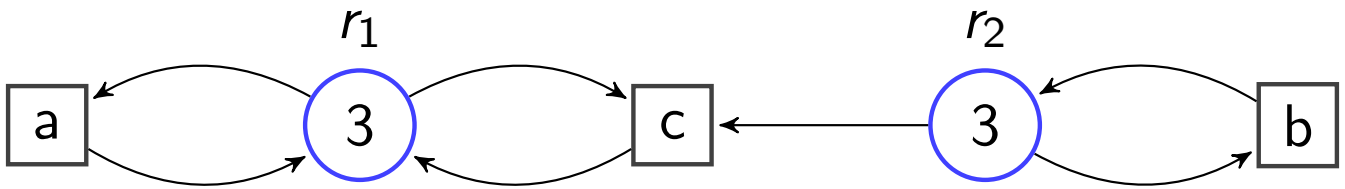
Simulations

fat $\rightarrow$ fact: Example



δ_a

r_1	r_1
i	$i-1$

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0	0	s

δ_b

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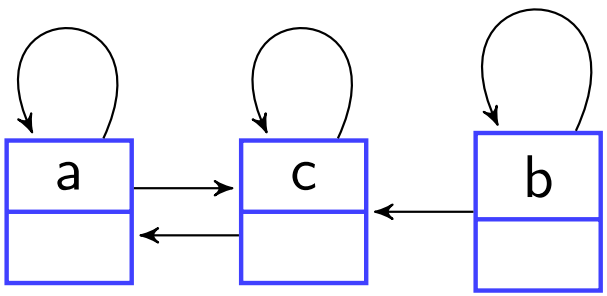

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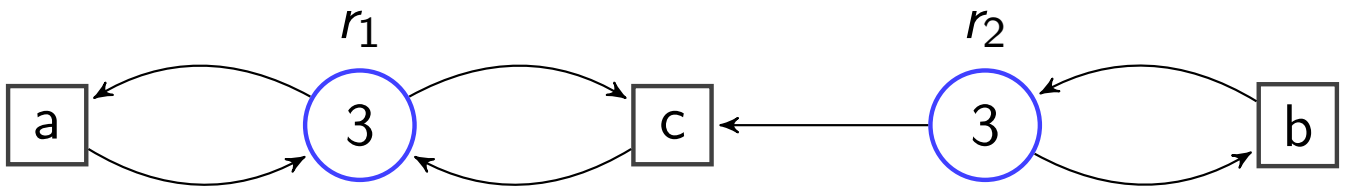

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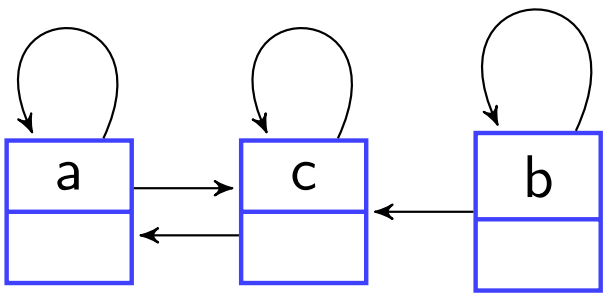

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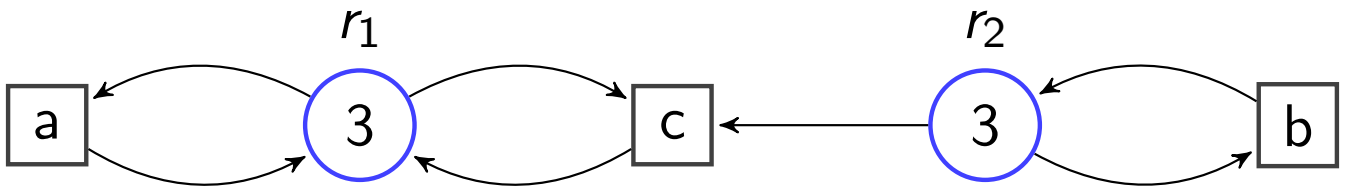

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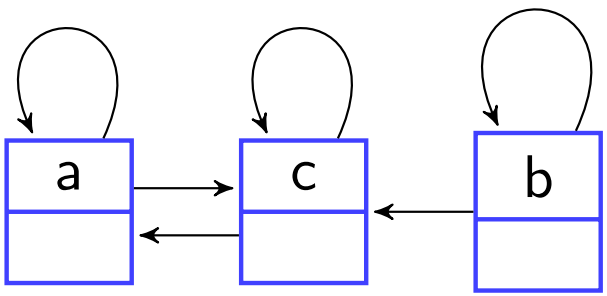

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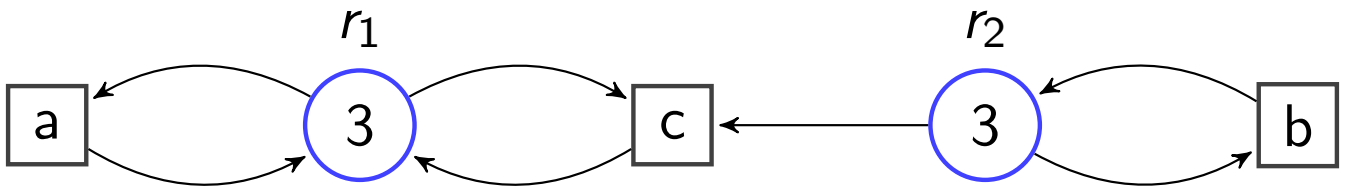

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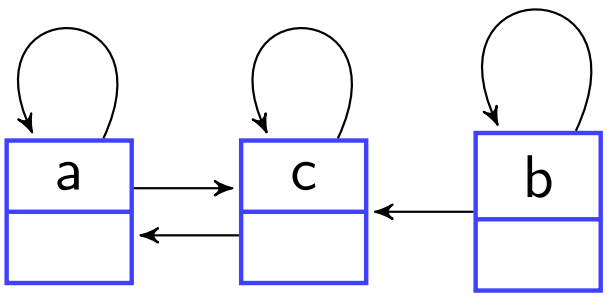

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Then there exists a fat-system $\tau' = (\Sigma, R, \mathcal{E}, X', \Delta')$ over σ covering τ.

Content

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In the sequel we restrain our attention to languages recognized by faca.

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For each faca $\mathcal{A} = (\Sigma, C, X, \Delta, I, F)$ there exists a faca $\mathcal{A}'$ over the same signature $\sigma = (\Sigma, C)$ with only one initial state s.t. $L(\mathcal{A}) = L(\mathcal{A}')$.

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$\Leftarrow$: Let $\mathcal{A}_2$ be a faca over σ_2 with initial state s_0^2, final states F_2. Construct $\mathcal{A}_1$ over σ_1 with the same values X that works like $\mathcal{A}_2$ for Actions in Σ_1. Let s be a final state, then $s_{|\Sigma_2 \setminus \Sigma_1} = s^2_{0|\Sigma_2 \setminus \Sigma_1}$. Pick $F_1 = \{s_{|\Sigma_1} \mid s \in F_2\}$. □

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Let $a \in \Sigma, s \in X^{C_2 a}, x \in X$ then

$$(s, x) \in \delta^2_a \in \Delta_2 \qquad \textit{iff} \qquad (s_{|C_1 a}, x) \in \delta^1_a \in \Delta_1$$

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Sources


V. Diekert and G. Rozenberg.
The Book of Traces.
World Scientific, 1995.


Nils Klarlund, Madhavan Mukund, and Milind A. Sohoni.
Determinizing asynchronous automata.
In *Proceedings of the 21st International Colloquium on Automata, Languages and Programming*, ICALP '94, pages 130–141, London, UK, UK, 1994. Springer-Verlag.


Antoni Mazurkiewicz.
Concurrent program schemes and their interpretations.
Technical report, DAIMI Aarhus University, 1977.
Report PB 78.


Wieslaw Zielonka.
Notes on finite asynchronous automata.
ITA, 21(2):99–135, 1987.