Petri Nets

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Seminar: **Automata Theory**

Chair of Software Engeneering

Albert-Ludwigs-Universität Freiburg

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Overview

1. **Motivation:**
   - Why not just using finite automata for everything?
   - What are Petri Nets and when do we use them?
2. **Introduction:**
   - How do Petri Nets work?
3. **Languages:**
   - What is the general expressiveness power of Petri Nets?
4. **Decidability and Complexity:**
   - What is the complexity of certain decision problems?
   - What if we assure certain conditions?

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Motivation

Consider:

- Two groups of workers produce goods of type **A** and type **B**.
- Two goods of type A and type B can be combined into type **C**.

The corresponding language:

$L = \{aabbc, ababc, abbac, babac, bbaac, baabc\}$

Motivation

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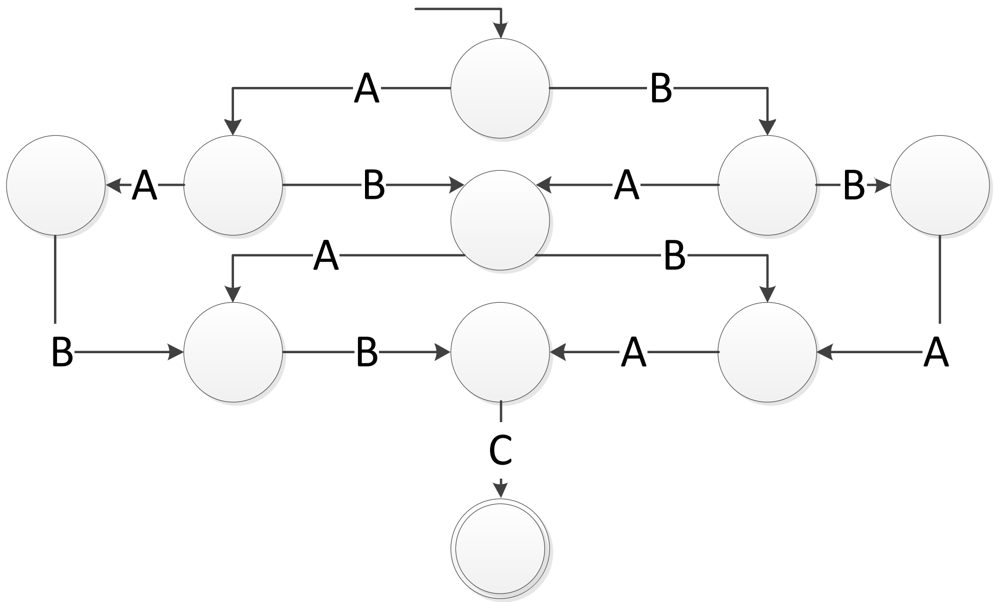
The corresponding finite automaton:



Problem Analysis

The problem:

- A and B are produced **concurrently** and **asynchronous.**
- The production has to be **synchronized** when producing a C**.**

The solution:

- Automaton model which allows for expressing these properties of a system or process more concise – **Petri Nets.**

Possible Solution

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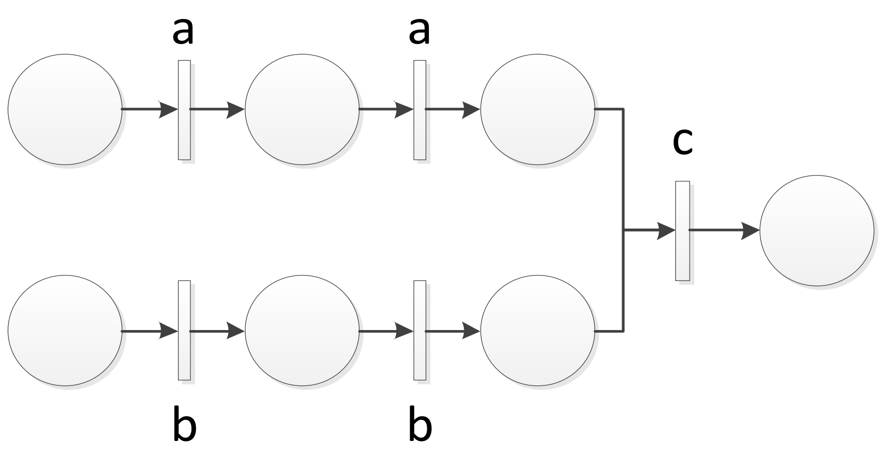
A Petri Net for the given problem:



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Structural Definitions

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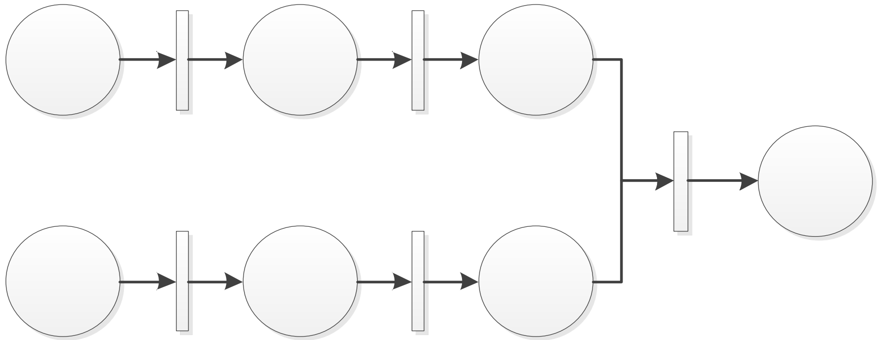

$$N = (P, T, F)$$

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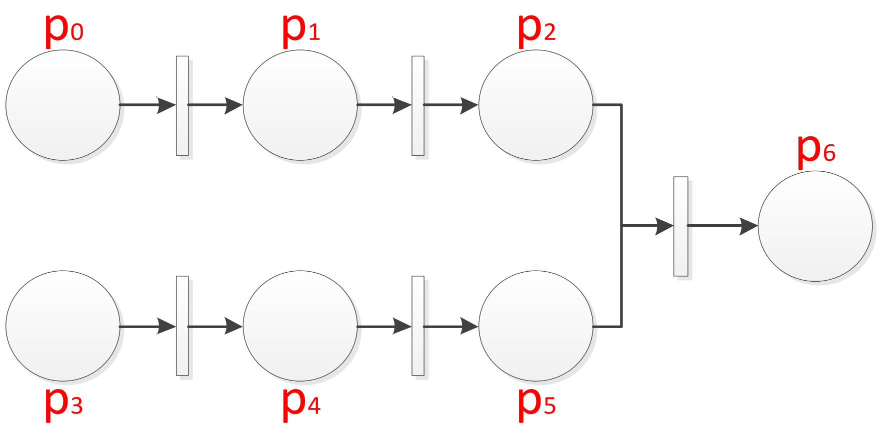

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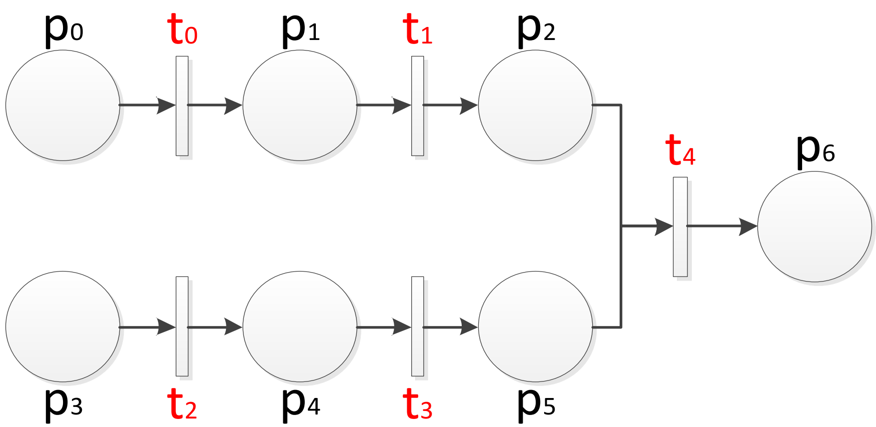

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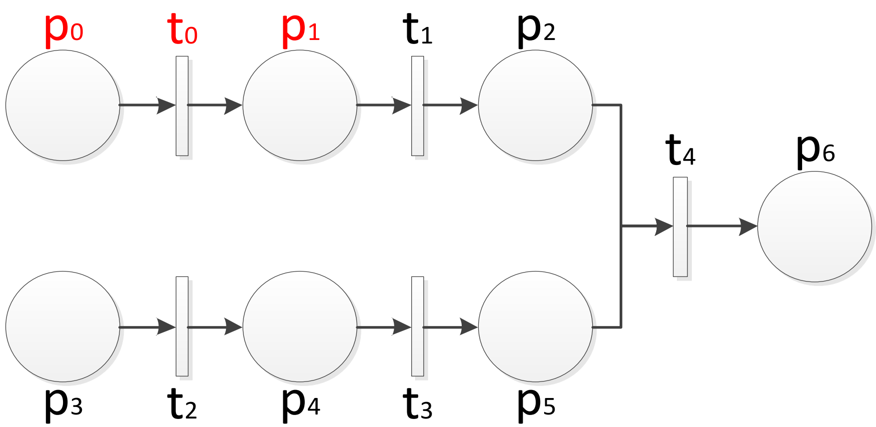

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$$F \subset (P \times T) \cup (T \times P)$$

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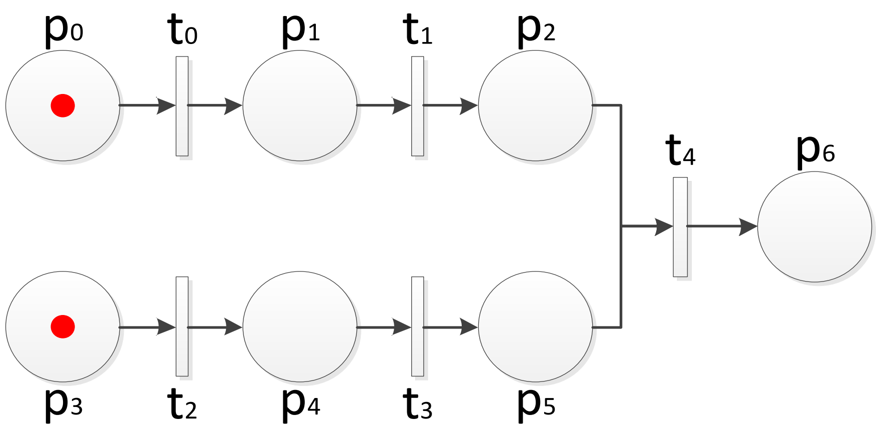

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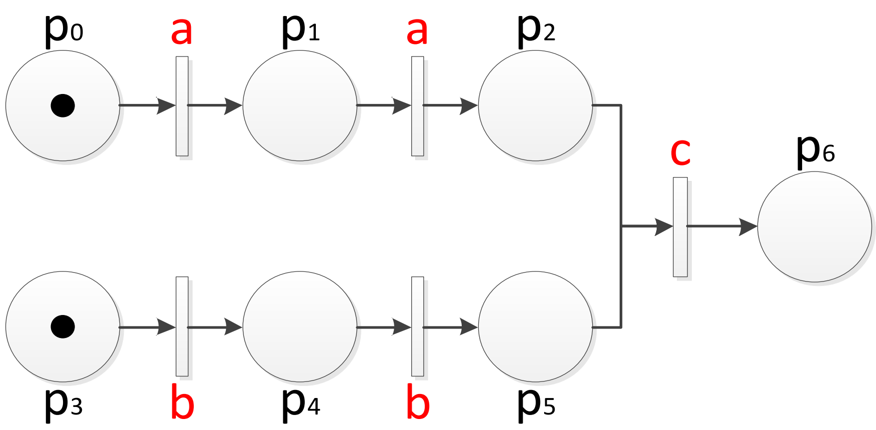

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$$\Sigma = \{a, b, c\}$$

$$l: T \rightarrow \Sigma$$

Behavior

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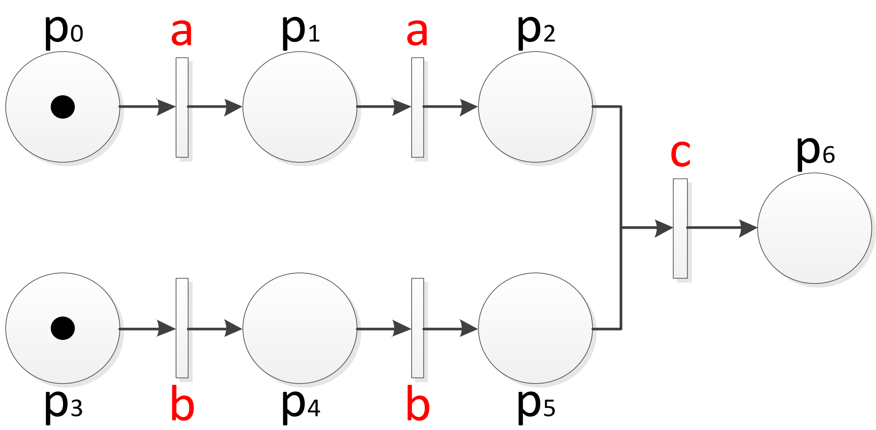

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$M_0 = (1, 0, 0, 1, 0, 0, 0)$

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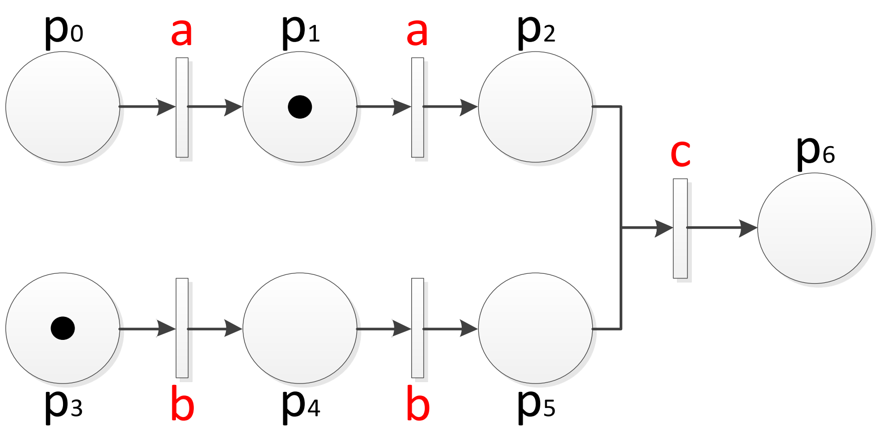

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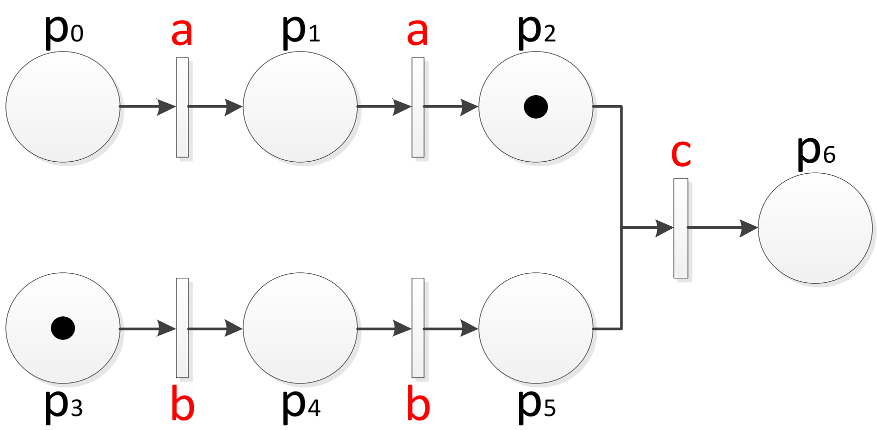

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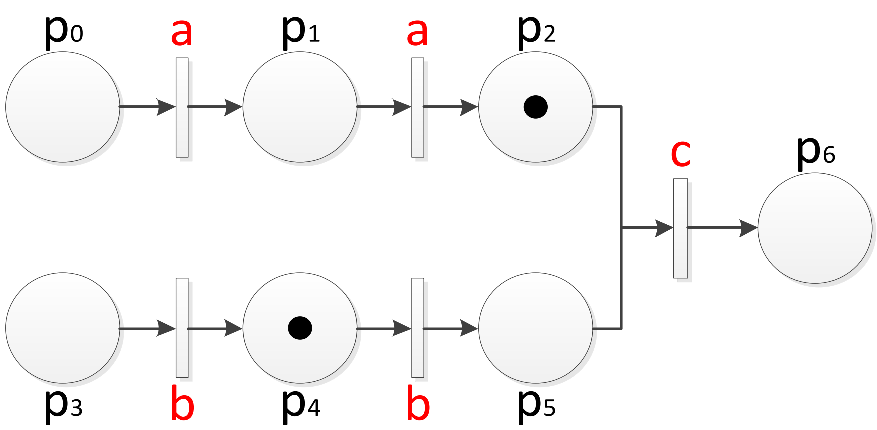

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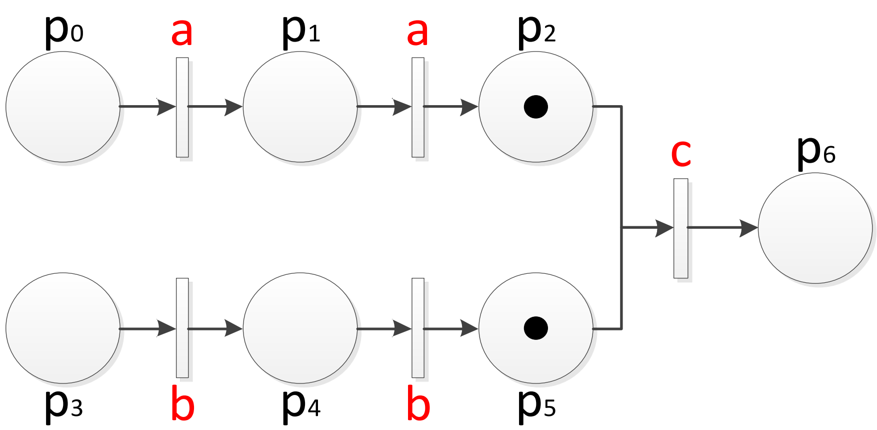

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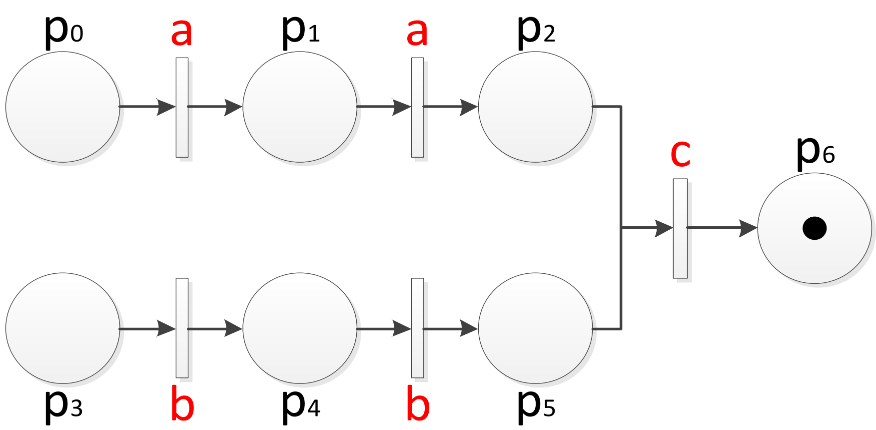

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$M_5 = (0, 0, 0, 0, 0, 0, 1)$

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Language Acceptance

Definition:

$\mathcal{N} = (N, M^I, M^F, l)$ with,

- $N = (P, T, F)$ a petri net,
- $l: T \rightarrow \Sigma$ a labeling function, Σ a non-empty alphabet,
- and M^I, M^F finite sets of initial and final markings respectively,

accepts a word $w \in \Sigma^*$ if there exists a firing sequence

$\tau = t_1, \dots, t_n$ from $m_i \in M^I$ to $m_j \in M^F$.

Petri Net Languages

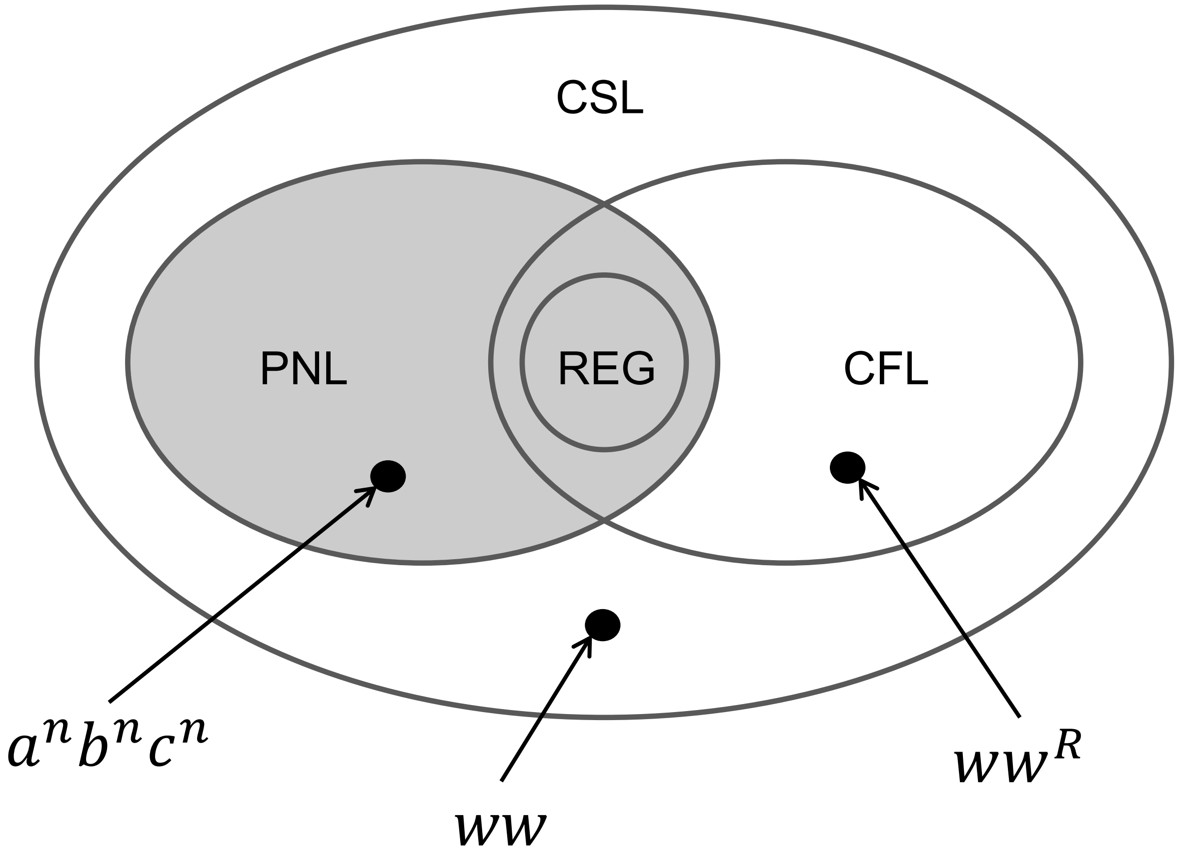

Figure taken from [3]

Languge - Example

Let $\Sigma = \{a, b, c\},\ L = \{a^n b^n c^n |\ n > 0\}$

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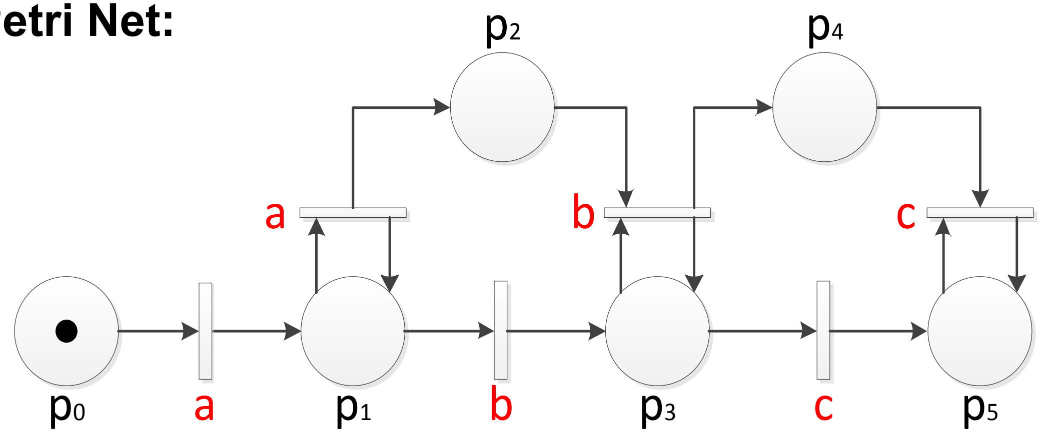

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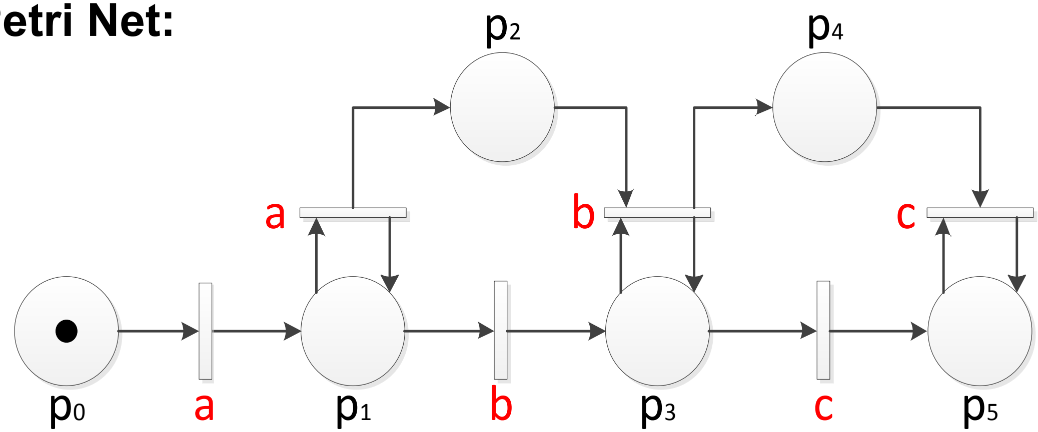

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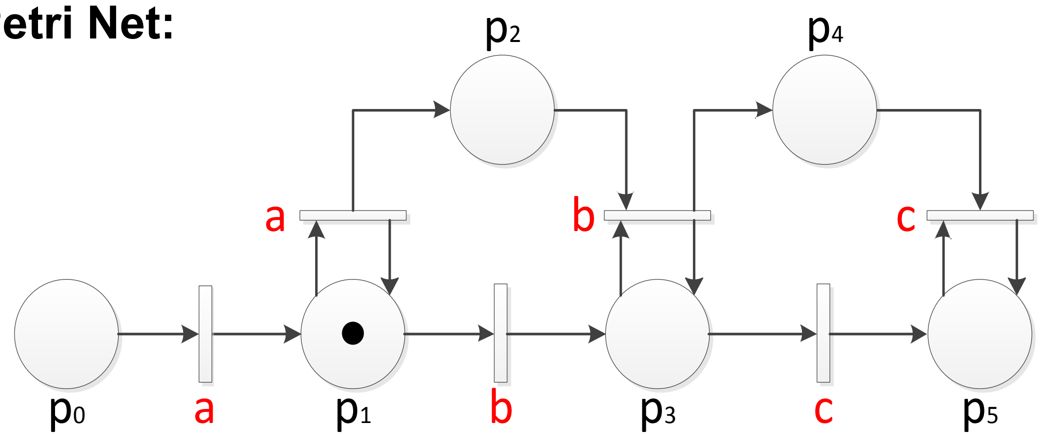

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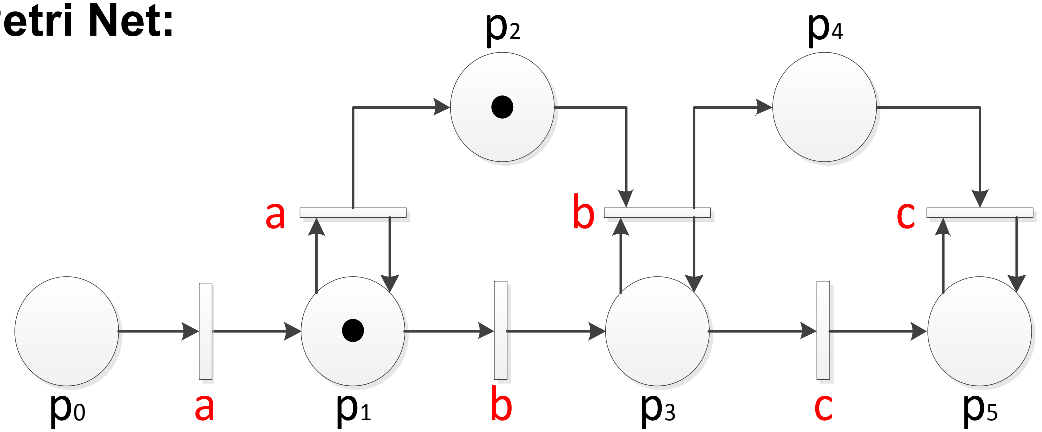

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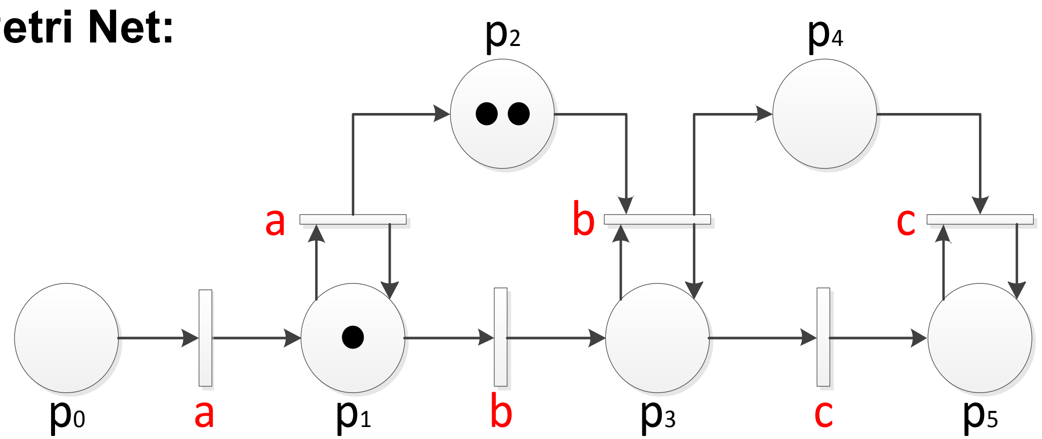

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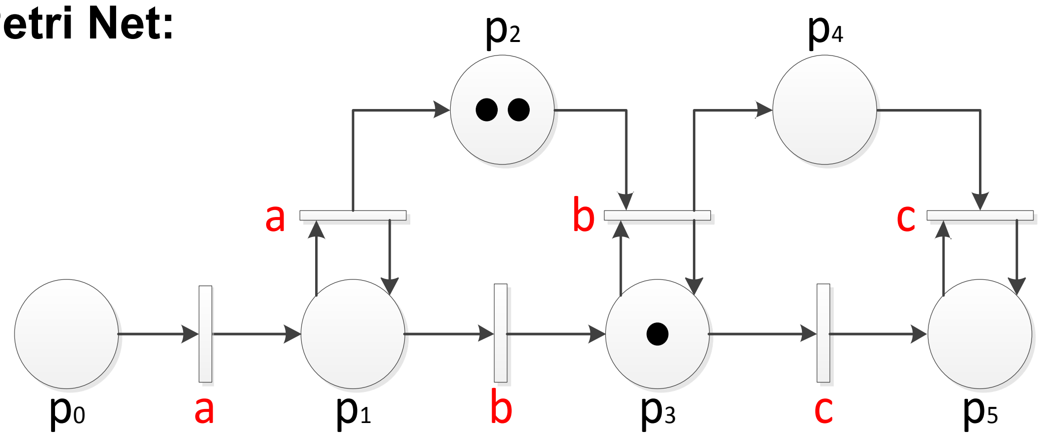

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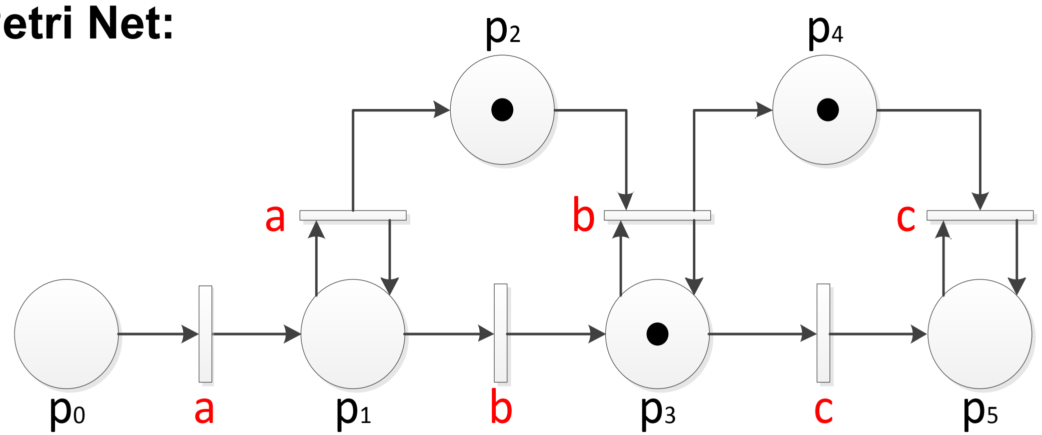

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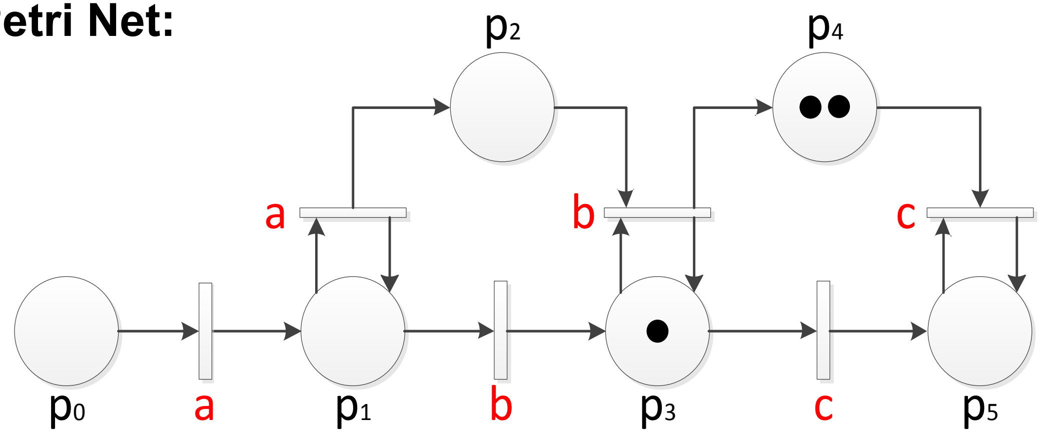

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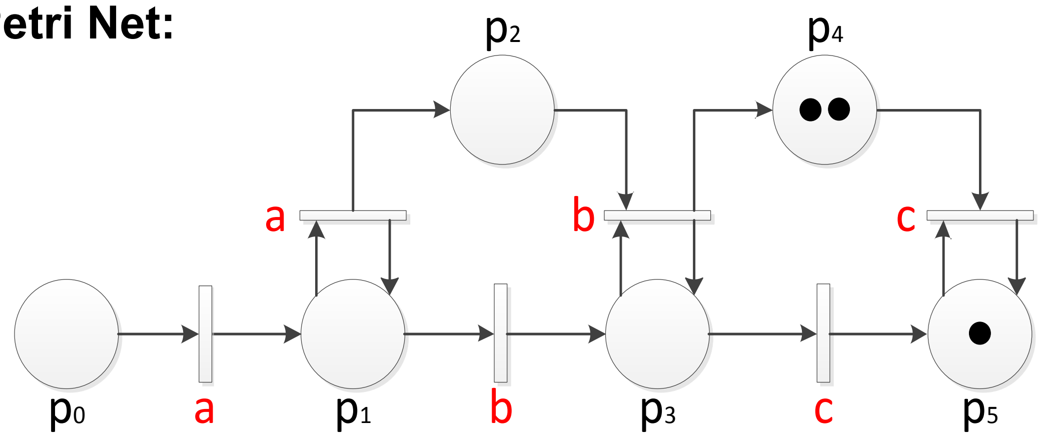

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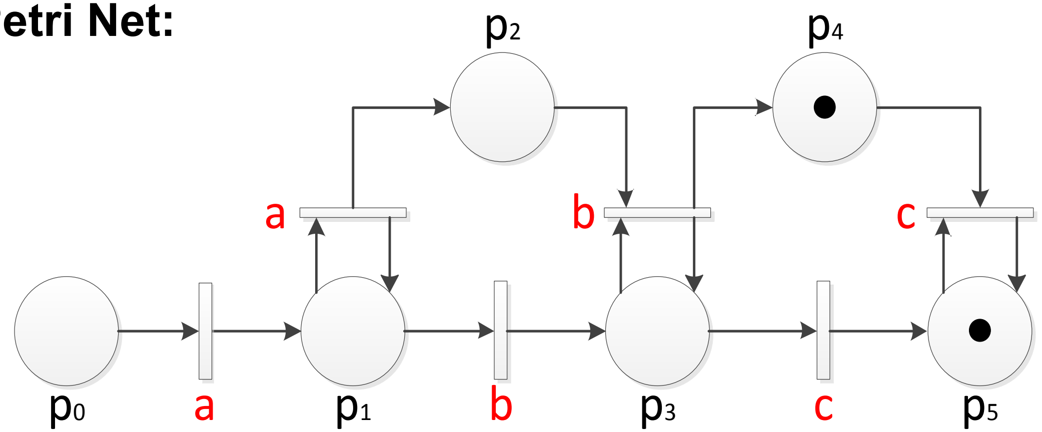

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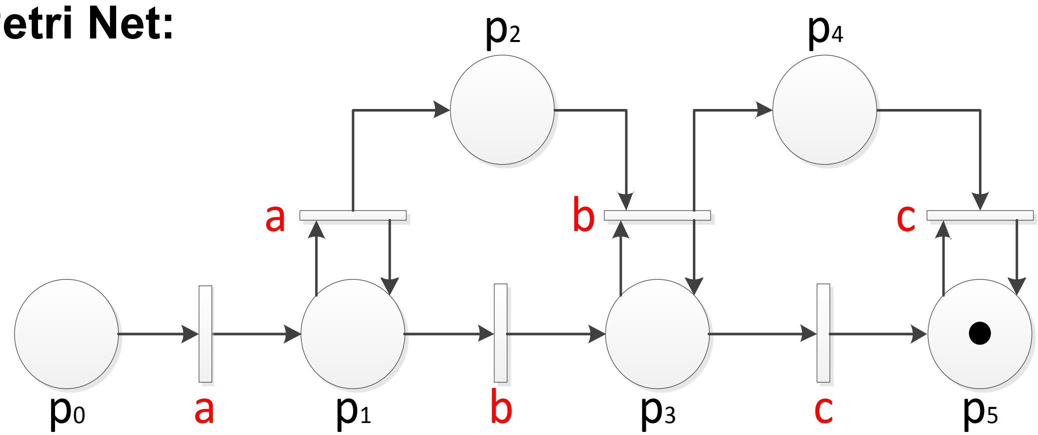

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Reachability Problem

Is a certain marking reachable in a Petri Net?

- I.e. can a system reach some bad state?

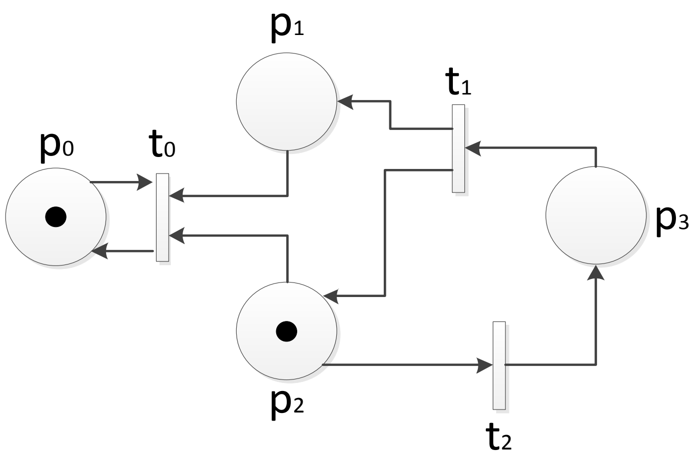
Analysis:

1. What about the reachability problem for Petri Nets in general?
2. Are there certain properties which allow for deciding the reachability problem efficiently?

General analysis tool: The reachability/coverability tree.

Reachability Tree

Petri Net:



Reachability Tree:

(1, 0, 1, 0)

Example taken from [2]

Reachability Tree

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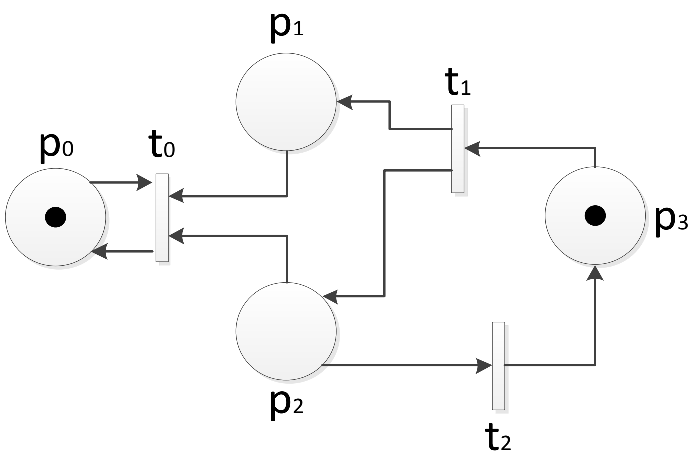

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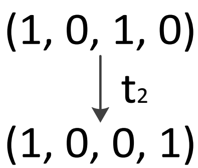

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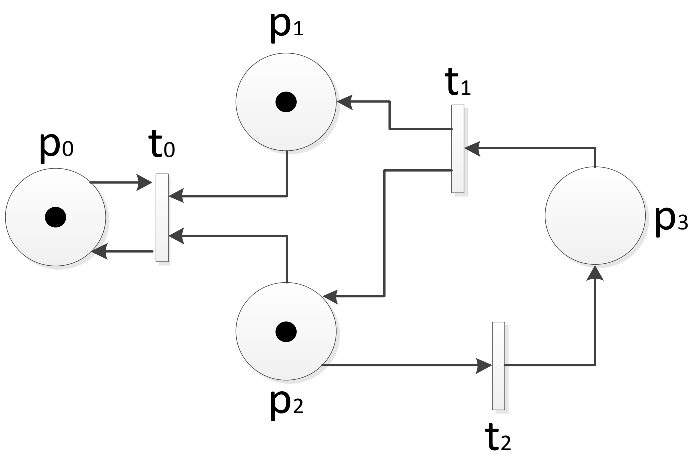

Reachability Tree:

(1, 0, 1, 0)

↓ t_2

(1, 0, 0, 1)

↓ t_1

(1, 1, 1, 0)

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Reachability Tree

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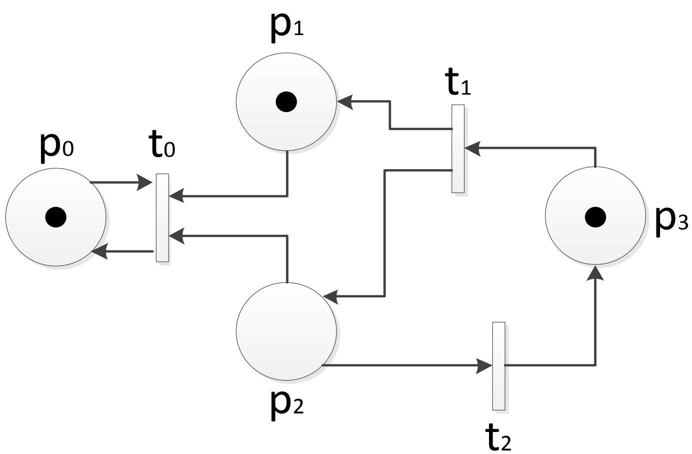

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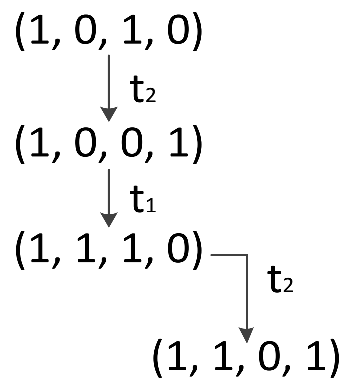

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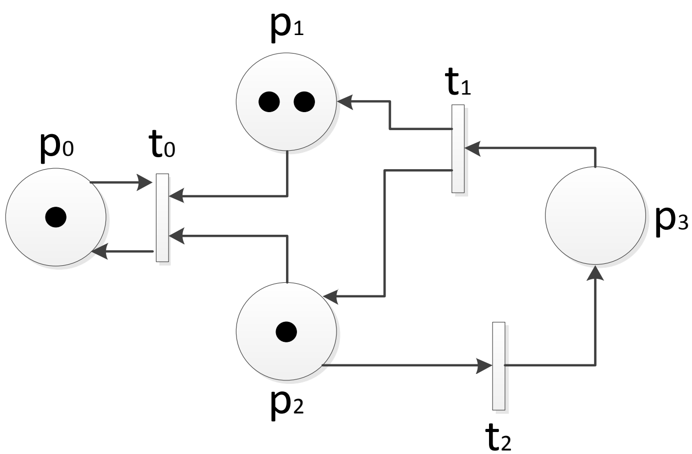

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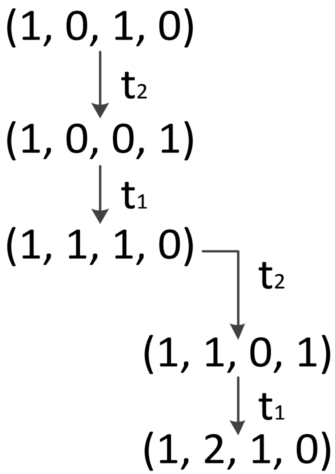

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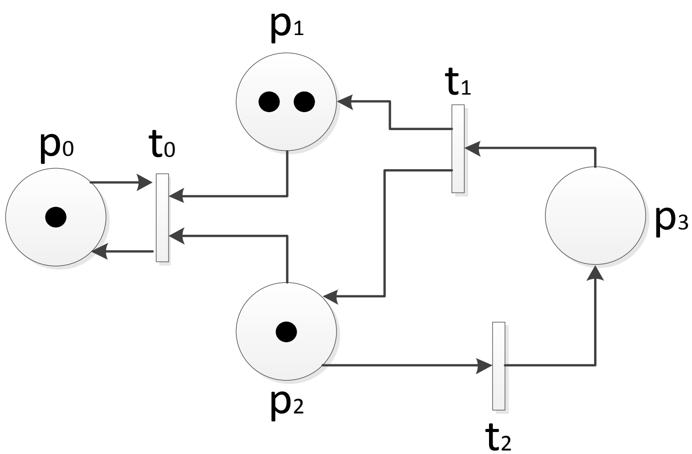
Coverability Tree

Petri Net:



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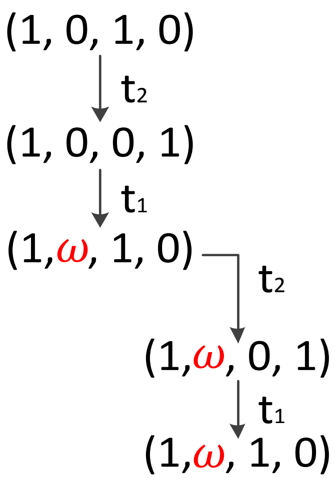

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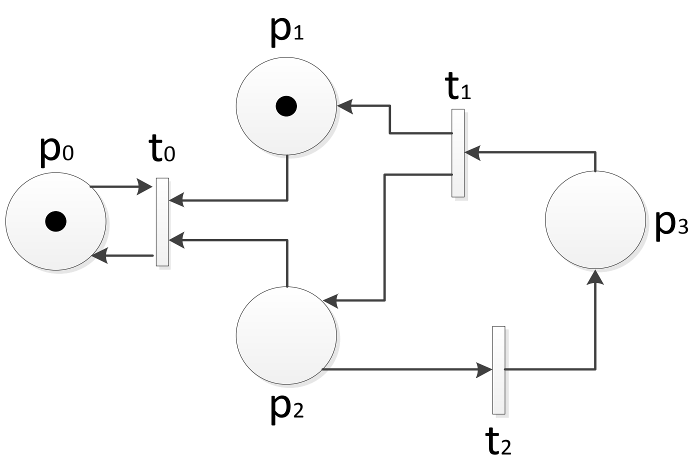

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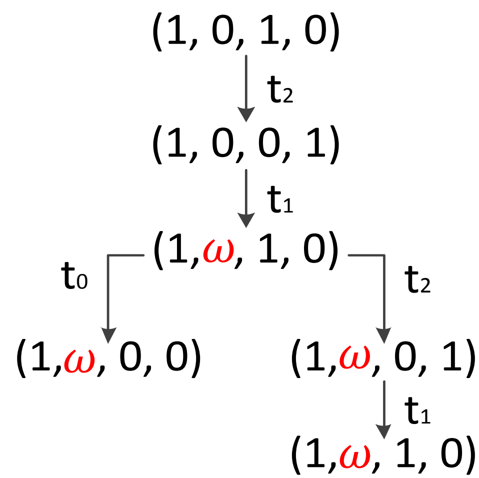
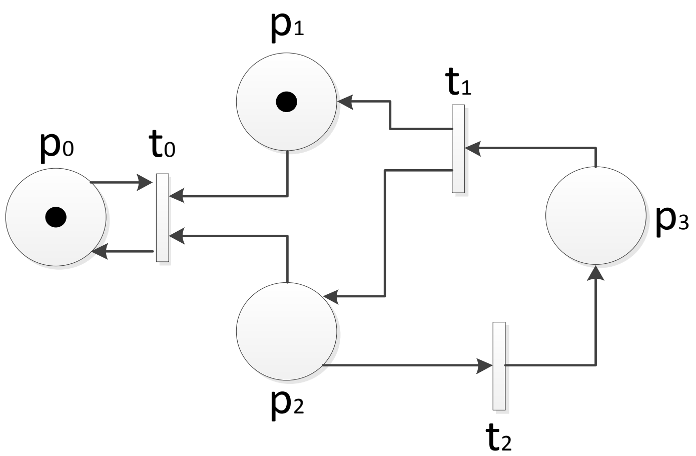


Example taken from [2]

Boundedness

Unbounded Petri Net:



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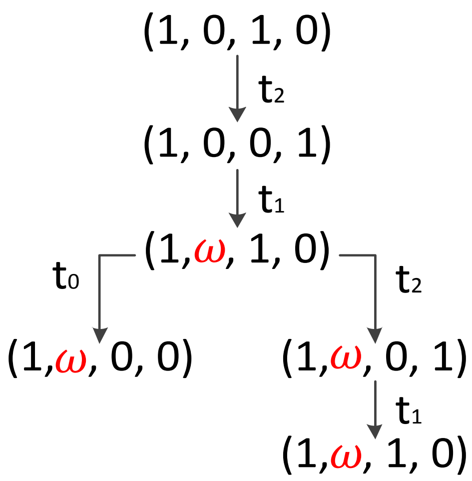


A Petri net ist unbounded if and only if there exists a reachable marking M and a sequence of transitions σ such that:

- $M \rightarrow_{\sigma} M + L$, where L is a non-zero label

Boundedness

A Petri net ist **unbounded** if and only if there exists a reachable marking M and a sequence of transitions σ such that:

$M \rightarrow_{\sigma} \; M + L$, where L is a non-zero label

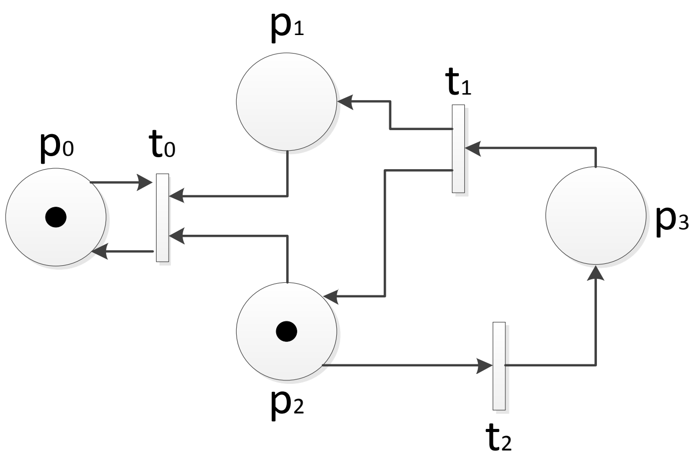
- Decidable in $2^{cn \log n}$

Subclasses of Petri Nets wrt. boundness:

- 1-bounded (or 1-safe)
- k-bounded (or bounded)

Conflicts

Petri Net:



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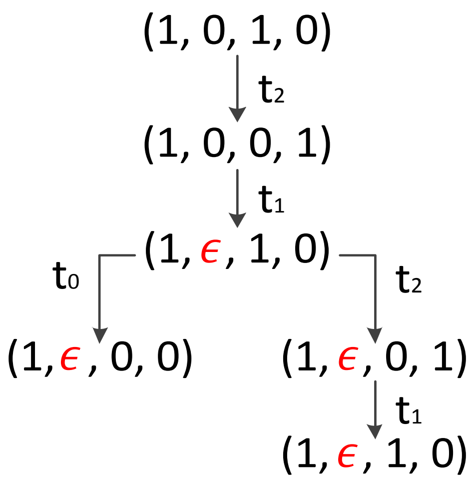
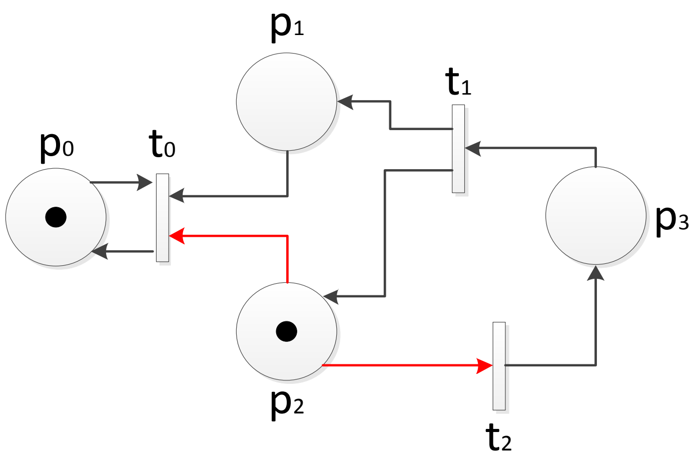
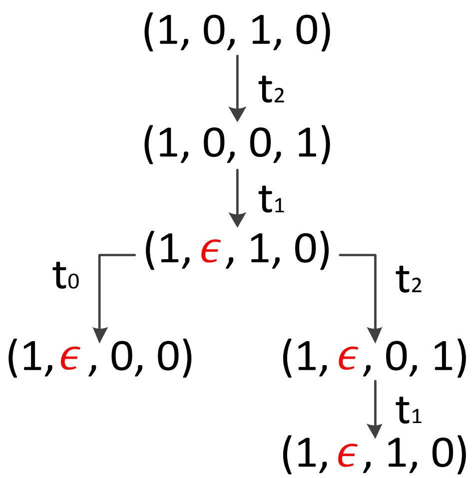


Conflicts

Conflicted Petri Net:



Coverability Tree:

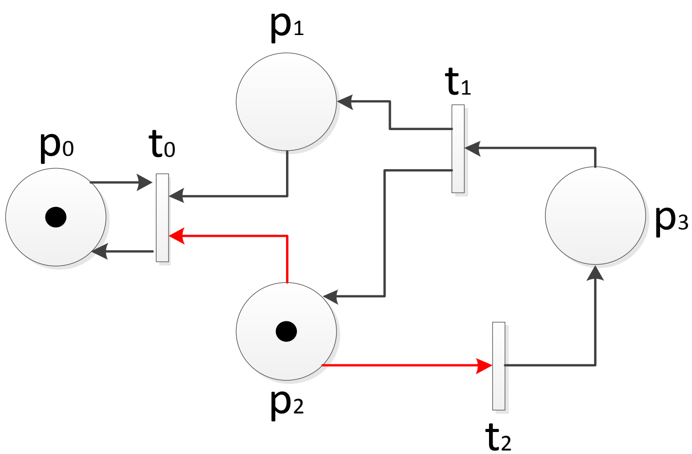


Informal Definition:

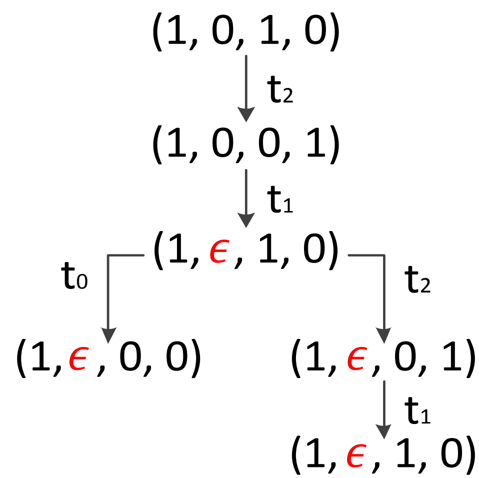
A Petri has a **conflicting configuration** if two or more transitions are enabled and firing one transition disables the other transitions.

Conflicts

Conflicted Petri Net:



Coverability Tree:



Informal Definition:

A Petri is **conflict free** if it has no conflicting configurations.

Reachability Problem

Is a certain marking reachable in a Petri Net?

- I.e. can a system reach some bad state?

Analysis:

1. What about the reachability problem for Petri Nets in general?
2. Are there certain properties which allow for deciding the reachability problem efficiently?

General analysis tool: The reachability/coverability tree.

Reachability Problem

Decide whether a marking M can be reached for a Petri Net N.

- **In general**, double exponential time.
- **1-safe**, PSPACE-complete.
- **Conflict-free**, NP-complete.
- **Bounded and conflict-free**, polynominal.

Conclusion

- Petri Nets allow for a **concise representation** of concurrent processes.
- In general, Petri Nets recognize **all regular languages**, **some context-free languages** and **some context-sensitive languages**.
- In general, certain **decision problems** are **harder** than in other automata models.
- The **expressiveness power** of Petri Nets can be **restricted** in order to **decide** certain **decision problems efficiently**.

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The End

Thank you for your attention!