



J. Hoenicke

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## Tutorials for Decision Procedures Exercise sheet 9

### Exercise 1: DP for quantifier-free $T_{\text{cons}} \cup T_E$

Apply the decision procedure for quantifier-free  $T_{\text{cons}} \cup T_E$  to decide satisfiability of the following  $\Sigma_{\text{cons}} \cup \Sigma_E$ -formulae:

- (a)  $y = \text{cons}(\text{cdr}(x), \text{car}(x)) \wedge x = \text{cons}(\text{car}(y), \text{cdr}(y))$
- (b)  $y = \text{cons}(\text{cdr}(x), \text{car}(x)) \wedge x = \text{cons}(\text{car}(y), \text{cdr}(y)) \wedge \text{car}(x) \neq \text{cdr}(x)$
- (c)  $\neg \text{atom}(x) \wedge y = \text{cons}(\text{cdr}(x), \text{car}(x)) \wedge z = \text{cons}(\text{cdr}(y), \text{car}(y)) \wedge z \neq x$

### Exercise 2: Satisfying Interpretation for $T_{\text{cons}}$

Take the result of applying the congruence closure algorithm on exercise 1.(a):

$$y = \text{cons}(\text{cdr}(x), \text{car}(x)) \wedge x = \text{cons}(\text{car}(y), \text{cdr}(y)).$$

Give a satisfying Interpretation  $I$ . Under this interpretation, what is the value of the term  $\text{cons}(x, \text{cons}(\text{car}(x), \text{car}(y)))$ ?

### Exercise 3: DP for quantifier-free $T_A$

Apply the decision procedure for quantifier-free  $T_A$  to decide satisfiability of the following  $\Sigma_A$ -formulae:

- (a)  $a \langle i \triangleleft e \rangle [j] = e \wedge j \neq i$
- (b)  $a \langle i \triangleleft e \rangle [j] = f \wedge a[j] \neq f$
- (c)  $a \langle i \triangleleft e \rangle [j] = f \wedge i = j \wedge e \neq f$
- (d)  $a \langle i \triangleleft e \rangle \langle j \triangleleft f \rangle [i] = g \wedge e \neq g$
- (e)  $a \langle i \triangleleft e \rangle \langle j \triangleleft f \rangle [i] = g \wedge e \neq g \wedge j \neq i$