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Tutorials for Decision Procedures Exercise sheet 10

Exercise 1: Decision Procedure for T_A

Apply the decision procedure for arrays to decide *validity* on the following T_A -formulae:

- (a) $j = k \rightarrow \forall i. a\langle j \triangleleft v \rangle[i] = a\langle k \triangleleft v \rangle[i]$
- (b) $(\forall i. a[i] = b[i]) \rightarrow (\forall i. a\langle j \triangleleft v \rangle[i] = b\langle j \triangleleft v \rangle[i])$
- (c) $\exists j. a\langle i \triangleleft v \rangle[j] = v$
- (d) $\forall j. a\langle i \triangleleft v \rangle[j] = v$

Exercise 2: Decision Procedure for $T_A^{\mathbb{Z}}$

Check *validity* of the formulae

- (a) $sorted(a, \ell, u) \rightarrow partitioned(a, \ell, k, k, u)$
- (b) $sorted(a, \ell, k) \wedge sorted(a, k, u) \rightarrow sorted(a, \ell, u)$

where *sorted* and *partitioned* are defined as usual:

$$sorted(a, \ell, u) : \forall i, j. \ell \leq i \leq j \leq u \rightarrow a[i] \leq a[j]$$

$$partitioned(a, \ell_1, u_1, \ell_2, u_2) : \forall i, j. \ell_1 \leq i \leq u_1 \wedge \ell_2 \leq j \leq u_2 \rightarrow a[i] \leq a[j]$$

Exercise 3: Correctness of DP for $T_A^{\mathbb{Z}}$

Let I be an interpretation. Prove for $F[\bar{i}] : expr \leq expr$ that $I \models F[\bar{i}] \rightarrow F[\bar{t}]$, where $\bar{i} = (i_1, \dots, i_n)$ and \bar{t} is the vector $\bar{t} = (t_1, \dots, t_n) \in \mathcal{I}^n$ with $\alpha_I[t_k] = proj_{\mathcal{I}}(\alpha_I[i_k])$ (in the notation of the book $\bar{t} = \text{proj}_{\mathcal{I}}(\bar{i})$). The expression *expr* is either a universal variable i_k or a *peexpr*. Note that \mathcal{I} contains all *peexpr* and that

$$proj_{\mathcal{I}}(v) = \begin{cases} \max\{\alpha_I[t] \mid t \in \mathcal{I} \wedge \alpha_I[t] \leq v\} & \text{if for some } t \in \mathcal{I}: \alpha_I[t] \leq v \\ \min\{\alpha_I[t] \mid t \in \mathcal{I}\} & \text{otherwise} \end{cases}$$