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## Tutorials for Decision Procedures Exercise sheet 3

### Exercise 1: Prenex Normal Form

Transform the following formula into prenex normal form:

$$F : \left( \forall z. \left( \left( \forall x. q(x, z) \right) \rightarrow p(x, g(y), z) \right) \right) \wedge \neg \left( \forall z. \neg \left( \forall x. q(f(x, y), z) \right) \right)$$

### Exercise 2: Correctness of PNF

Show that the formula  $F$  from exercise 1 and the formula  $F'$  in prenex form that you computed are equivalent by proving  $F \leftrightarrow F'$  with a semantic tableaux.

### Exercise 3: Semantic Tableaux

Use the semantic tableaux method to prove the validity of the following formulae.

- (a)  $(\forall x. (p(x) \rightarrow q(a))) \wedge (\exists x. p(x)) \rightarrow q(a)$
- (b)  $(\forall x. p(f(x))) \wedge (\forall y. (q(y) \rightarrow \neg p(f(y)))) \rightarrow \neg q(b)$
- (c)  $(\forall x, y. (p(x, y) \vee p(y, x))) \rightarrow \forall z. p(z, z)$
- (d)  $\forall y. \exists x. (p(x) \rightarrow p(y))$
- (e)  $\exists x. \forall y. (p(x) \rightarrow p(y))$