Tutorials for Decision Procedures
Exercise sheet 5

Exercise 1: Induction in $T_{PA}$
Prove the $T_{PA}$-validity of the following formula using the semantic tableaux.

$$\forall x. 0 + x = x$$

Write down each proof step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as $T_{PA}$-valid. Note, that you may not assume commutativity, associativity, etc. Only use the Peano-axioms and the axioms from $T_E$. You need the induction axiom.

Exercise 2: Semantic Argument in $T_R$
Show the $T_R$-validity of the following formula using the semantic argument.

$$\forall x. x \cdot x \geq 0$$

Write down every step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as $T_R$-valid. Additionally, you may use the following derived facts without proving them:

$$\forall x. 0 \geq x \rightarrow -x \geq 0$$
$$\forall x. (-x) \cdot (-x) = x \cdot x$$

Exercise 3: Integer Arithmetic
Consider the $T_Z$-formula $F : \exists x. \forall y. \neg(y + 1 = x)$.

(a) Convert $F$ into an equisatisfiable $T_N$-formula $G$.

(b) Prove unsatisfiability of $G$ using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

(c) Prove validity of the $T_N$-formula $\exists x. \forall y. \neg(y + 1 = x)$. 