Decision Procedures

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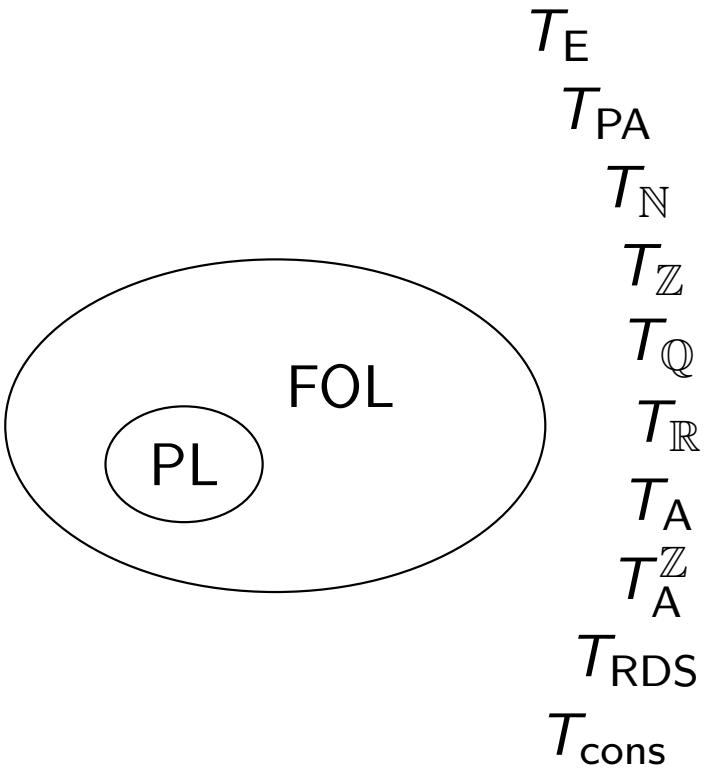
Conclusion

Topics

Lecture	Topics
1–3	Propositional Logic
4–5	First-Order Logic
6–8	First-Order Theories
9–10	Quantifier Elimination for $T_{\mathbb{Z}}$ and $T_{\mathbb{Q}}$
11–12	Dutertre–de Moura Algorithm ($T_{\mathbb{Q}}$)
13–14	Congruence Closure Algorithm (T_{E}, T_{cons}, T_{A})
15–16	DP for Array Property Fragment
17–18	Nelson-Oppen
19–20	DPLL(T) with Learning
21–22	Program Correctness
23	Interpolation

Logics

PL	Propostional Logic	Lec. 1–3
FOL	First-Order Logic	Lec. 4–5
T_x	Theories	Lec. 6–8



Theories and their DPs

Theory	Full	Array Prop.	Quant. free	Conj. quant. free
T_{E}	✘	-	✔ (19–20)	✔ (13)
T_{PA}	✘	-	✘	✘
$T_{\mathbb{Z}}$	✔ (10)	-	✔	✔
$T_{\mathbb{Q}}$	✔ (9)	-	✔ (19–20)	✔ (11–12)
$T_{\mathbb{R}}$	✔ (-)	-	✔	✔
T_{A}	✘	✔ (15–16)	✔	✔ (15)
$T_{\mathsf{A}}^{\mathbb{Z}}$	✘	✔ (15–16)	✔	✔
T_{RDS}	✘	-	✔	✔
T_{cons}	✘	-	✔	✔ (14)
$T_1 \cup T_2$	-	-	✔ (-)	✔ (17–18)

Propositional Logic

- What is an atom, a literal, a formula.
- What is an interpretation?
- What does $I \models F$ mean, how do we compute it.
- What is satisfiability, validity.
- What is the duality between satisfiable and valid?
- What is the semantic argument?
- Write down the proof rules.
- How can we prove $P \wedge Q \rightarrow P \vee \neg Q$?
- What is $\Leftrightarrow$ (equivalent) and $\Rightarrow$ (implies).
- What Normal Forms do you know (NNF, DNF, CNF)?
- How to convert formulae into normal form.

DPLL for Propositional Logic

- What is a Decision Procedure?
- What is equisatisfiability; why is it useful?
- How to convert to CNF with polynomial time complexity?
- What is a clause?
- What does DPLL stand for?
- What is Boolean Constraint Propagation (BCP) (aka. Unit Propagation).
- What is Pure Literal Propagation (PL).
- Why is the DPLL algorithm correct?
- What is the worst case time complexity of DPLL?

First-Order Logic

- What is a variable, a constant, a function (symbol), a predicate (symbol), a term, an atom, a literal, a formula?
- How do first-order logic and predicate logic relate?
- What is an interpretation in FOL?
- Why is D_I non-empty?
- What does α_I assign?
- What is an x-variant of an interpretation?
- How do we compute whether $I \models F$?
- What is satisfiability, validity?
- What are the additional rules in the Semantic Argument (version of lecture 4)?
- Soundness and Completeness of semantic argument.
- What is a Hintikka set?
- Normal forms. What is PNF (prenex normal form)?
- Is validity for FOL decidable?

First-Order Theories

- What is a theory?
- What is a signature Σ?
- What do T-valid and emph T-satisfiable mean?
- What is T-equivalent?
- What is a decision procedure for a theory?
- What is a fragment of a theory?
- What are the most common fragments (quantifier-free, conjunctive)?
- What theories do you know?
- What are their axioms?
- What fragments of these theories are decidable?
- Bonus Question: Is there any closed formulae in T_{PA} that is satisfiable but not valid? What about $T_{\mathbb{Z}}$, $T_{\mathbb{Q}}$?

Quantifier Elimination

- What is Quantifier Elimination?
- Does $T_{\mathbb{Z}}$ admit quantifier elimination? What does it mean?
- Why is it enough to eliminate one existential quantifiers over a quantifier-free formula?
- How can we eliminate more than one quantifier?
- What is $\widehat{T_{\mathbb{Z}}}$?
- What is Cooper's method?
- What is Ferrante and Rackoff's method ($T_{\mathbb{Q}}$)?
- What is the Array Property Fragment?
- What do all quantifier elimination methods of the lecture have in common?
- What is the complexity of quantifier elimination?
- Why is quantifier elimination a decision procedure?

Simplex Based Algorithm

- Which theory does the Algorithm of Dutertre and de Moura decide?
- How does the algorithm work?
- How can we convert an arbitrary formula to the required format for the algorithm?
- What is the tableaux?
- What is a pivot step?
- Does the algorithm terminate?
- What is the complexity?

Congruence Closure Algorithm

- What is the congruence closure algorithm?
- How does it work for T_E?
- What are the data structures; what are the operations?
- What complexity does the algorithm have?
- What are the extensions for T_{cons}?
- What is the complexity?
- How did we prove correctness of the decision procedure?

Decision Procedure for T_A

- How does the DP for quantifier-free fragment of T_A work?
- What is the complexity?
- What is $T_A^=$?

Array Property Fragment

- What is the Array Property Fragment of $T_A / T_A^=$?
- Why are there so many restrictions?
- What are the transformation steps?
- How are quantifier eliminated?
- What is λ and why is it necessary?
- Why is the decision procedure correct?
- What is the Array Property Fragment of $T_A^{\mathbb{Z}}$?
- What are differences to T_A?
- Why do we not need λ for $T_A^{\mathbb{Z}}$?
- Why is the decision procedure correct?
- How can we check this fragment?

Nelson-Oppen

- What is the Nelson-Oppen procedure?
- For what theories does it work? For which fragment of the theory?
- What is a stably infinite theory?
- Why is it important that theories are stably infinite?
- What are the two phases of Nelson-Oppen?
- What is the difference between the non-deterministic and deterministic variant of Nelson-Oppen?
- What is a convex theory?
- What is the emphcomplexity of the deterministic version for convex/non-convex theories?

DPLL(T)

- How can we extend the DPLL algorithm to decide T-satisfiability.
- What is a minimal unsatisfiable core?
- How can we compute it efficiently?
- What is the relation between min. unsat. core and conflict clause?
- Why is the algorithm correct, why does it terminate?
- How can we extend it two more than one theory?
- What is the relation to Nelson-Oppen?

Program Correctness

- What is a specification?
- What types of specification are in a typical program? (Precondition, postcondition, loop invariants, assertions)
- When is a procedure correct (partial/total correctness)?
- What is a basic path? Why is it useful?
- How do we prove correctness of a basic path?
- What is a verification condition?
- What is the weakest precondition?
- How do we compute weakest precondition?
- What is a P-invariant annotation, what is a P-inductive annotation?
- Why are we interested in P-inductive annotations?
- What is a ranking function? Why do we need it?
- What is a well-founded relation?
- How do we prove total correctness?

General hints for exam

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- You should learn definitions (formally).
  This includes the rules (semantic argument, DPLL with learning).
- You should understand them (informally).
- You should know important theorems.
- Knowing the proofs is a plus. Don't loose yourself in the details!
- You should be able to apply the decision procedures.
  Do the exercises! Invent some new exercises and solve them!
- You should know some examples/counter-examples,
  e.g., why is λ necessary?
- When you feel well prepared, check if you can answer the questions in this slide set.
- When learning, do not leave out a whole topic completely!
- Learn in a group. Ask question to each other and answer them as if you were in the exam.

Organisation

- There will be only oral exams for this lecture.
- You should have officially registered at the Prüfungsamt.
- The exams will be in September.