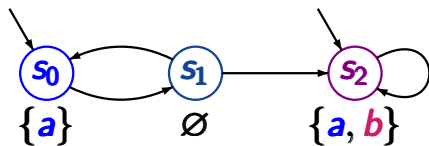


# Correct or wrong ?

LTLSP3.1-7

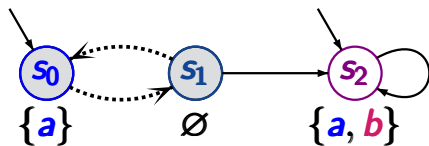


$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

# Correct or wrong ?

LTLSP3.1-7

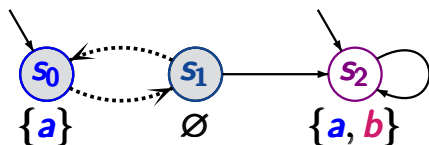


$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

# Correct or wrong ?

LTLSF3.1-7



$$AP = \{a, b\}$$

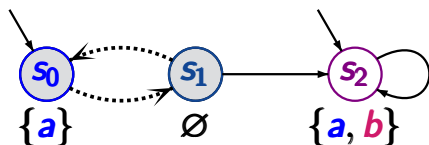
path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$\text{trace}(\pi) = (\{a\} \emptyset)^\omega$$

$\pi \models a \cup b$  ?

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

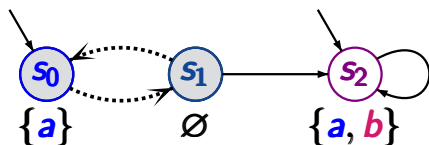
$$\text{trace}(\pi) = (\{a\} \emptyset)^\omega$$

$\pi \not\models a \cup b$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

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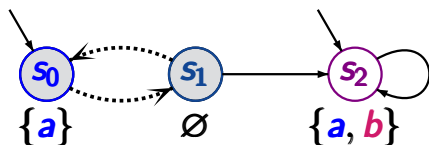
$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b) ?$$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$\text{trace}(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

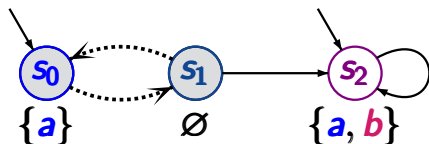
as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

# Correct or wrong ?

LTLSP3.1-7



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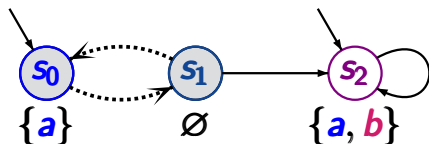
$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b ?$$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$trace(\pi) = (\{a\} \emptyset)^\omega$$

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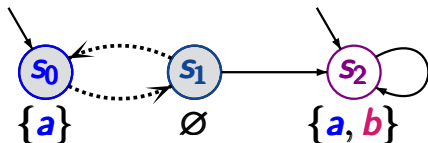
$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$



# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$\text{trace}(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

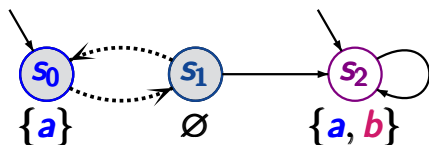
$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$

$$\pi \models \square a ?$$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$trace(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b$$

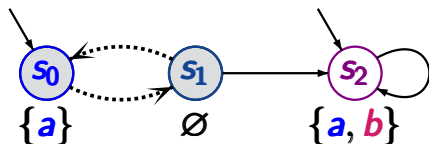
as  $s_0 \models \neg b$

$$\pi \not\models \square a$$

as  $s_1 \not\models a$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$trace(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$

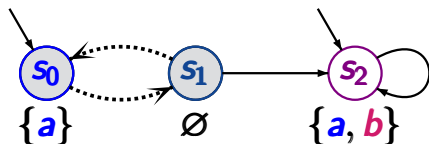
$$\pi \not\models \square a$$

as  $s_1 \not\models a$

$$\pi \models \square \diamond a ?$$

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$trace(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$

$$\pi \not\models \square a$$

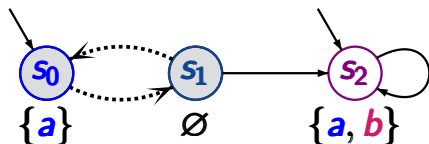
as  $s_1 \not\models a$

$$\pi \models \square \diamond a$$

as  $\square \diamond \hat{=}$  infinitely often

# Correct or wrong ?

LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$trace(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$

$$\pi \not\models \square a$$

as  $s_1 \not\models a$

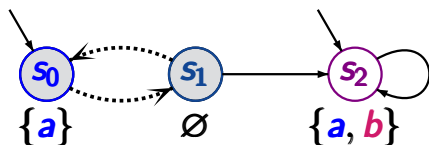
$$\pi \models \square \diamond a$$

as  $\square \diamond \hat{=}$  infinitely often

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LTLSP3.1-7



$$AP = \{a, b\}$$

path  $\pi = s_0 s_1 s_0 s_1 s_0 s_1 \dots$

$$\text{trace}(\pi) = (\{a\} \emptyset)^\omega$$

$$\pi \not\models a \cup b$$

as  $s_0 \not\models b$  and  $s_1 \not\models a \vee b$

$$\pi \models \diamond b \rightarrow (a \cup b)$$

as  $\pi \not\models \diamond b$

$$\pi \models \bigcirc \bigcirc \neg b$$

as  $s_0 \models \neg b$

$$\pi \not\models \square a$$

as  $s_1 \not\models a$

$$\pi \models \square \diamond a$$

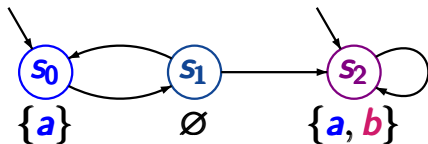
as  $\square \diamond \hat{=}$  infinitely often

$$\pi \not\models \diamond \square a$$

as  $\diamond \square \hat{=}$  eventually forever

# Which formulas hold for $\mathcal{T}$ ?

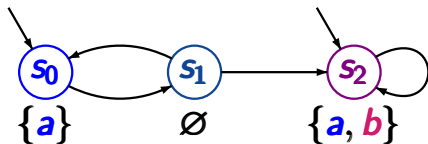
LTLSF3.1-11



$$AP = \{a, b\}$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



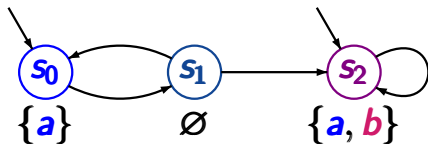
$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$



# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



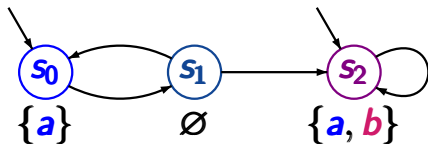
$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

$$\text{as } s_0 \models a \text{ and } s_2 \models a$$

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LTLSF3.1-11



$$AP = \{a, b\}$$

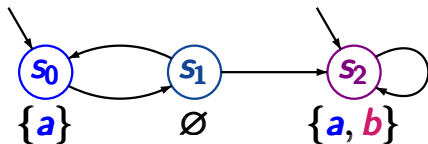
$$\mathcal{T} \models a$$

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$$\mathcal{T} \models \diamond \square a$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

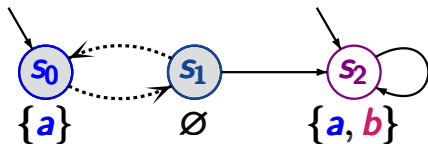
$$\mathcal{T} \models a$$

$$\text{as } s_0 \models a \text{ and } s_2 \models a$$

$$\mathcal{T} \not\models \diamond \square a$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

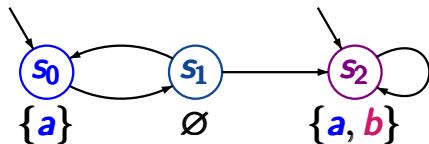
$$\text{as } s_0 \models a \text{ and } s_2 \models a$$

$$\mathcal{T} \not\models \diamond \Box a$$

$$\text{as } s_0 s_1 s_0 s_1 \dots \not\models \diamond \Box a$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

as  $s_0 \models a$  and  $s_2 \models a$

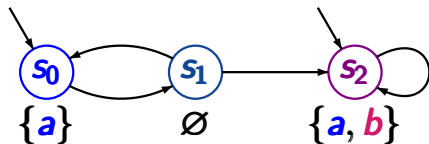
$$\mathcal{T} \not\models \diamond \Box a$$

as  $s_0 s_1 s_0 s_1 \dots \not\models \diamond \Box a$

$$\mathcal{T} \models \diamond \Box b \vee \Box \diamond (\neg a \wedge \neg b)$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

$$\text{as } s_0 \models a \text{ and } s_2 \models a$$

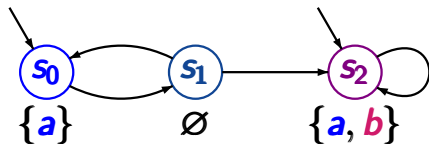
$$\mathcal{T} \not\models \diamond \Box a$$

$$\text{as } s_0 s_1 s_0 s_1 \dots \not\models \diamond \Box a$$

$$\mathcal{T} \models \diamond \Box b \vee \Box \diamond (\neg a \wedge \neg b) \quad \text{as } s_2 \models b, s_1 \not\models a, b$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

$$\text{as } s_0 \models a \text{ and } s_2 \models a$$

$$\mathcal{T} \not\models \diamond \Box a$$

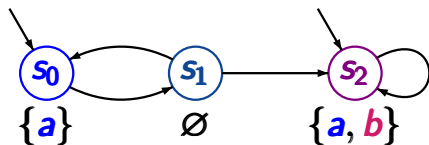
$$\text{as } s_0 s_1 s_0 s_1 \dots \not\models \diamond \Box a$$

$$\mathcal{T} \models \diamond \Box b \vee \Box \diamond (\neg a \wedge \neg b) \quad \text{as } s_2 \models b, s_1 \not\models a, b$$

$$\mathcal{T} \models \Box (a \rightarrow (\bigcirc \neg a \vee b))$$

# Which formulas hold for $\mathcal{T}$ ?

LTLSF3.1-11



$$AP = \{a, b\}$$

$$\mathcal{T} \models a$$

as  $s_0 \models a$  and  $s_2 \models a$ 

$$\mathcal{T} \not\models \diamond \Box a$$

as  $s_0 s_1 s_0 s_1 \dots \not\models \diamond \Box a$ 

$$\mathcal{T} \models \diamond \Box b \vee \Box \diamond (\neg a \wedge \neg b)$$

as  $s_2 \models b$ ,  $s_1 \not\models a, b$ 

$$\mathcal{T} \models \Box (a \rightarrow (\bigcirc \neg a \vee b))$$

as  $s_2 \models b$ ,  $s_0 \models \bigcirc \neg a$



# Correct or wrong?

LTLSF3.1-12

For each path  $\pi$  we have:  $\pi \models \varphi$  or  $\pi \models \neg\varphi$

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**correct**, since  $\pi \models \neg\varphi$  iff  $\pi \not\models \varphi$

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For each state  $s$  we have:  $s \models \varphi$  or  $s \models \neg\varphi$

# Correct or wrong?

LTLSF3.1-12

For each path  $\pi$  we have:  $\pi \models \varphi$  or  $\pi \models \neg\varphi$

**correct**, since  $\pi \models \neg\varphi$  iff  $\pi \not\models \varphi$

For each state  $s$  we have:  $s \models \varphi$  or  $s \models \neg\varphi$

**wrong.**

# Correct or wrong?

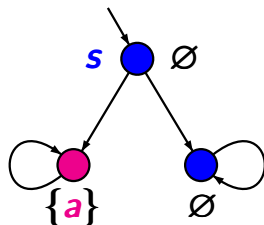
LTLSF3.1-12

For each path  $\pi$  we have:  $\pi \models \varphi$  or  $\pi \models \neg\varphi$

correct, since  $\pi \models \neg\varphi$  iff  $\pi \not\models \varphi$

For each state  $s$  we have:  $s \models \varphi$  or  $s \models \neg\varphi$

wrong.



$s \not\models \diamond a$  and  $s \not\models \neg \diamond a$



LTL formulas over  $AP = \{wait_1, crit_1, wait_2, crit_2\}$

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- the mutual exclusion property

$$\varphi_{mutex} = ?$$



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$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

LTL formulas over  $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

- “every process enters the critical section infinitely often”

$$\varphi_{live} = ?$$

LTL formulas over  $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

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$$\varphi_{live} = \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

LTL formulas over  $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

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- “every process enters the critical section infinitely often”

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- starvation freedom  
“every waiting process finally enters its critical section”

$$\varphi_{sf} = ?$$

LTL formulas over  $AP = \{wait_1, crit_1, wait_2, crit_2\}$

- the mutual exclusion property

$$\varphi_{mutex} = \Box(\neg crit_1 \vee \neg crit_2)$$

- “every process enters the critical section infinitely often”

$$\varphi_{live} = \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

- starvation freedom

“every waiting process finally enters its critical section”

$$\varphi_{sf} = \Box(wait_1 \rightarrow \Diamond crit_1) \wedge \Box(wait_2 \rightarrow \Diamond crit_2)$$

Provide an LTL formula over  $AP = \{a, b\}$  for ... LTLSF3.1-17

Provide an LTL formula over  $AP = \{a, b\}$  for ...

LTLSF3.1-17

- set of all words  $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$  such that:

$$\forall i \geq 0. ( a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} )$$

- set of all words  $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$  such that:

$$\forall i \geq 0. ( a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} )$$

$$\forall j \geq 0. ( b \in A_j \vee a \notin A_{j+1} )$$



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$$\forall i \geq 0. ( a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} )$$

$$\forall j \geq 0. ( b \in A_j \vee a \notin A_{j+1} )$$

$$\hat{=} \text{Words}( \Box( b \vee \bigcirc \neg a ) )$$

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$$\forall j \geq 0. ( b \in A_j \vee a \notin A_{j+1} )$$

$$\cong \text{Words}( \Box( b \vee \bigcirc \neg a ) )$$

- set of all words of the form

$$\{b\}^{n_1} \{a\} \{b\}^{n_2} \{a\} \{b\}^{n_3} \{a\} \dots$$

where  $n_1, n_2, n_3, \dots \geq 0$

- set of all words  $A_0 A_1 A_2 \dots \in (2^{AP})^\omega$  such that:

$$\forall i \geq 0. ( a \in A_i \implies i \geq 1 \wedge b \in A_{i-1} )$$

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$$\cong \text{Words}( \Box( b \vee \bigcirc \neg a ) )$$

- set of all words of the form

$$\{b\}^{n_1} \{a\} \{b\}^{n_2} \{a\} \{b\}^{n_3} \{a\} \dots$$

where  $n_1, n_2, n_3, \dots \geq 0$

$$\cong \text{Words}( \Box( (b \wedge \neg a) \cup (a \wedge \neg b) ) )$$

# Correct or wrong?

LTLSF3.1-26

$$\diamond(\varphi \vee \psi) \equiv \diamond\varphi \vee \diamond\psi$$

# Correct or wrong?

LTLSF3.1-26

$$\diamond(\varphi \vee \psi) \equiv \diamond\varphi \vee \diamond\psi$$

**correct**

$$\diamond(\varphi \vee \psi) \equiv \diamond\varphi \vee \diamond\psi$$

correct

---

$$\diamond(\varphi \wedge \psi) \equiv \diamond\varphi \wedge \diamond\psi$$

# Correct or wrong?

LTLSF3.1-26

$$\diamond(\varphi \vee \psi) \equiv \diamond\varphi \vee \diamond\psi$$

correct

---

$$\diamond(\varphi \wedge \psi) \equiv \diamond\varphi \wedge \diamond\psi$$

wrong,  
e.g.,



$$\models \diamond b \wedge \diamond a$$
$$\not\models \diamond(b \wedge a)$$

# Correct or wrong?

LTLSF3.1-26

$$\diamond(\varphi \vee \psi) \equiv \diamond\varphi \vee \diamond\psi$$

correct

---

$$\diamond(\varphi \wedge \psi) \equiv \diamond\varphi \wedge \diamond\psi$$

wrong,

e.g.,



$$\models \diamond b \wedge \diamond a$$

$$\not\models \diamond(b \wedge a)$$

---

similarly:  $\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$

$$\Box(\varphi \vee \psi) \not\equiv \Box\varphi \vee \Box\psi$$



# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\psi \equiv \diamond\psi$$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

correct

$$\begin{array}{l} \exists k \geq 0 \quad \exists j \geq k \quad \text{s.t.} \quad A_j A_{j+1} A_{j+2} \dots \models \varphi \\ \text{iff} \quad \quad \quad \exists j \geq 0 \quad \text{s.t.} \quad A_j A_{j+1} A_{j+2} \dots \models \varphi \end{array}$$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

correct      analogously:       $\square\square\varphi \equiv \square\varphi$

$$\begin{array}{l} \exists k \geq 0 \quad \exists j \geq k \quad \text{s.t. } A_j A_{j+1} A_{j+2} \dots \models \varphi \\ \text{iff} \quad \quad \quad \exists j \geq 0 \quad \text{s.t. } A_j A_{j+1} A_{j+2} \dots \models \varphi \end{array}$$

$$\begin{array}{l} \forall k \geq 0 \quad \forall j \geq k \quad \text{s.t. } A_j A_{j+1} A_{j+2} \dots \models \varphi \\ \text{iff} \quad \quad \quad \forall j \geq 0 \quad \text{s.t. } A_j A_{j+1} A_{j+2} \dots \models \varphi \end{array}$$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

**correct**      analogously:  $\square\square\varphi \equiv \square\varphi$

---

$$\bigcirc\square\varphi \equiv \square\bigcirc\varphi$$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\psi \equiv \diamond\psi$$

**correct**      analogously:       $\square\square\psi \equiv \square\psi$

---

$$\bigcirc\square\psi \equiv \square\bigcirc\psi$$

**correct**

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

correct      analogously:  $\square\square\varphi \equiv \square\varphi$

---

$$\bigcirc\square\varphi \equiv \square\bigcirc\varphi \stackrel{\text{def}}{=} \psi$$

correct

note that:

$A_0 A_1 A_2 \dots \models \psi$  iff  $A_i A_{i+1} \dots \models \varphi$  for all  $i \geq 1$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

**correct**      analogously:  $\square\square\varphi \equiv \square\varphi$

---

$$\bigcirc\square\varphi \equiv \square\bigcirc\varphi$$

**correct**

---

$$\diamond\square\varphi \equiv \square\diamond\varphi$$

# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

**correct**      analogously:       $\square\square\varphi \equiv \square\varphi$

---

$$\bigcirc\square\varphi \equiv \square\bigcirc\varphi$$

**correct**

---

$$\diamond\square\varphi \equiv \square\diamond\varphi$$

$\square\diamond \hat{=}$  infinitely often  
 $\diamond\square \hat{=}$  eventually forever



# Correct or wrong?

LTLSF3.1-27

$$\diamond\diamond\varphi \equiv \diamond\varphi$$

**correct** analogously:  $\square\square\varphi \equiv \square\varphi$

---

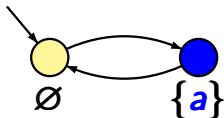
$$\bigcirc\square\varphi \equiv \square\bigcirc\varphi$$

**correct**

---

$$\diamond\square\varphi \equiv \square\diamond\varphi$$

**wrong**



$\square\diamond \hat{=}$  infinitely often

$\diamond\square \hat{=}$  eventually forever

$\models \square\diamond a$

$\not\models \diamond\square a$

$$\varphi \mathbf{W} \psi \stackrel{\text{def}}{=} (\varphi \mathbf{U} \psi) \vee \square \varphi$$

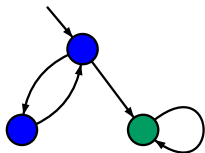
deriving “always” and “until” from “weak until”:

$$\square \varphi \equiv \varphi \mathbf{W} \textit{false}$$

$$\varphi \mathbf{U} \psi \equiv (\varphi \mathbf{W} \psi) \wedge \diamond \psi$$

Does  $\mathcal{T} \models aWb$  hold ?

LTLSF3.1-32

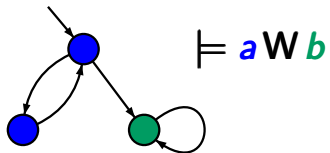


●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

Does  $\mathcal{T} \models aWb$  hold ?

LTLSF3.1-32

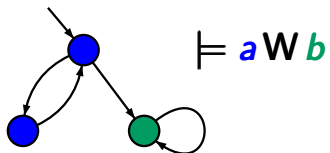


●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

Does  $\mathcal{T} \models aWb$  hold ?

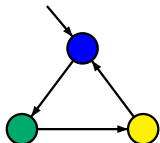
LTLSF3.1-32



●  $\hat{=} \{a\}$

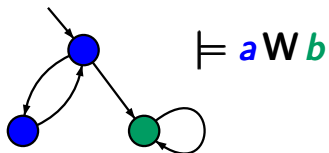
●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$



Does  $\mathcal{T} \models aWb$  hold ?

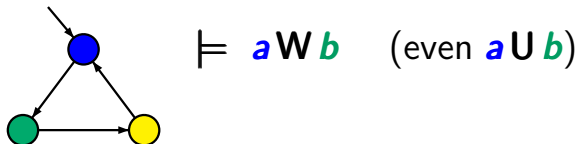
LTLSF3.1-32



●  $\hat{=} \{a\}$

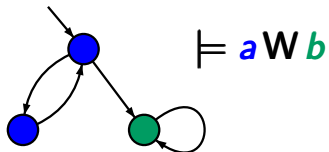
●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$



Does  $\mathcal{T} \models aWb$  hold ?

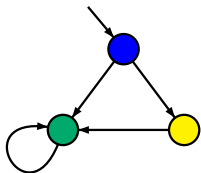
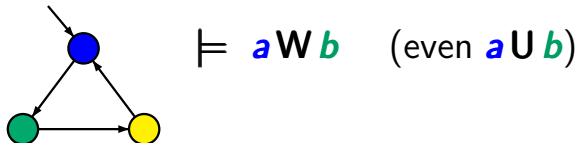
LTLSF3.1-32



●  $\hat{=} \{a\}$

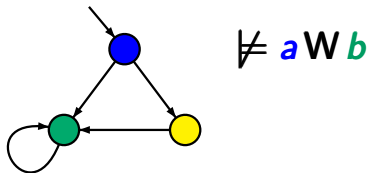
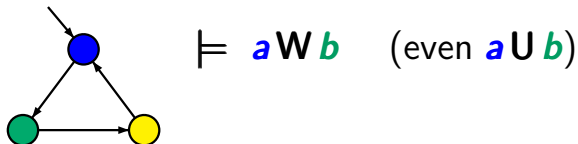
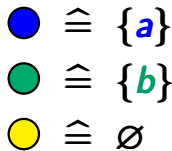
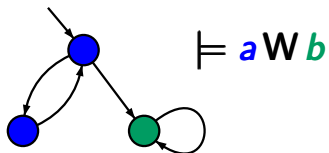
●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$



Does  $\mathcal{T} \models aWb$  hold ?

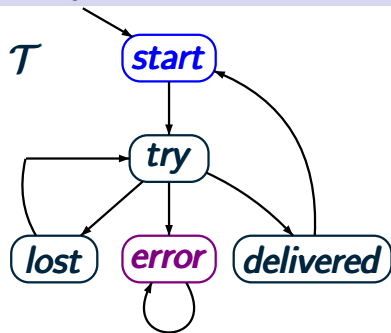
LTLSF3.1-32





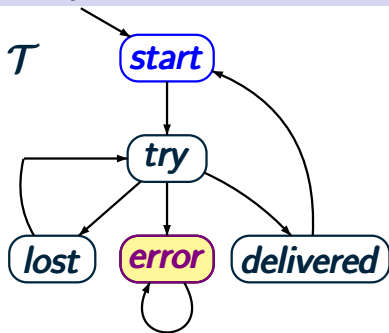
# Example: CTL semantics

CTLSS4.1-16



$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$  ?

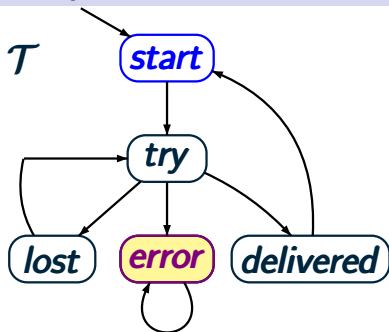
$\phi_1 = \exists \diamond \forall \square \neg \text{start}$



$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$  ?

$$\phi_1 = \exists \diamond \forall \square \neg \text{start}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

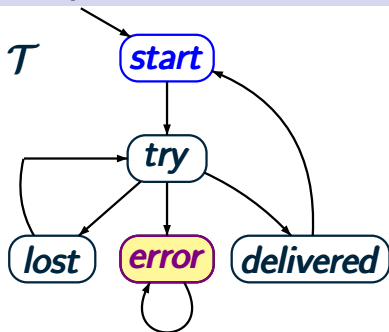


$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$  ?

$$\phi_1 = \exists \diamond \forall \square \neg \text{start} \rightsquigarrow \exists \diamond \text{error}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \diamond \forall \square \neg \text{start}) = ?$$



$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$  ?

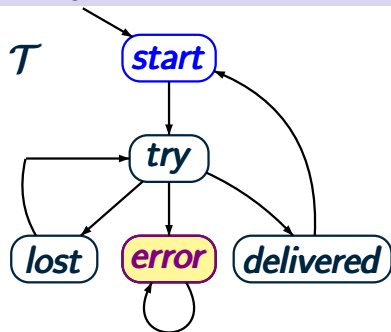
$$\phi_1 = \exists \diamond \forall \square \neg \text{start} \rightsquigarrow \exists \diamond \text{error}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \diamond \forall \square \neg \text{start}) = \text{Sat}(\exists \diamond \text{error}) = \text{“all states”}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start} \quad \checkmark$$

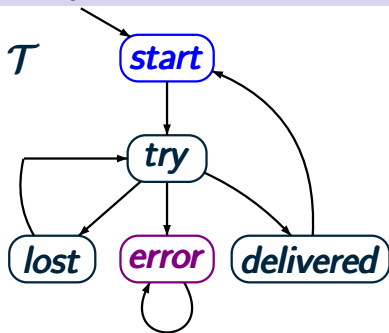
$$\phi_1 = \exists \diamond \forall \square \neg \text{start} \rightsquigarrow \exists \diamond \text{error}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \diamond \forall \square \neg \text{start}) = \text{Sat}(\exists \diamond \text{error}) = \text{“all states”}$$

# Example: CTL semantics

CTLSS4.1-16



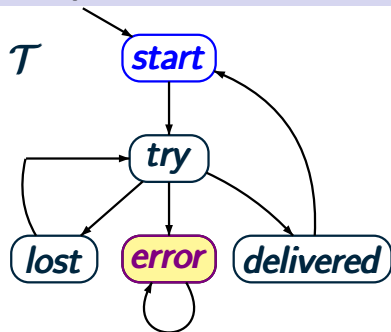
$$\mathcal{T} \models \exists \Diamond \Box \neg \textit{start}$$

$$\mathcal{T} \models \forall \bigcirc \bigcirc \bigcirc \Box \neg \textit{start} ?$$

$$\Phi_2 = \forall \bigcirc \exists \bigcirc \Box \neg \textit{start}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

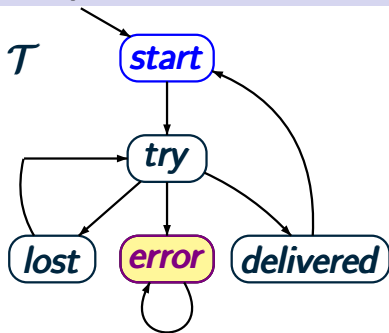
$$\mathcal{T} \models \forall \bigcirc \exists \bigcirc \bigcirc \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \bigcirc \exists \bigcirc \Box \neg \text{start}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \neg \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \neg \exists \Diamond \neg \Box \neg \text{start} ?$$

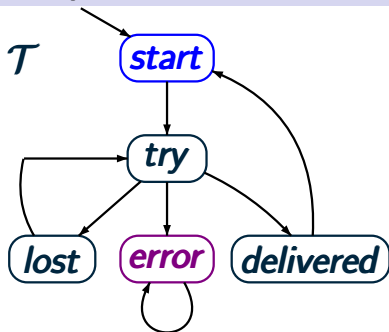
$$\Phi_2 = \forall \Box \neg \exists \Diamond \neg \Box \neg \text{start} \rightsquigarrow \forall \Box \neg \exists \Diamond \text{error}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$



# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \neg \Box \neg \text{start}$$

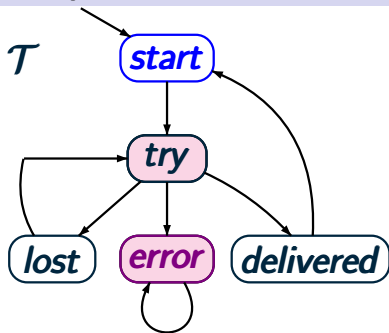
$$\mathcal{T} \models \forall \Diamond \exists \Diamond \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \Diamond \exists \Diamond \Box \neg \text{start} \rightsquigarrow \forall \Diamond \exists \Diamond \Box \text{error}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Diamond \Box \neg \text{start} \text{ ?}$$

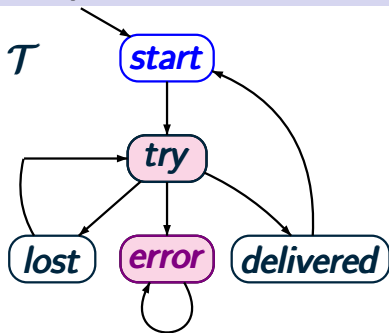
$$\Phi_2 = \forall \Diamond \Box \neg \text{start}$$

$$\rightsquigarrow \forall \Diamond \Box \text{error}$$

$$\begin{aligned} \text{Sat}(\forall \Box \neg \text{start}) &= \{\text{error}\} \\ \text{Sat}(\exists \Diamond \Box \neg \text{start}) &= \{\text{error}, \text{try}\} \end{aligned}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \neg \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Diamond \exists \Diamond \Box \neg \text{start} ?$$

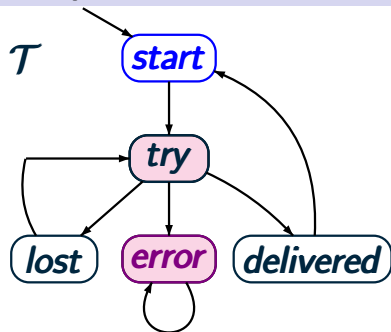
$$\Phi_2 = \forall \Diamond \exists \Diamond \Box \neg \text{start} \rightsquigarrow \forall \Diamond (\text{error} \vee \text{try})$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Diamond \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Diamond \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \Diamond \Box \neg \text{start} \quad \rightsquigarrow \quad \forall \Diamond (\text{error} \vee \text{try})$$

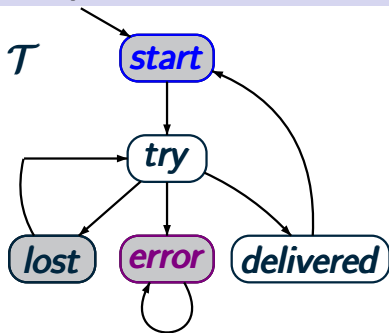
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Diamond \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

$$\text{Sat}(\forall \Diamond \Box \neg \text{start}) = ?$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \bigcirc \bigcirc \Box \neg \text{start} \quad \checkmark$$

$$\Phi_2 = \forall \bigcirc \bigcirc \Box \neg \text{start} \quad \rightsquigarrow \forall \bigcirc (\bigcirc (\text{error} \vee \text{try}))$$

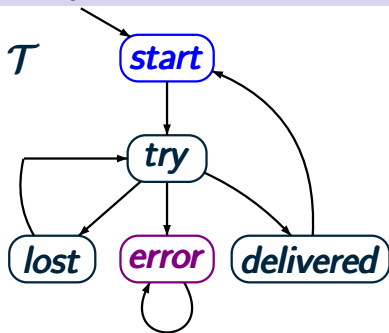
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \bigcirc \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

$$\text{Sat}(\forall \bigcirc \bigcirc \Box \neg \text{start}) = \{\text{error}, \text{lost}, \text{start}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

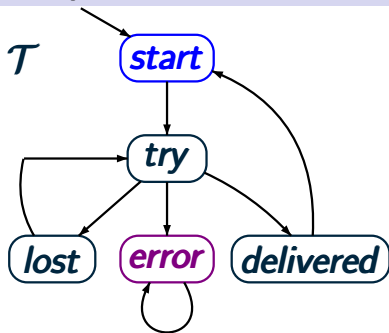
$$\mathcal{T} \models \forall \Box \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \exists \Diamond \Box \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \forall \Box \neg \text{start}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \forall \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \neg \text{start}$$

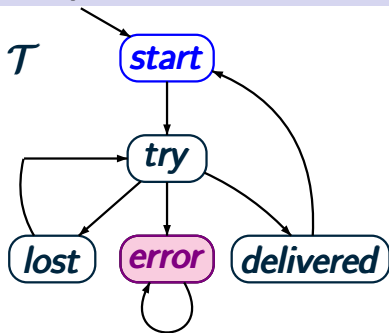
$$\mathcal{T} \models \exists \Box \forall \Diamond \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \forall \Diamond \neg \text{start}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \forall \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \forall \Diamond \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \forall \Diamond \neg \text{start} \rightsquigarrow \exists \Box \forall \Diamond \neg \text{error}$$

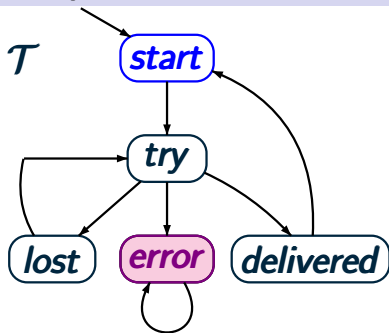
$$\text{Sat}(\forall \Box \neg \text{start}) = \{ \text{error} \}$$

$$\text{Sat}(\forall \Box \forall \Diamond \neg \text{start}) = ?$$



# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \forall \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \forall \Diamond \neg \text{start} ?$$

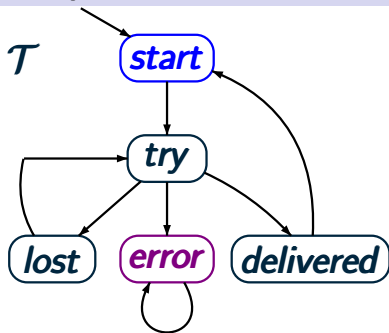
$$\Phi_3 = \exists \Box \forall \Diamond \neg \text{start} \rightsquigarrow \exists \Box \text{error}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Box \forall \Diamond \neg \text{start}) = \{\text{error}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \exists \Diamond \Box \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \exists \Diamond \Box \neg \text{start} \rightsquigarrow \exists \Box \text{error}$$

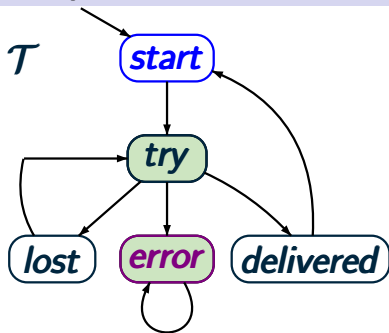
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Box \exists \Diamond \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Box \exists \Diamond \Box \neg \text{start}) = ?$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \neg \square \neg \text{start}$$

$$\mathcal{T} \models \forall \Diamond \neg \square \neg \text{start}$$

$$\mathcal{T} \models \exists \Diamond \neg \square \neg \text{start} ?$$

$$\Phi_3 = \exists \Diamond \neg \square \neg \text{start} \rightsquigarrow \exists \Diamond \text{error}$$

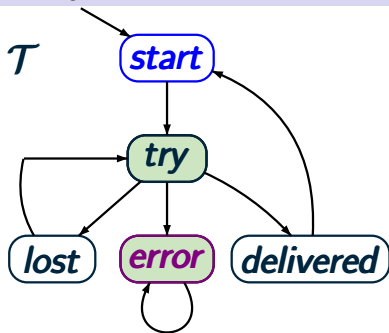
$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Diamond \neg \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Diamond \neg \square \neg \text{start}) = \{\text{error}, \text{try}\}$$

# Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \not\models \exists \Diamond \Box \neg \text{start}$$

$$\Phi_3 = \exists \Diamond \forall \Box \neg \text{start} \rightsquigarrow \exists \Diamond \text{error}$$

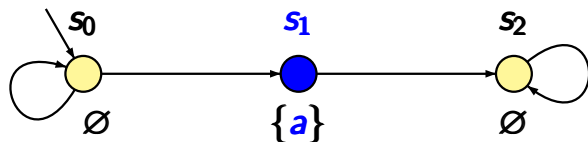
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Diamond \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Diamond \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

# Example: CTL semantics

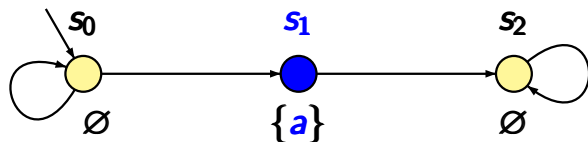
CTLSS4.1-17



does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

# Example: CTL semantics

CTLSS4.1-17

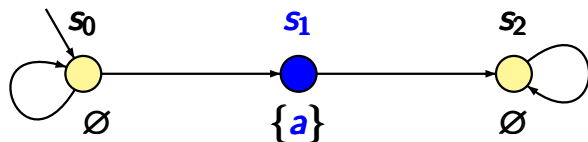


does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

# Example: CTL semantics

CTLSS4.1-17



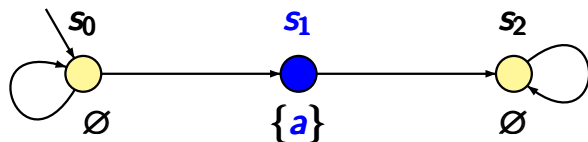
does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

$$\text{Sat}(\forall \square \neg a) = \{s_2\}$$

# Example: CTL semantics

CTLSS4.1-17



does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

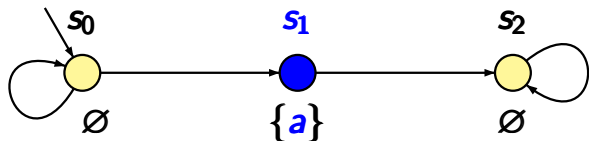
$$\text{Sat}(\forall \square \neg a) = \{s_2\}$$

$$\text{Sat}(\exists \bigcirc \forall \square \neg a) = \{s_2, s_1\}$$



# Example: CTL semantics

CTLSS4.1-17



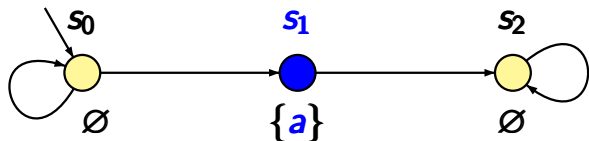
does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

does  $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$  hold ?

# Example: CTL semantics

CTLSS4.1-17

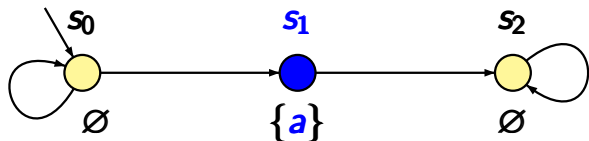


does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

does  $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$  hold ?

*answer:* yes



does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

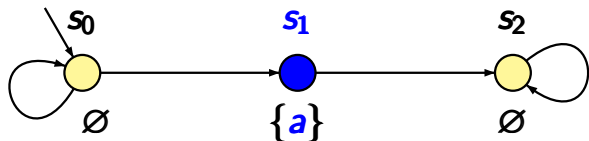
does  $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$  hold ?

*answer:* yes

$$\text{Sat}(\exists \bigcirc \neg a) = \{s_0, s_1, s_2\}$$

# Example: CTL semantics

CTLSS4.1-17



does  $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$  hold ?

*answer:* no

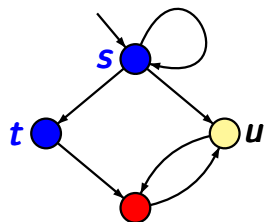
does  $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$  hold ?

*answer:* yes

$$\begin{aligned} \text{Sat}(\exists \bigcirc \neg a) &= \{s_0, s_1, s_2\} \\ \text{Sat}(\forall \square \exists \bigcirc \neg a) &= \{s_0, s_1, s_2\} \end{aligned}$$

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

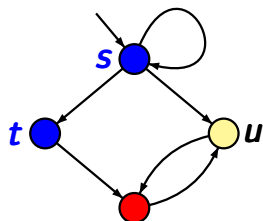
●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$  ?

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

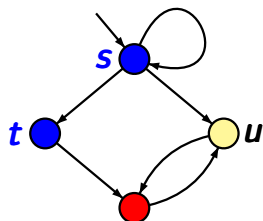
●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \text{ U } b)$

✓ as  $s \models \exists (a \text{ U } b)$

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

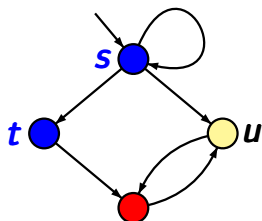
●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

✓ as  $s s s \dots \models \square \exists (a \cup b)$

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

✓ as  $s s s \dots \models \square \exists (a \cup b)$

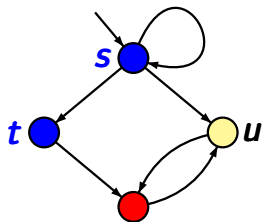
$\mathcal{T} \models \exists ((\exists \bigcirc a) \cup b)$

?



# Example: CTL semantics

CTLSS4.1-18



$$\bullet \hat{=} \{a\}$$

$$\bullet \hat{=} \{b\}$$

$$\bullet \hat{=} \emptyset$$

$$\mathcal{T} \models \exists \square \exists (a \cup b)$$

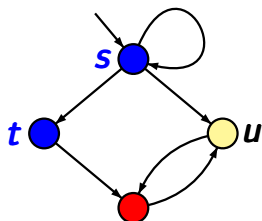
$$\checkmark \text{ as } s s s \dots \models \square \exists (a \cup b)$$

$$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b)$$

$$\text{as } t \not\models \exists \bigcirc a, u \not\models \exists \bigcirc a$$

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

✓ as  $s s s \dots \models \square \exists (a \cup b)$

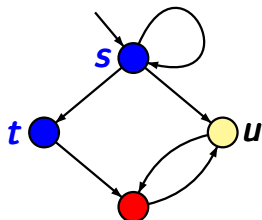
$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b)$

as  $t \not\models \exists \bigcirc a$ ,  $u \not\models \exists \bigcirc a$

$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b))$  ?

# Example: CTL semantics

CTLSS4.1-18



●  $\hat{=} \{a\}$

●  $\hat{=} \{b\}$

●  $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

✓ as  $s s s \dots \models \square \exists (a \cup b)$

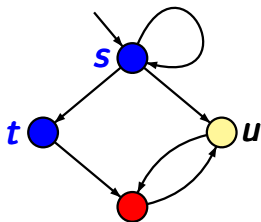
$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b)$

as  $t \not\models \exists \bigcirc a$ ,  $u \not\models \exists \bigcirc a$

$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b))$  ✓

# Example: CTL semantics

CTLSS4.1-18



$$\bullet \hat{=} \{a\}$$

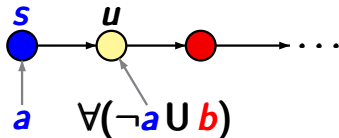
$$\bullet \hat{=} \{b\}$$

$$\bullet \hat{=} \emptyset$$

$$\mathcal{T} \models \exists \square \exists (a \cup b) \quad \checkmark \quad \text{as } s s s \dots \models \square \exists (a \cup b)$$

$$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b) \quad \text{as } t \not\models \exists \bigcirc a, u \not\models \exists \bigcirc a$$

$$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b)) \quad \checkmark$$



$$\models a \cup \forall (\neg a \cup b)$$

Let  $\mathcal{T}$  be a transition system and  $\phi$  a CTL formula.  
Is the following statement correct ?

if  $\mathcal{T} \not\models \neg\phi$  then  $\mathcal{T} \models \phi$

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*answer:* no

Let  $\mathcal{T}$  be a transition system and  $\phi$  a CTL formula.  
Is the following statement correct ?

if  $\mathcal{T} \not\models \neg\phi$  then  $\mathcal{T} \models \phi$

*answer:* no

$\mathcal{T} \models \phi$  iff  $s_0 \models \phi$  for all initial states  $s_0$

---

# Correct or wrong?

CTLSS4.1-19

Let  $\mathcal{T}$  be a transition system and  $\phi$  a CTL formula.  
Is the following statement correct ?

if  $\mathcal{T} \not\models \neg\phi$  then  $\mathcal{T} \models \phi$

*answer:* no

$\mathcal{T} \models \phi$  iff  $s_0 \models \phi$  for all initial states  $s_0$

---

$\mathcal{T} \not\models \neg\phi$  iff there exists an initial state  $s_0$  with  
 $s_0 \not\models \neg\phi$



# Correct or wrong?

CTLSS4.1-19

Let  $\mathcal{T}$  be a transition system and  $\phi$  a CTL formula.  
Is the following statement correct ?

if  $\mathcal{T} \not\models \neg\phi$  then  $\mathcal{T} \models \phi$

*answer:* no

$\mathcal{T} \models \phi$  iff  $s_0 \models \phi$  for all initial states  $s_0$

---

$\mathcal{T} \not\models \neg\phi$  iff there exists an initial state  $s_0$  with  
 $s_0 \not\models \neg\phi$

iff there exists an initial state  $s_0$  with  
 $s_0 \models \phi$

# Correct or wrong?

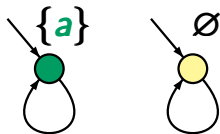
CTLSS4.1-19

Let  $\mathcal{T}$  be a transition system and  $\phi$  a CTL formula.  
Is the following statement correct ?

if  $\mathcal{T} \not\models \neg\phi$  then  $\mathcal{T} \models \phi$

*answer:* no

transition system  $\mathcal{T}$  with 2 initial states:



$\mathcal{T} \not\models \exists\Box a$

$\mathcal{T} \not\models \neg\exists\Box a$

# Correct or wrong?

CTLSS4.1-23

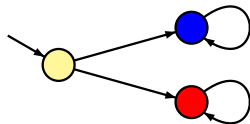
$$\exists \diamond (a \wedge b) \equiv \exists \diamond a \wedge \exists \diamond b$$

# Correct or wrong?

CTLSS4.1-23

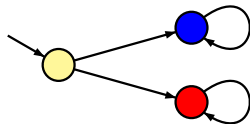
$$\exists \diamond (a \wedge b) \equiv \exists \diamond a \wedge \exists \diamond b$$

wrong, e.g.,



$$\exists \Diamond(a \wedge b) \equiv \exists \Diamond a \wedge \exists \Diamond b$$

wrong, e.g,



---

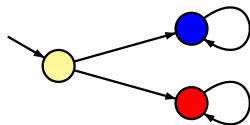
$$\forall \Diamond(a \wedge b) \equiv \forall \Diamond a \wedge \forall \Diamond b$$

# Correct or wrong?

CTLSS4.1-23

$$\exists \Diamond(a \wedge b) \equiv \exists \Diamond a \wedge \exists \Diamond b$$

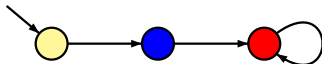
wrong, e.g.,



---

$$\forall \Diamond(a \wedge b) \equiv \forall \Diamond a \wedge \forall \Diamond b$$

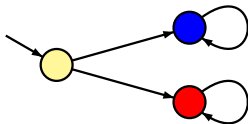
wrong, e.g.,



# Correct or wrong?

$$\exists \Diamond(a \wedge b) \equiv \exists \Diamond a \wedge \exists \Diamond b$$

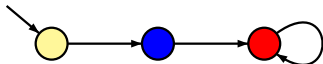
wrong, e.g.,



---

$$\forall \Diamond(a \wedge b) \equiv \forall \Diamond a \wedge \forall \Diamond b$$

wrong, e.g.,



but:

$$\forall \Box(\Phi_1 \wedge \Phi_2) \equiv \forall \Box \Phi_1 \wedge \forall \Box \Phi_2$$

$$\exists \Diamond(\Phi_1 \vee \Phi_2) \equiv \exists \Diamond \Phi_1 \vee \exists \Diamond \Phi_2$$

# Correct or wrong?

CTLSS4.1-24

$$\forall \alpha \forall \beta a \equiv \forall \beta \forall \alpha a$$



# Correct or wrong?

CTLSS4.1-24

$$\forall x \forall y a \equiv \forall y \forall x a$$

correct.

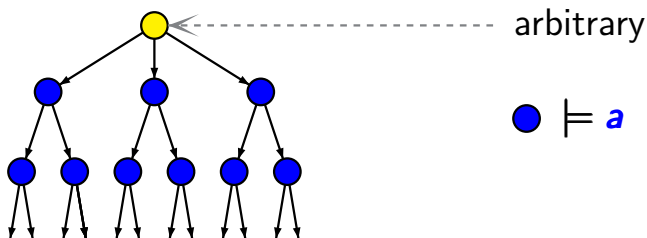
# Correct or wrong?

CTLSS4.1-24

$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

**correct.**

both formulas require computation trees  
of the form:



# Correct or wrong?

CTLSS4.1-24

$$\forall \circ \forall \square a \equiv \forall \square \forall \circ a$$

correct.

---

$$\exists \circ \exists \square a \equiv \exists \square \exists \circ a$$

# Correct or wrong?

CTLSS4.1-24

$$\forall \circ \forall \square a \equiv \forall \square \forall \circ a$$

correct.

---

$$\exists \circ \exists \square a \equiv \exists \square \exists \circ a$$

wrong,

# Correct or wrong?

CTLSS4.1-24

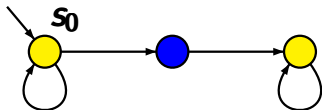
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

---

$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



# Correct or wrong?

CTLSS4.1-24

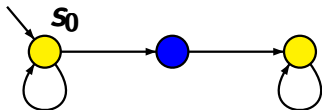
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

---

$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \square \exists \bigcirc a$$

# Correct or wrong?

CTLSS4.1-24

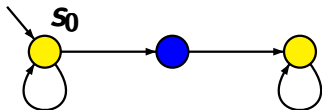
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

---

$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

note:  $Sat(\exists \square a) = \emptyset$

# Correct or wrong?

CTLSS4.1-24

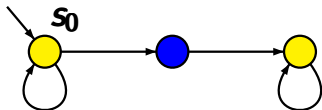
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

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$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

$$s_0 \models \exists \square \exists \bigcirc a$$



# Correct or wrong?

CTLSS4.1-24

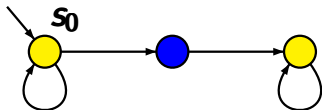
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

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wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

$$s_0 \models \exists \square \exists \bigcirc a$$

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# Correct or wrong?

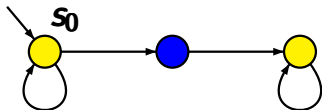
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correct.

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$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

$$s_0 \models \exists \square \exists \bigcirc a$$

$$s_0 \models \exists \bigcirc a$$

$$\implies s_0 s_0 s_0 \dots \models \square \exists \bigcirc a$$

# Correct or wrong?

CTLSS4.1-24

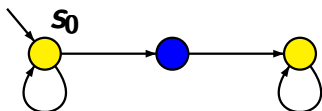
$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

---

$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

$$s_0 \models \exists \square \exists \bigcirc a$$

$$s_0 \models \exists \bigcirc a$$

$$\implies s_0 s_0 s_0 \dots \models \square \exists \bigcirc a$$

$$\implies s_0 \models \exists \square \exists \bigcirc a$$

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

answer: **no.**

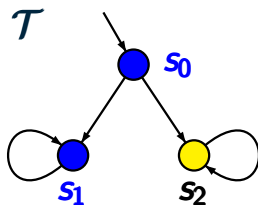
Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

answer: **no.**



Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

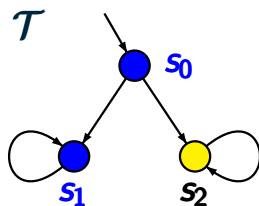
answer: **no.**



$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

answer: **no.**



$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$

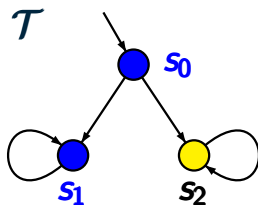
note:  $\pi = s_0 s_2 s_2 s_2 \dots$  is a path in  $\mathcal{T}$  with

$trace(\pi) = \{a\} \emptyset \emptyset \emptyset \dots \notin Words(\Diamond(a \wedge \bigcirc a))$



Does  $\forall \Diamond(a \wedge \exists \bigcirc a) \equiv \Diamond(a \wedge \bigcirc a)$  hold ?

answer: **no.**

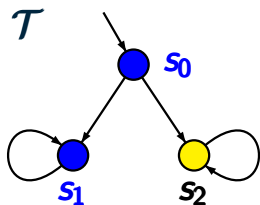


$$\mathcal{T} \not\models \Diamond(a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \Diamond(a \wedge \exists \bigcirc a)$$

Does  $\forall \diamond (a \wedge \exists \bigcirc a) \equiv \diamond (a \wedge \bigcirc a)$  hold ?

answer: **no.**



$$\mathcal{T} \not\models \diamond (a \wedge \bigcirc a)$$

$$\mathcal{T} \models \forall \diamond (a \wedge \exists \bigcirc a)$$

$$\text{Sat}(\exists \bigcirc a) = \{s_0, s_1\}$$

$$\text{Sat}(\forall \diamond (a \wedge \exists \bigcirc a)) = \{s_0, s_1\}$$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
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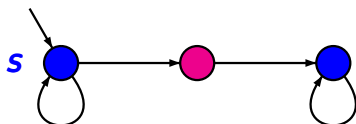
**wrong.**

# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.

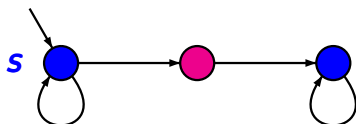


# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  $\pi \models \square \diamond a$

wrong.



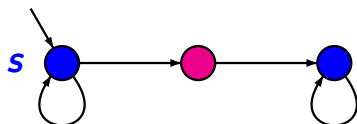
$s \models \exists \square \exists \diamond a$

# Correct or wrong?

COMPARISON4.2-10

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  $\pi \models \square \diamond a$

wrong.



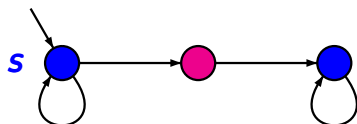
$s \models \exists \square \exists \diamond a$

note that:  $s \models \exists \diamond a$

# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.



$s \models \exists \square \exists \diamond a$

note that:  $s \models \exists \diamond a$

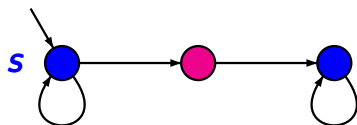
thus:  $s s s \dots \models \square \exists \diamond a$



# Correct or wrong?

$s \models \exists \square \exists \diamond a$  iff there is a path  $\pi \in \text{Paths}(s)$  with  
 $\pi \models \square \diamond a$

wrong.



$s \models \exists \square \exists \diamond a$

note that:  $s \models \exists \diamond a$

thus:  $s s s \dots \models \square \exists \diamond a$

but there is no path where  $\square \diamond a$  holds

# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

# Correct or wrong?

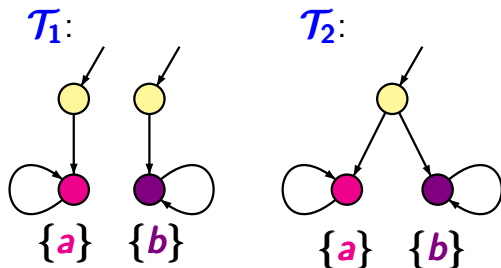
If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

wrong.

# Correct or wrong?

If  $\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent TS then for all CTL formulas  $\phi$ :  $\mathcal{T}_1 \models \phi$  iff  $\mathcal{T}_2 \models \phi$

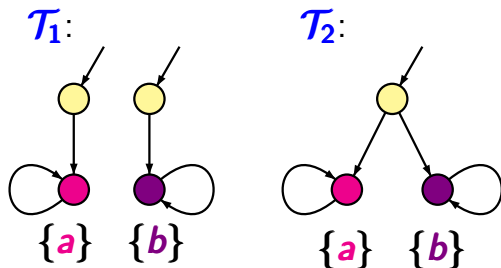
wrong.



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wrong.



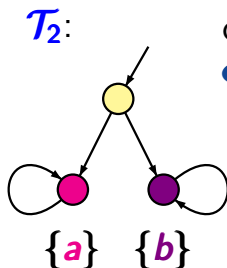
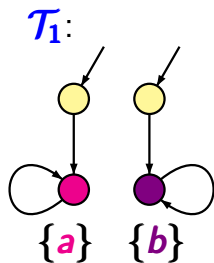
$\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent

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COMPARISON4.2-12

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wrong.



consider the CTL formula  
 $\phi = \exists \bigcirc a \wedge \exists \bigcirc b$

$$\mathcal{T}_1 \not\models \phi$$

$$\mathcal{T}_2 \models \phi$$

$\mathcal{T}_1$  and  $\mathcal{T}_2$  are trace equivalent