Contents & Goals

Last Lecture:
• Started DC Syntax and Semantics: Symbols, State Assertions

This Lecture:
• Educational Objectives:
  - Capabilities for following tasks/questions.
  - Read (and at best also write) Duration Calculus terms.
• Content:
  - Duration Calculus continued

Duration Calculus: Overview

We will introduce three (or five) syntactical "levels":

(i) Symbols:
  - f, g,
  - true, false,
  - =, <, >, ≤, ≥, x, y, z, X, Y, Z, d

(ii) State Assertions:
  - \( P ::= 0 \ |
  - 1 \ |
  - X = d \ |
  - \neg P \ |
  - 1 \ |
  - P_1 \land P_2 \ |

(iii) Terms:
  - \( \theta ::= x \ |
  - \ell \ |
  - \int P \ |
  - f(\theta_1, \ldots, \theta_n) \ |

(iv) Formulae:
  - \( F ::= p(\theta_1, \ldots, \theta_n) \ |
  - \neg F_1 \ |
  - F_1 \land F_2 \ |
  - \forall x \cdot F_1 \ |
  - F_1 ; F_2 \ |

(v) Abbreviations:
  - \[ P \], \[ P \] \leq t, ♦ F, □ F

Terms: Syntax

• Duration terms (DC terms or just terms) are defined by the following grammar:

\[ \theta ::= x \ |
- \ell \ |
- \int P \ |
- f(\theta_1, \ldots, \theta_n) \ |
\]

where

- \( x \) is a global variable,
- \( \ell \) and \( \int \) are special symbols,
- \( P \) is a state assertion, and
- \( f \) a function symbol (of arity \( n \)).

• \( \ell \) is called length operator,
- \( \int \) is called integral operator

• Notation: we may write function symbols in infix notation as usual, i.e. write \( \theta_1 + \theta_2 \) instead of \( + (\theta_1, \theta_2) \).

Definition 1.

\[ \text{[Rigid]} \]

A term without length and integral symbols is called rigid.
Abbreviations:

Note: quantification only over $\forall$, $\exists$, $\forall$, $\exists$, $\forall$, $\exists$, $\forall$, $\exists$.

Terms: Syntax

Duration Calculus: Overview

Terms: Remarks

Remark 2.5.

The semantics of terms is insensitive against changes of the interpretation at individual time points.

Remark 2.6.

The set of formulae is defined by the following grammar:

$F, \Diamond, \Box, P, \langle, \rangle, t \leq t_{\text{notover}}(\text{state variables}), \text{second-order}$
Formulae: Priority Groups

To avoid parentheses, we define the following five priority groups from highest to lowest priority:

• ¬ (negation)
• ; (chop)
• ∧, ∨ (and/or)
• =⇒, ⇐⇒ (implication/equivalence)
• ∃, ∀ (quantifiers)

Examples:

• ¬ F ; F ∨ H
• ∀ x • F ∧ G

Formulae: Semantics

The semantics of a formula is a function $I/C_2 F/C_3 : \text{Val} \times \text{Int} \rightarrow \{tt, ff\}$ i.e. $I/C_2 F/C_3 (V, [b,e])$ is the truth value of $F$ under interpretation $I$ and valuation $V$ in the interval $[b,e]$.

This value is defined inductively on the structure of $F$:

$I/C_2 p(θ_1, . . . ,θ_n)/C_3 (V, [b,e]) = \hat{p}(I/C_2 θ_1/C_3 (V, [b,e]), . . . , I/C_2 θ_n/C_3 (V, [b,e]))$

$I/C_2 ¬ F_1/C_3 (V, [b,e]) = tt$ iff $I/C_2 F_1/C_3 (V, [b,e]) = ff$

$I/C_2 F_1 ∧ F_2/C_3 (V, [b,e]) = tt$ iff $I/C_2 F_1/C_3 (V, [b,e]) = I/C_2 F_2/C_3 (V, [b,e]) = tt$

$I/C_2 ∀ x • F_1/C_3 (V, [b,e]) = tt$ iff for all $a ∈ \mathbb{R}$, $I/C_2 F_1[x:=a]/C_3 (V, [b,e]) = tt$

Formulae: Example

$F := \int L=0 ; \int L=1$

Examples:

$F\left[x:=θ_1\right]/C_3 (V, [0, 2]) = \text{tt}$

References