Real-Time Systems

Lecture 09: PLC Automata

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Contents & Goals

Last Lecture:

- DC Implementables.
- A controller for the gas burner.

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is the "philosophy" of PLC? What did we generalise/abstract them to?
  - What's an example for giving a DC semantics for a constructive formalism?
  - How does the proposed approach work, from requirements to a correct implementation with DC?
- **Content:**
  - Continue Implementables Example
  - Programmable Logic Controllers (PLC) ("Speicherprogrammierbare Steuerungen" (SPS))
  - PLC Automata
  - An overapproximating DC semantics for PLCA
  - An reaction time theorem for PLCA

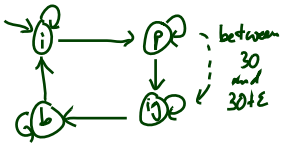
Example: Gas Burner

Recall: Control Automata

Model of Gas Burner controller as a system of four control automata:

- H : Boolean,
  representing **heat request**, (input)
- F : Boolean,
  representing **flame**, (input)
- C with $\mathcal{D}(C) = \{\text{idle}, \text{purge}, \text{ignite}, \text{burn}\}$,
  representing the **controller**, (local)
- G : Boolean,
  representing **gas valve**. (output)

Gas Burner Controller Specification



$$\lceil\rceil \vee \lceil \mathsf{idle} \rceil ; \mathit{true}, \quad \lceil\rceil \vee \lceil \neg H \rceil ; \mathit{true}, \quad \lceil\rceil \vee \lceil \neg F \rceil ; \mathit{true}, \quad \lceil\rceil \vee \lceil \neg G \rceil ; \mathit{true} \qquad \text{(Init-1 - 4)}$$

$$\lceil \mathsf{idle} \rceil \longrightarrow \lceil \mathsf{idle} \vee \mathsf{purge} \rceil \qquad \text{(Seq-1)}$$

$$\lceil \mathsf{purge} \rceil \longrightarrow \lceil \mathsf{purge} \vee \mathsf{ignite} \rceil \qquad \text{(Seq-2)}$$

$$\lceil \mathsf{ignite} \rceil \longrightarrow \lceil \mathsf{ignite} \vee \mathsf{burn} \rceil \qquad \text{(Seq-3)}$$

$$\lceil \mathsf{burn} \rceil \longrightarrow \lceil \mathsf{burn} \vee \mathsf{idle} \rceil \qquad \text{(Seq-4)}$$

$$\lceil \mathsf{purge} \rceil \xrightarrow{30+\varepsilon} \lceil \neg \mathsf{purge} \rceil \qquad \text{(Prog-1)}$$

$$\lceil \mathsf{ignite} \rceil \xrightarrow{0.5+\varepsilon} \lceil \neg \mathsf{ignite} \rceil \qquad \text{(Prog-2)}$$

$$\lceil \mathsf{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg \mathsf{idle} \rceil \qquad \text{(Syn-1)}$$

$$\lceil \mathsf{burn} \wedge (\neg H \vee \neg F) \rceil \xrightarrow{\varepsilon} \lceil \neg \mathsf{burn} \rceil \qquad \text{(Syn-2)}$$

$$\lceil G \wedge (\mathsf{idle} \vee \mathsf{purge}) \rceil \xrightarrow{\varepsilon} \lceil \neg G \rceil \qquad \text{(Syn-3)}$$

$$\lceil \neg G \wedge (\mathsf{ignite} \vee \mathsf{burn}) \rceil \xrightarrow{\varepsilon} \lceil G \rceil \qquad \text{(Syn-4)}$$

$$\lceil \neg \mathsf{idle} \rceil ; \lceil \mathsf{idle} \wedge \neg H \rceil \longrightarrow \lceil \mathsf{idle} \rceil \qquad \text{(Stab-1)}$$

$$\lceil \mathsf{idle} \wedge \neg H \rceil \longrightarrow_0 \lceil \mathsf{idle} \rceil \qquad \text{(Stab-1-init)}$$

$$\lceil \neg \mathsf{purge} \rceil ; \lceil \mathsf{purge} \rceil \xrightarrow{\leq 30} \lceil \mathsf{purge} \rceil \qquad \text{(Stab-2)}$$

$$\lceil \neg \mathsf{ignite} \rceil ; \lceil \mathsf{ignite} \rceil \xrightarrow{\leq 0.5} \lceil \mathsf{ignite} \rceil \qquad \text{(Stab-3)}$$

$$\lceil \neg \mathsf{burn} \rceil ; \lceil \mathsf{burn} \wedge H \wedge F \rceil \longrightarrow \lceil \mathsf{burn} \rceil \qquad \text{(Stab-4)}$$

$$\lceil F \rceil ; \lceil \neg F \wedge \neg \mathsf{ignite} \rceil \longrightarrow \lceil \neg F \rceil \qquad \text{(Stab-5)}$$

$$\lceil \neg F \wedge \neg \mathsf{ignite} \rceil \longrightarrow_0 \lceil \neg F \rceil \qquad \text{(Stab-5-init)}$$

$$\lceil G \rceil ; \lceil \neg G \wedge (\mathsf{idle} \vee \mathsf{purge}) \rceil \longrightarrow \lceil \neg G \rceil \qquad \text{(Stab-6)}$$

$$\lceil \neg G \wedge (\mathsf{idle} \vee \mathsf{purge}) \rceil \longrightarrow_0 \lceil \neg G \rceil \qquad \text{(Stab-6-init)}$$

$$\lceil \neg G \rceil ; \lceil G \wedge (\mathsf{ignite} \vee \mathsf{burn}) \rceil \longrightarrow \lceil G \rceil \qquad \text{(Stab-7)}$$

Gas Burner Controller Correctness Proof

$$\text{GB-Ctrl} := \text{Init-1} \wedge \cdots \wedge \text{Stab-7} \wedge \varepsilon > 0$$

Recall:

$$\text{Req} :\Longleftrightarrow \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

and (cf. [Olderog and Dierks, 2008])

$$\models \text{Req-1} \implies \text{Req}$$

for the **simplified**

$$\text{Req-1} := \Box(\ell \leq 30 \implies \textstyle\int L \leq 1).$$

Here we show

$$\models \text{GB-Ctrl} \wedge A(\varepsilon) \implies \text{Req-1}.$$

Lemma 3.15

$$\models \text{GB-Ctrl} \implies \Box \left(\begin{array}{rrl} & (\lceil \text{idle} \rceil & \implies \int G \leq \varepsilon) \\ \wedge & (\lceil \text{purge} \rceil & \implies \int G \leq \varepsilon) \\ \wedge & (\lceil \text{ignite} \rceil & \implies \ell \leq 0.5 + \varepsilon) \\ \wedge & (\lceil \text{burn} \rceil & \implies \int \neg F \leq 2\varepsilon) \end{array} \right)$$

- $I, [b,e] \models \lceil \text{burn} \rceil$

(sys-2) $\lceil \text{burn} \wedge (\neg H \wedge \neg F) \rceil \xrightarrow{\varepsilon} \lceil \neg \text{burn} \rceil$

(Sys-5) $\lceil F \rceil ; \lceil \neg F \wedge \neg \text{ignite} \rceil \longrightarrow \lceil \neg F \rceil$

$I, [b,e] \models \Box(\lceil \neg F \rceil \Rightarrow \ell \leq \varepsilon)$
$\wedge \neg \Diamond (\lceil F \rceil ; \lceil \neg F \rceil ; \lceil F \rceil)$

$\lceil F \rceil$
$\lceil \neg F \rceil$
$\lceil F \rceil ; \lceil \neg F \rceil$
$\lceil \neg F \rceil ; \lceil F \rceil$
$\lceil \neg F \rceil ; \lceil F \rceil ; \lceil \neg F \rceil$

Lemma 3.16

$$\models \exists \varepsilon \bullet \text{GB-Ctrl} \implies \underbrace{\Box(\ell \leq 30 \implies \int L \leq 1)}_{\text{Req-1}}$$

Proof Sketch:

Choose $I, V, [b,e]$ s.t. $I, V, [b,e] \models \text{GB-Ctrl} \wedge \ell \leq 30$.

Distinguish 5 cases:

$I, V, [b,e] \models \lceil \rceil$
$\vee (\lceil \text{idle} \rceil ; \text{true} \wedge \ell \leq 30)$ (1)
$\vee (\lceil \text{purge} \rceil ; \text{true} \wedge \ell \leq 30)$ (4)
$\vee (\lceil \text{ignite} \rceil ; \text{true} \wedge \ell \leq 30)$ (3)
$\vee (\lceil \text{burn} \rceil ; \text{true} \wedge \ell \leq 30)$ (2)

Lemma 3.16 Cont'd

- Case 0: $\mathcal{I}, \mathcal{V}, [b, e] \models \lceil\rceil$ ✓
- Case 1: $\mathcal{I}, \mathcal{V}, [b, e] \models \lceil \mathsf{idle} \rceil$; $true \wedge \ell \leq 30$

$$\lceil \mathsf{idle} \rceil \longrightarrow \lceil \mathsf{idle} \vee \mathsf{purge} \rceil \qquad \text{(Seq-1)}$$

$$\lceil \neg \mathsf{purge} \rceil ; \lceil \mathsf{purge} \rceil \overset{\leq 30}{\longrightarrow} \lceil \mathsf{purge} \rceil \qquad \text{(Stab-2)}$$

$I, V, [b,e] \models \lceil idle \rceil \vee \lceil idle \rceil ; \lceil purge \rceil$

3.15 ↳ $I, V, [b,e] \models \int l \leq \varepsilon \vee \int l \leq \varepsilon \; ; \int l \leq \varepsilon$

↳ $I, V, [b,e] \models \int l \leq 2 \cdot \varepsilon$

Thus $\varepsilon \leq 0.5$ is sufficient for Req-1 in this case.

Lemma 3.16 Cont'd

- Case 2: $\mathcal{I}, \mathcal{V}, [b, e] \models \lceil \mathsf{burn} \rceil$; $true \wedge \ell \leq 30$

$$\lceil \mathsf{burn} \rceil \longrightarrow \lceil \mathsf{burn} \vee \mathsf{idle} \rceil \qquad \text{(Seq-4)}$$

$I, V, [b,e] \models (\lceil burn \rceil \vee \lceil burn \rceil ; \lceil idle \rceil ; true) \wedge \ell \leq 30$

3.15, Case 1 ↳ $I, V, [b,e] \models (\int l \leq 2\varepsilon \vee \int l \leq 2\varepsilon \; ; \int l \leq 2\varepsilon) \wedge \ell \leq 30$

↳ $I, V, [b,e] \models \int l \leq 4 \cdot \varepsilon$

Thus $\varepsilon \leq 0.25$ sufficient for Req-1.

Lemma 3.16 Cont'd

- Case 3: $\mathcal{I}, \mathcal{V}, [b, e] \models \lceil \mathsf{ignite} \rceil$; $true \wedge \ell \leq 30$

$$\lceil \mathsf{ignite} \rceil \longrightarrow \lceil \mathsf{ignite} \vee \mathsf{burn} \rceil \qquad \text{(Seq-3)}$$

$\mathcal{I}, \mathcal{V}, [b,e] \models \lceil \text{ignite} \rceil \vee \lceil \text{ignite} \rceil ; \lceil \text{burn} \rceil ; true$

3.5, Case 2: $\mathcal{I}, \mathcal{V}, [b,e] \models \int L \leq 0.5 + \varepsilon \vee \left(\int L \leq 0.5 + \varepsilon ; \int L \leq 4\varepsilon \right)$

$\mathcal{I}, \mathcal{V}, [b,e] \models \int L \leq 0.5 + 5\varepsilon$

So $\boxed{\varepsilon \leq 0.1}$ sufficient for Req. 1.

Lemma 3.16 Cont'd

- Case 4: $\mathcal{I}, \mathcal{V}, [b, e] \models \lceil \mathsf{purge} \rceil$; $true \wedge \ell \leq 30$

$$\lceil \mathsf{purge} \rceil \longrightarrow \lceil \mathsf{purge} \vee \mathsf{ignite} \rceil \qquad \text{(Seq-2)}$$

$\mathcal{I}, \mathcal{V}, [b,e] \models \lceil \text{purge} \rceil \vee \lceil \text{purge} \rceil ; \lceil \text{ignite} \rceil ; true$

3.5, Case 3: $\mathcal{I}, \mathcal{V}, [b,e] \models \int L \leq \varepsilon \vee \left(\int L \leq \varepsilon ; \int L \leq 0.5 + 5\varepsilon \right)$

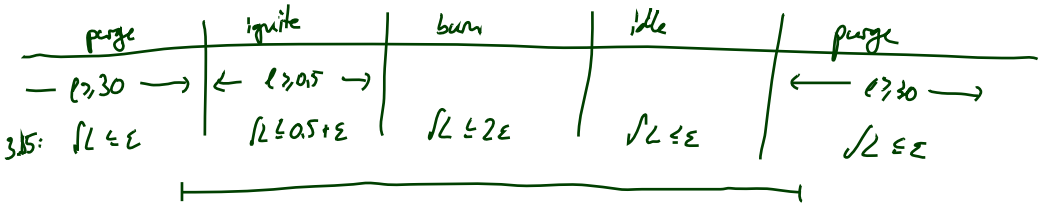
$\mathcal{I}, \mathcal{V}, [b,e] \models \int L \leq 0.5 + 6\varepsilon$

Thus $\boxed{\varepsilon \leq \frac{1}{12}}$ is sufficient for Req. 1 in this case.

Correctness Result

Theorem 3.17.

$$\models \left(\text{GB-Ctrl} \wedge \varepsilon \leq \frac{1}{12}\right) \implies \text{Req}$$



↳ $\int L \leq 0.5 + 6\varepsilon$ in the worst case

↳ $A(\varepsilon) :\Leftrightarrow \varepsilon \leq \frac{1}{12}$

Discussion

- We used only

  'Seq-1', 'Seq-2', 'Seq-3', 'Seq-4',
  'Prog-2', 'Syn-2', 'Syn-3',
  'Stab-2', 'Stab-5', 'Stab-6'.

What about

$$\text{Prog-1} = \lceil \text{purge} \rceil \xrightarrow{30+\varepsilon} \lceil \neg\text{purge} \rceil$$

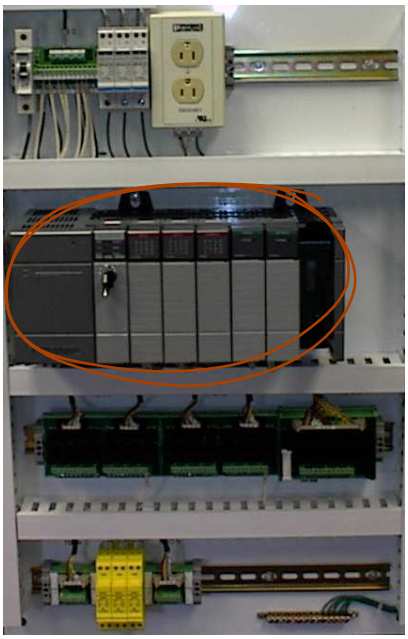
$\lceil \text{idle} \wedge H \rceil \xrightarrow{\varepsilon} \lceil \neg \text{idle} \rceil$

for instance?

Nein, there is the requirement (not explicitly noted down) that the system <u>does</u> something finally, e.g. get the heating going on regular.

What is a PLC?

How do PLC look like?



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What's special about PLC?

PLC - programmable logic controller
SPS - Speicherprogrammierbare Steuerung

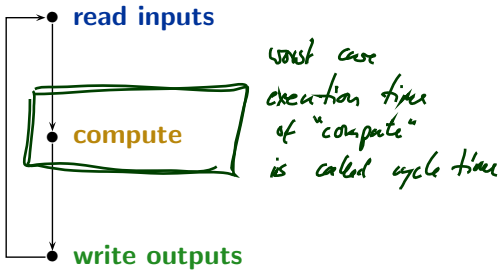
- microprocessor, memory, **timers**
- digital (or analog) I/O ports
- possibly RS 232, fieldbuses, networking
- robust hardware
- reprogrammable
- **standardised programming model** (IEC 61131-3)

Where are PLC employed?

- mostly **process automatisation**
  - production lines
  - packaging lines
  - chemical plants
  - power plants
  - electric motors, pneumatic or hydraulic cylinders
  - ...
- not so much: **product automatisation**, there
  - tailored or OTS controller boards
  - embedded controllers
  - ...

How are PLC programmed?

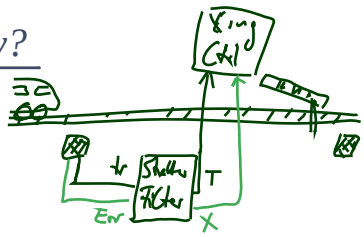
- PLC have in common that they operate in a cyclic manner:



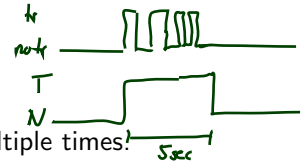
- Cyclic operation is repeated until external interruption (such as shutdown or reset).
- Cycle time: typically a few milliseconds. [Lukoschus, 2004]

- Programming for PLC means providing the "**compute**" part.
- Input/output values are available via designated local variables.

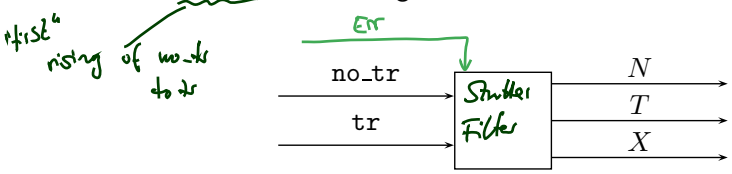
How are PLC programmed, practically?



- **Example:** reliable, stutter-free train sensor.
  - Assume a track-side sensor with outputs:
    - `no_tr` — "no passing train"
    - `tr` — "a train is passing"
  - Assume that a change from "`no_tr`" to "`tr`" signals arrival of a train. (No spurious sensor values.)

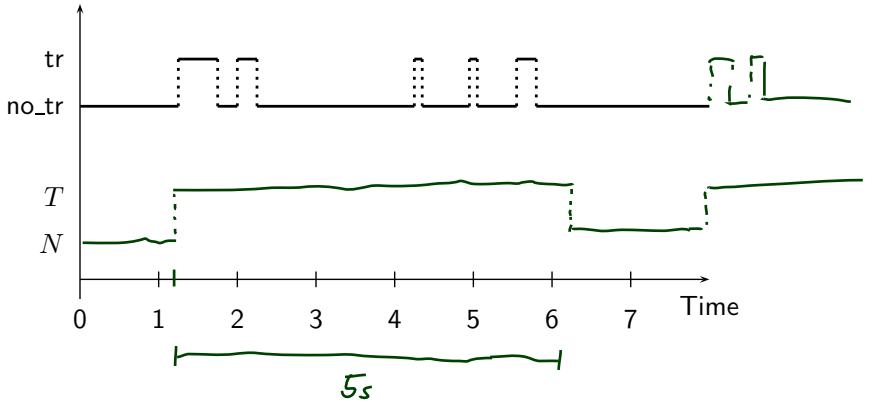


  - **Problem:** the sensor may **stutter**, i.e. oscillate between "`no_tr`" and "`tr`" multiple times!
  - **Idea:** a stutter **filter** with outputs N and T, for "no train" and "train passing" (and possibly X, for error).
    After arrival of a train, ignore "`no_tr`" for 5 seconds.



Example: Stutter Filter

- **Idea:** After arrival of a train, ignore "no_tr" for 5 seconds.



How are PLC programmed, practically?

```
 1: PROGRAM PLC_PRG_FILTER
 2: VAR
 3:     state : INT := 0; (* 0:=N, 1:=T, 2:=X *)
 4:     tmr   : TON;
 5: ENDVAR
 6:
 7: IF state = 0 THEN
 8:     %output := N;
 9:     IF %input = tr THEN
10:         state := 1;
11:         %output := T;
12:     ELSIF %input = Error THEN
13:         state := 2;
14:         %output := X;
15:     ENDIF
16: ELSIF state = 1 THEN
17:     tmr( IN := TRUE, PT := t#5.0s );
18:     IF (%input = no_tr AND NOT tmr.Q) THEN
19:         state := 0;
20:         %output := N;
21:         tmr( IN := FALSE, PT := t#0.0s );
22:     ELSIF %input = Error THEN
23:         state := 2;
24:         %output := X;
25:         tmr( IN := FALSE, PT := t#0.0s );
26:     ENDIF
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```

How are PLC programmed, practically?

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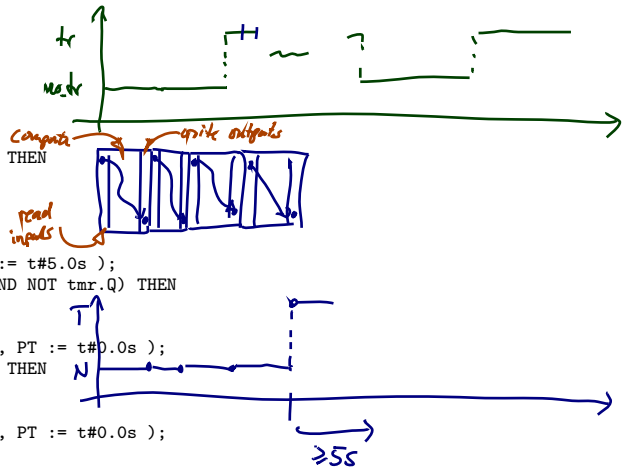
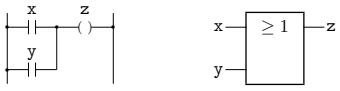
26:     ENDIF
27: ENDIF
```



Alternative Programming Languages by IEC 61131-3

```
LD  x
OR  y
ST  z
```
Instruction List

```
z := x OR y
```
Structured Text



(Relay) Ladder Diagram Function Block Diagram

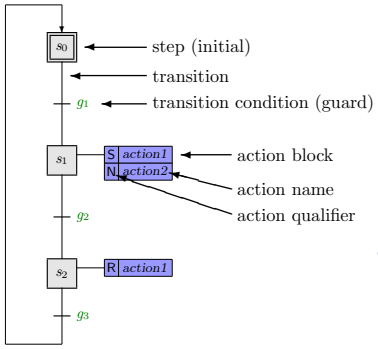
[Lukoschus, 2004]

Figure 2.2: Implementations of the operation "x becomes $y \vee z$"

Tied together by

- Sequential Function Charts (SFC)

Unfortunate: deviations in semantics... [Bauer, 2003]



[Lukoschus, 2004]

Figure 2.3: Elements of sequential function charts

Why study PLC?

- **Note:**
  the discussion here is **not limited** to PLC and IEC 61131-3 languages.
- Any programming language on an operating system with **at least one** real-time clock will do.
  (Where a **real-time clock** is a piece of hardware such that,
  - we can program it to wait for t time units,
  - we can query whether the set time has elapsed,
  - if we program it to wait for t time units,
    it does so with negligible deviation.)
- And strictly speaking, we don't even need "full blown" operating systems.
- PLC are just a formalisation on a good level of abstraction:
  - there are inputs **somehow** available as local variables,
  - there are outputs **somehow** available as local variables,
  - **somehow**, inputs are polled and outputs updated atomically,
  - there is **some** interface to a real-time clock.

PLC Automata

PLC Automata

Definition 5.2. A **PLC-Automaton** is a structure

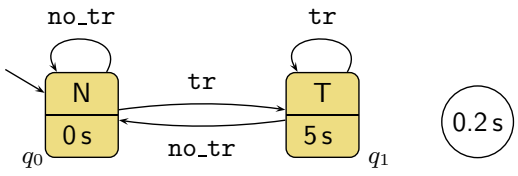
$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$$

where

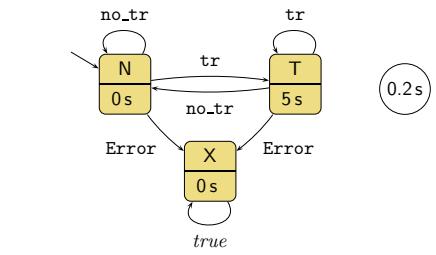
- $(q \in)\ Q$ is a finite set of **states**, $q_0 \in Q$ is the **initial state**,
- $(\sigma \in)\ \Sigma$ is a finite set of **inputs**,
- $\delta : Q \times \Sigma \to Q$ is the **transition function** (!),
- $S_t : Q \to \mathbb{R}_0^+$ assigns a **delay time** to each state,
- $S_e : Q \to 2^\Sigma$ assigns a set of **delayed inputs** to each state,
- Ω is a finite, non-empty set of **outputs**,
- $\omega : Q \to \Omega$ assigns an **output** to each state,
- ε is an **upper time bound** for the execution cycle.

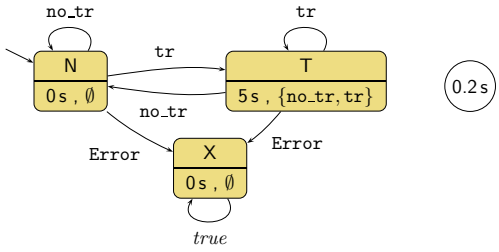
PLC Automata Example: Stuttering Filter

$$\begin{aligned}\mathcal{A} = (&Q = \{q_0, q_1\},\\ &\Sigma = \{\texttt{tr}, \texttt{no\_tr}\},\\ &\delta = \{(q_0, \texttt{tr}) \mapsto q_1, (q_0, \texttt{no\_tr}) \mapsto q_0, (q_1, \texttt{tr}) \mapsto q_1, (q_1, \texttt{no\_tr}) \mapsto q_0\},\\ &q_0 = q_0,\\ &\varepsilon = 0.2,\\ &S_t = \{q_0 \mapsto 0, q_1 \mapsto 5\},\\ &S_e = \{q_0 \mapsto \emptyset, q_1 \mapsto \Sigma\},\\ &\Omega = \{N, T\},\\ &\omega = \{q_0 \mapsto N, q_1 \mapsto T\})\end{aligned}$$



PLC Automata Example: Stuttering Filter with Exception





PLC Automaton Semantics

```
 1: PROGRAM PLC_PRG_FILTER
 2: VAR
 3:   state : INT := 0; (* 0:=N, 1:=T, 2:=X *)
 4:   tmr   : TP;
 5: ENDVAR
 6:
 7: IF state = 0 THEN
 8:     %output := N;
 9:     IF %input = tr THEN
10:         state := 1;
11:         %output := T;
12:     ELSIF %input = Error THEN
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14:         %output := X;
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16: ELSIF state = 1 THEN
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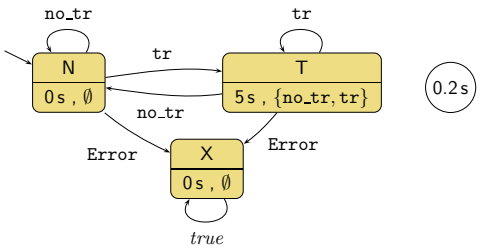
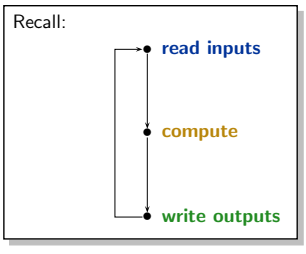
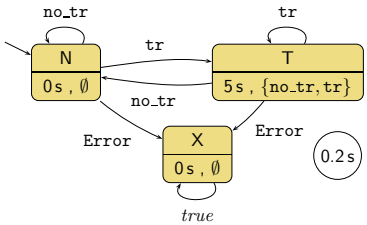
27: ENDIF
```





PLCA Semantics: Examples

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 1: PROGRAM PLC_PRG_FILTER
 2: VAR
 3:   state : INT := 0; (* 0:=N, 1:=T, 2:=X *)
 4:   tmr   : TP;
 5: ENDVAR
 6:
 7: IF state = 0 THEN
 8:     %output := N;
 9:     IF %input = tr THEN
10:         state := 1;
11:         %output := T;
12:     ELSIF %input = Error THEN
13:         state := 2;
14:         %output := X;
15:     ENDIF
16: ELSIF state = 1 THEN
17:     tmr( IN := TRUE, PT := t#5.0s );
18:     IF (%input = no_tr AND NOT tmr.Q) THEN
19:         state := 0;
20:         %output := N;
21:         tmr( IN := FALSE, PT := t#0.0s );
22:     ELSIF %input = Error THEN
23:         state := 2;
24:         %output := X;
25:         tmr( IN := FALSE, PT := t#0.0s );
26:     ENDIF
27: ENDIF
```
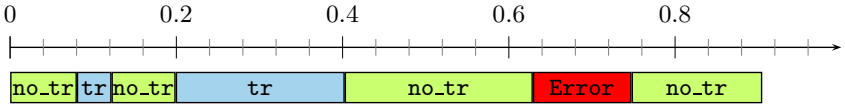



We assess correctness in terms of cycle time...

...but where does the cycle time come from?

- First of all, ST on the hardware **has** a cycle time
  - so we can **measure** it — if it is larger than ε, don't use this program on this controller
  - we can **estimate** (approximate) the **WCET** (worst case execution time) — if it's larger than ε, don't use it, if it's smaller we're safe

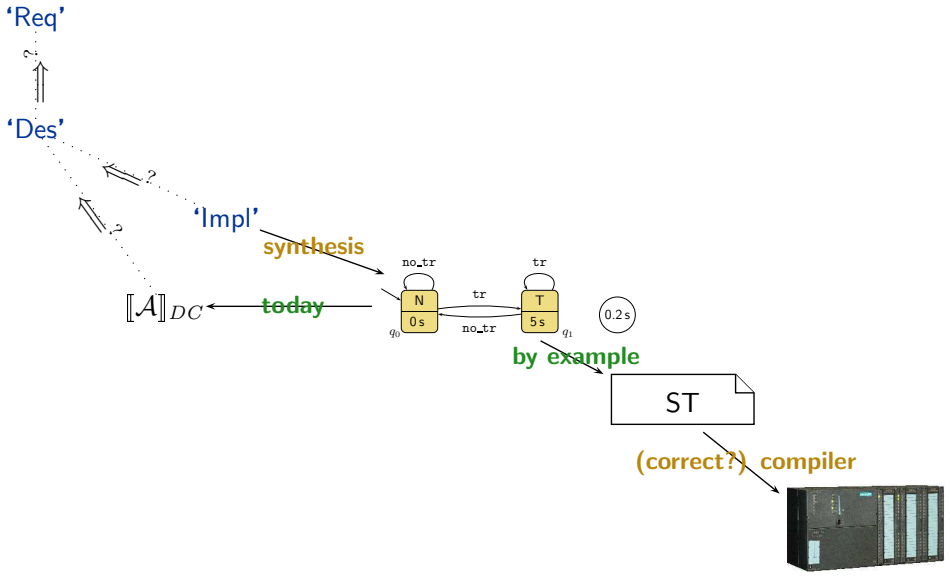
    (Major obstacle: caches, out-of-order execution.)

- Some PLC have a **watchdog**:
  - set it to ε,
  - if the current "computing" cycle takes longer,
  - then the watchdog forces the PLC into an error state and signals the error condition

And what does this have to with DC?

Wait, what is the Plan?

Full DC | DC Implementables | PLC-Automata | IEC 61131-3 | Binary



An Overapproximating DC Semantics for PLC Automata

Interesting Overall Approach

- Define PLC Automaton syntax (abstract and concrete).
- Define PLC Automaton semantics by translation to ST (structured text).
- Give DC **over-approximation** of PLC Automaton semantics.
- Assess correctness of over-approximation against DC **requirements**.

- **In other words:** we'll define $[\![\mathcal{A}]\!]_{DC}$ such that

$$\text{“}\mathcal{I} \in [\![\mathcal{A}]\!]\text{”} \implies \mathcal{I} \models [\![\mathcal{A}]\!]_{DC}$$

  but not necessarily the other way round.
- **In even other words:** $\text{“}[\![\mathcal{A}]\!]\text{”} \subseteq \{\mathcal{I} \mid \mathcal{I} \models [\![\mathcal{A}]\!]_{DC}\}$.

Observables

- Consider
$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega).$$

- The DC formula $[\![\mathcal{A}]\!]_{DC}$ we construct ranges over the observables
  - $\mathsf{In}_{\mathcal{A}}$, $\mathcal{D}(\mathsf{In}_{\mathcal{A}}) = \Sigma$ — values of the **inputs**
  - $\mathsf{St}_{\mathcal{A}}$, $\mathcal{D}(\mathsf{St}_{\mathcal{A}}) = Q$ — current **local state**
  - $\mathsf{Out}_{\mathcal{A}}$, $\mathcal{D}(\mathsf{Out}_{\mathcal{A}}) = \Omega$ — values of the **outputs**

Overview

$$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$$

- A arbitrary with $\emptyset \neq A \subseteq \Sigma$,
- $\lceil q \wedge A \rceil$ abbreviates $\lceil \mathsf{St}_{\mathcal{A}} = q \wedge \mathsf{In}_{\mathcal{A}} \in A \rceil$,
- $\delta(q, A)$ abbreviates $\mathsf{St}_{\mathcal{A}} \in \{\delta(q, a) \mid a \in A\}$.

- **Initial State**:
$$\lceil\rceil \vee \lceil q_0 \rceil \,;\, true \tag{DC-1}$$

- **Effect of Transitions, untimed**:
$$\lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \longrightarrow \lceil q \vee \delta(q, A) \rceil \tag{DC-2}$$

- **Cycle time**:
$$\lceil q \wedge A \rceil \xrightarrow{\varepsilon} \lceil q \vee \delta(q, A) \rceil \tag{DC-3}$$

- **Delays**:
$$S_t(q) > 0 \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil \xrightarrow{\leq S_t(q)} \lceil q \vee \delta(q, A \setminus S_e(q)) \rceil \tag{DC-4}$$
$$S_t(q) > 0 \implies \lceil \neg q \rceil \,;\, \lceil q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \xrightarrow{\leq S_t(q)} \lceil q \vee \delta(q, A \setminus S_e(q)) \rceil \tag{DC-5}$$

Overview

$\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$

- A arbitrary with $\emptyset \neq A \subseteq \Sigma$,
- $\lceil q \wedge A \rceil$ abbreviates $\lceil \mathsf{St}_{\mathcal{A}} = q \wedge \mathsf{In}_{\mathcal{A}} \in A \rceil$,
- $\delta(q, A)$ abbreviates $\mathsf{St}_{\mathcal{A}} \in \{\delta(q, a) \mid a \in A\}$.

- **Progress from non-delayed inputs**:

$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \Box(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \quad \text{(DC-6)}$$

$$S_t(q) = 0 \wedge q \notin \delta(q, A) \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \longrightarrow \lceil \neg q \rceil \quad \text{(DC-7)}$$

- **Progress from delayed inputs**:

$$S_t(q) > 0 \wedge q \notin \delta(q, A) \implies \Box(\lceil q \rceil^{S_t(q)} \,;\, \lceil q \wedge A \rceil \implies \ell < S_t(q) + 2\varepsilon) \quad \text{(DC-8)}$$

$$S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \implies \Box(\lceil q \wedge A \rceil \implies \ell < 2\varepsilon) \quad \text{(DC-9)}$$

$$S_t(q) > 0 \wedge A \cap S_e(q) = \emptyset \wedge q \notin \delta(q, A) \implies \lceil \neg q \rceil \,;\, \lceil q \wedge A \rceil^{\varepsilon} \longrightarrow \lceil \neg q \rceil \quad \text{(DC-10)}$$

Behaviour of the Output and System Start

$$\Box(\lceil q \rceil \implies \lceil \omega(q) \rceil) \quad \text{(DC-11)}$$

$$\lceil q_0 \wedge A \rceil \longrightarrow_0 \lceil q_0 \vee \delta(q_0, A) \rceil \quad \text{(DC-2')}$$

$$S_t(q_0) > 0 \implies \lceil q_0 \wedge A \rceil \xrightarrow{\leq S_t(q_0)}_0 \lceil q_0 \vee \delta(q_0, A \setminus S_e(q_0)) \rceil \quad \text{(DC-4')}$$

$$S_t(q_0) > 0 \implies \lceil q_0 \rceil \,;\, \lceil q_0 \wedge A \rceil^{\varepsilon} \xrightarrow{\leq S_t(q_0)}_0 \lceil q_0 \vee \delta(q_0, A \setminus S_e(q_0)) \rceil \quad \text{(DC-5')}$$

$$S_t(q_0) = 0 \wedge q_0 \notin \delta(q_0, A) \implies \lceil q_0 \wedge A \rceil^{\varepsilon} \longrightarrow_0 \lceil \neg q_0 \rceil \quad \text{(DC-7')}$$

$$S_t(q_0) > 0 \wedge A \cap S_e(q_0) = \emptyset \wedge q_0 \notin \delta(q_0, A) \implies \lceil q_0 \wedge A \rceil^{\varepsilon} \longrightarrow_0 \lceil \neg q_0 \rceil \quad \text{(DC-10')}$$

DC Semantics of PLC Automata

Definition 5.3.
The **Duration Calculus semantics** of a PLC Automaton $\mathcal{A}$ is

$$[\![\mathcal{A}]\!]_{DC} := \bigwedge_{\substack{q \in Q,\\ \emptyset \neq A \subseteq \Sigma}} \text{DC-1} \wedge \cdots \wedge \text{DC-11} \wedge \text{DC-2}' \wedge \text{DC-4}' \wedge \text{DC-5}' \wedge \text{DC-7}' \wedge \text{DC-10}'.$$

Claim:

- Let $P_{\mathcal{A}}$ be the ST program semantics of $\mathcal{A}$.
- Let π be a recording over time of then inputs, local states, and outputs of a PLC device running $P_{\mathcal{A}}$.
- Let $\mathcal{I}_\pi$ be an encoding of π as an interpretation of $\text{In}_{\mathcal{A}}$, $\text{St}_{\mathcal{A}}$, and $\text{Out}_{\mathcal{A}}$.
- Then $\mathcal{I}_\pi \models [\![\mathcal{A}]\!]_{DC}$.
- But not necessarily the other way round.

One Application: Reaction Times

One Application: Reaction Times

- Given a PLC-Automaton, one often wants to know whether it guarantees properties of the form

$$\lceil \mathsf{St}_{\mathcal{A}} \in Q \wedge \mathsf{In}_{\mathcal{A}} = \mathit{emergency\_signal} \rceil \xrightarrow{0.1} \lceil \mathsf{St}_{\mathcal{A}} = \mathit{motor\_off} \rceil$$

("**whenever** the **emergency signal** is observed,
the PLC Automaton switches the **motor off** **within at most** 0.1 seconds")

- Which is (**why?**) far from obvious from the PLC Automaton in general.
- We will give a theorem, that allows us to compute an upper bound on such reaction times.
- Then in the above example, we could simply compare this upper bound one against the required 0.1 seconds.

The Reaction Time Problem in General

- Let
  - $\Pi \subseteq Q$ be a set of **start states**,
  - $A \subseteq \Sigma$ be a set of **inputs**,
  - $c \in \mathsf{Time}$ be a **time bound**, and
  - $\Pi_{target} \subseteq Q$ be a set of **target states**.
- Then we seek to establish properties of the form

$$\lceil \mathsf{St}_{\mathcal{A}} \in \Pi \wedge \mathsf{In}_{\mathcal{A}} \in A \rceil \xrightarrow{c} \lceil \mathsf{St}_{\mathcal{A}} \in \Pi_{target} \rceil,$$

abbreviated as

$$\lceil \Pi \wedge A \rceil \xrightarrow{c} \lceil \Pi_{target} \rceil.$$

Reaction Time Theorem Premises

- Actually, the reaction time theorem addresses **only** the **special case**

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \underbrace{\delta^n(\Pi, A)}_{=\Pi_{target}} \rceil$$

for PLC Automata with

$$\delta(\Pi, A) \subseteq \Pi.$$

- Where the transition function is canonically **extended** to **sets** of start states and inputs:

$$\delta(\Pi, A) := \{\delta(q, a) \mid q \in \Pi \wedge a \in A\}.$$

Reaction Time Theorem (Special Case $n = 1$)

Theorem 5.6. Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$, $\Pi \subseteq Q$, and $A \subseteq \Sigma$ with

$$\delta(\Pi, A) \subseteq \Pi.$$

Then

$$\lceil \Pi \wedge A \rceil \xrightarrow{c} \lceil \underbrace{\delta(\Pi, A)}_{=\Pi_{target}} \rceil$$

where

$$c := \varepsilon + \max(\{0\} \cup \{s(\pi, A) \mid \pi \in \Pi \setminus \delta(\Pi, A)\})$$

and

$$s(\pi, A) := \begin{cases} S_t(\pi) + 2\varepsilon & \text{, if } S_t(\pi) > 0 \text{ and } A \cap S_e(\pi) \neq \emptyset \\ \varepsilon & \text{, otherwise.} \end{cases}$$

Reaction Time Theorem (General Case)

Theorem 5.8. Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, \varepsilon, S_t, S_e, \Omega, \omega)$, $\Pi \subseteq Q$, and $A \subseteq \Sigma$ with

$$\delta(\Pi, A) \subseteq \Pi.$$

Then for all $n \in \mathbb{N}_0$,

$$\lceil \Pi \wedge A \rceil \xrightarrow{c_n} \lceil \underbrace{\delta^n(\Pi, A)}_{=\Pi_{target}} \rceil$$

where

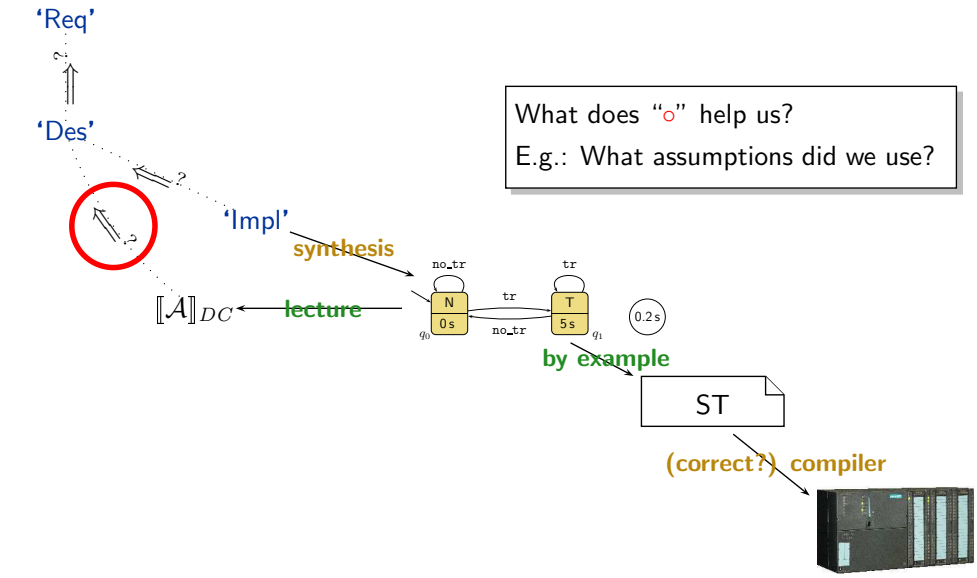
$$c_n := \varepsilon + \max\left(\{0\} \cup \left\{ \sum_{i=1}^{k} s(\pi_i, A) \;\middle|\; \begin{array}{l} 1 \leq k \leq n \wedge \\ \exists\, \pi_1, \ldots, \pi_k \in \Pi \setminus \delta^n(\Pi, A) \\ \quad \forall\, j \in \{1, \ldots, k-1\} : \\ \qquad \pi_{j+1} \in \delta(\pi_j, A) \end{array} \right\} \right)$$

and $s(\pi, A)$ as before.

Methodology: Overview

Methodology

Full DC | DC Implementables | PLC-Automata | IEC 61131-3 | Binary



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