Recall: Putting It All Together

Let $A = (L, B, X, I, E, \ell_{ini})$ be a timed automaton, $\ell \in L$ a location.

- $R(A)$ can be constructed effectively.
- There are finitely many locations in $L$ (by definition).
- There are finitely many regions by Lemma 4.28.
- So $Conf(R(A))$ is finite (by construction).
- It is decidable whether the configuration of a region $\emptyset$ is empty or whether there exists a sequence $\langle \ell_{ini}, [\nu_{ini}] \rangle \xrightarrow{\lambda_1} \langle \ell_1, [\nu_1] \rangle \xrightarrow{\lambda_2} \langle \ell_2, [\nu_2] \rangle \ldots \xrightarrow{\lambda_n} \langle \ell_n, [\nu_n] \rangle$ such that $\ell_n = \ell$ (reachability in graphs).

So we have

Theorem 4.33.

[Decidability] The location reachability problem for timed automata is decidable.

The Constraint Reachability Problem

- Given: A timed automaton $A$, one of its control locations $\ell$, and a clock constraint $\varphi$.
- Question: Is a configuration $\langle \ell, \nu \rangle$ reachable where $\nu | ; = \varphi$, i.e. is there a transition sequence of the form $\langle \ell_{ini}, \nu_{ini} \rangle \xrightarrow{\lambda_1} \langle \ell_1, \nu_1 \rangle \xrightarrow{\lambda_2} \langle \ell_2, \nu_2 \rangle \ldots \xrightarrow{\lambda_n} \langle \ell_n, \nu_n \rangle = \langle \ell, \nu \rangle$ in the labelled transition system $T(A)$ with $\nu | ; = \varphi$?

Note: We just observed that $R(A)$ loses some information about the clock valuation that are possible in/from a region.

Theorem 4.34.

The constraint reachability problem for timed automata is decidable.

The Delay Operation

- Let $[\nu]$ be a clock region.
- We set $\text{delay}([\nu]) = \{ \nu' + t | \nu' \sim = \nu \land t \in \text{Time} \}$.
The zone $A$ is an equivalence class of clocks with the property that $\forall x, y \in A \ (x \leq y) = \Delta$.

2) $x > 1$ and $x < 1$ are not one-on-one correspondences between clock constraints and zones. There's no one-on-one correspondence between clock constraints and zones. Therefore, we cannot represent the zone $A$ using a simple interval $[0, \infty)$.

3) $x < 0$ implies that $x$ is in the negative half-plane.

The zone $A$ is an equivalence class of clocks with the property that $\forall x, y \in A \ (x \leq y) = \Delta$. Therefore, we cannot represent the zone $A$ using a simple interval $[0, \infty)$.
To this end: remove all upper bounds. Algorithm general: \( \phi \in \nu \rightarrow C^2 \in \nu \in \phi \rightarrow C^3 \).

\( L \) by a clock reset. \( \phi \) of given configuration \( \text{Post } e \). \( \ell, z \) is not already reachable in \( \nu \rightarrow C^3 \). \( \phi \mid_{C^2} \). \( \{ \nu \rightarrow C^3 \} = \{ \nu \rightarrow C^3 \} \). \( \ell, z \) with source. \( \text{Post } e \) off. \( \ell, z \) \( \ell, z \) is reachable from a configuration \( \nu \rightarrow C^3 \). Wanted: \( \nu \rightarrow C^3 \). Good news! \( \phi \rightarrow C^3 \).

\( \nu \rightarrow C^3 \) are found. \( \nu \rightarrow C^3 \) are reachable from a configuration \( \nu \rightarrow C^3 \). Wanted: \( \nu \rightarrow C^3 \). Good news! \( \phi \rightarrow C^3 \).

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Difference Bound Matrices

Extended Timed Automata

Pros and cons

Then we can efficiently compute

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\phi = \bigwedge_{i} \bigvee_{j} c_{i,j} \sim x_{j}
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...because given

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