Real-Time Systems

Lecture 14: Extended Timed Automata

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Contents & Goals

Last Lecture:

- Decidability of the location reachability problem: zones.
- Extended TA: data variables.

This Lecture:

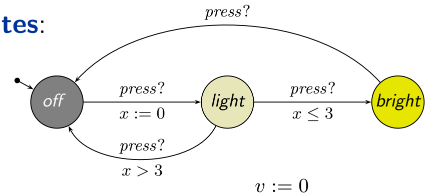
- **Educational Objectives:** Capabilities for following tasks/questions.
  - By what are TA extended? Why is that useful?
  - What's an urgent/committed location? What's the difference?
  - What's an urgent channel?
  - Where has the notion of "input action" and "output action" correspondences in the formal semantics?
- **Content:**
  - Extended TA:
    - Structuring Facilities
    - Restriction of Non-Determinism
  - The Logic of Uppaal

Extended Timed Automata

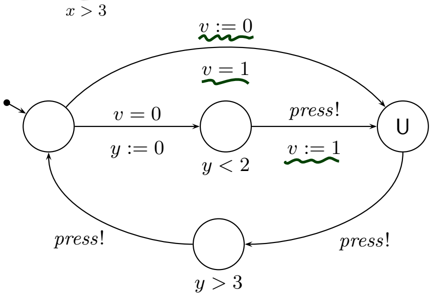
Recall: Example (Partly Already Seen in Uppaal Demo)

Templates:

- $\mathcal{L}$:



- $\mathcal{U}$:



Extensions:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Location, Urgent Channel

System:

Recall: Data Variables and Expressions

- Let $(v, w \in)\ V$ be a set of (integer) variables.

  $(\psi_{int} \in)\ \Psi(V)$: **integer expressions** over V using func. symb. $+, -, \dots$

  $(\varphi_{int} \in)\ \Phi(V)$: **integer** (or **data**) **constraints** over V
  using **integer expressions**, predicate symbols $=, <, \leq, \dots$, and
  boolean logical connectives. (incl. $\vee, \neg, \wedge, \Rightarrow, \Leftarrow, \dot{\vee}, \dots$)
- Recall: $\Phi(X)$ for clocks X (simple clock constraint)
- Let $(x, y \in)\ X$ be a set of clocks.

  $(\varphi \in)\ \Phi(X, V)$: (**extended**) **guards**, defined by

  $$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

  where $\varphi_{clk} \in \Phi(X)$ is a simple clock constraint (as defined before)
  and $\varphi_{int} \in \Phi(V)$ an **integer (or data) constraint**.

$x - y < 0$

Examples: Extended guard or not extended guard? Why?

(a) $\underbrace{x < y}_{\in \Phi(X)} \wedge \underbrace{v > 2}_{\in \Phi(V)}$, ✓ (b) $\underbrace{x < y}_{\in \Phi(X)} \underset{\uparrow\ \text{no}}{\vee} \underbrace{v > 2}_{\in \Phi(V)}$, ✗ (c) $\underbrace{v < 1 \vee v > 2}_{\in \Phi(V)}$, ✓ (d) $\underbrace{x < v}_{\notin \Phi(X),\ \notin \Phi(V)}$ ✗

Modification or Reset Operation

- **New**: a **modification** or **reset operation** is

  $$x := 0, \qquad x \in X,$$

  or

  $$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$
- By $R(X, V)$ we denote the set of all resets.
- By $\vec{r}$ we denote a finite list $\langle r_1, \dots, r_n \rangle$, $n \in \mathbb{N}_0$,
  of reset operations $r_i \in R(X, V)$;
  $\langle\rangle$ is the empty list.
- By $R(X, V)^*$ we denote the set of all such lists of reset operations.

Modification or Reset Operation

OLD:
$(\ell, \alpha, \varphi, Y, \ell')$
NEW:
$(\ell, \alpha, \varphi, \vec{r}, \ell')$

- **New**: a **modification** or **reset** (**operation**) is

$$x := 0, \qquad x \in X,$$

or

$$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$

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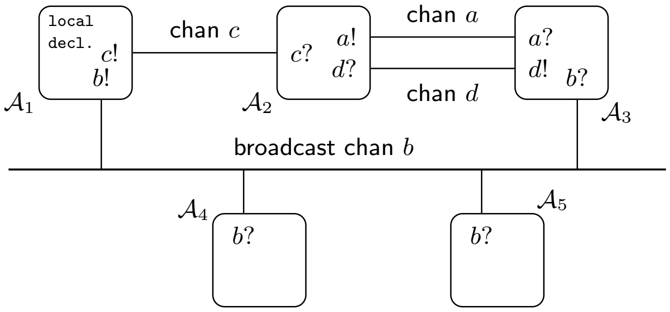
Examples: Modification or not? Why?

(a) $x := y$, (b) $x := v$, (c) $v := x$, (d) $v := w$, (e) $v := 0$

(a) clock, not 0 ✗
(b) not 0 ✗
(c) $\notin \Psi(V)$ ✗
(d) $\in V$, $\in \Psi(V)$ ✓
(e) $\in \Psi(V)$ ✓

Structuring Facilities

`global decl.:  clocks, variables, channels, constants`

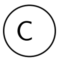


- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels: chan c and broadcast chan b.
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.

Restricting Non-determinism

- **Urgent locations** — enforce local immediate progress.

- **Committed locations** — enforce **atomic** immediate progress.

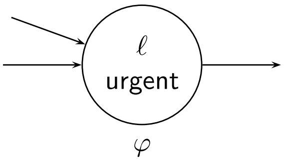


- **Urgent channels** — enforce cooperative immediate progress.

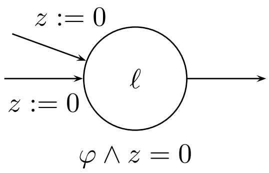
`urgent chan press;`

Urgent Locations: Only an Abbreviation...

Replace



with



where z is a fresh clock:

- reset z on all in-going egdes,
- add $z = 0$ to invariant.

$N=3$
$|U|=20$
each aut.
at least one

- 1
- 3 ✓
-
- 20

Question: How many fresh clocks do we need in the worst case for a network of N extended timed automata?

Extended Timed Automata

Definition 4.39. An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where L, B, X, I, ℓ_{ini} are as in Def. 4.3, except that location invariants in I are **downward closed**, and where

- $C \subseteq L$: **committed locations**,
- $U \subseteq B$: **urgent channels**,
- V: a set of data variables,
- $E \subseteq L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$: a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \mathsf{chan}(\alpha) \in U \implies \varphi = \mathit{true}.$$

Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a list $\vec{r}$ of **reset operations**.

Operational Semantics of Networks

Definition 4.40. Let $\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$, $1 \leq i \leq n$, be extended timed automata with pairwise disjoint sets of clocks X_i.

The operational semantics of $\mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n})$ (closed!) is the labelled transition system

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n}))$$
$$= (\mathit{Conf}, \mathsf{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} \mid \lambda \in \mathsf{Time} \cup \{\tau\}\}, C_{ini})$$

where

- $X = \bigcup_{i=1}^{n} X_i$ and $V = \bigcup_{i=1}^{n} V_i$, $\mathcal{D}(V)$
- $\mathit{Conf} = \{\langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : X \cup V \to \mathsf{Time},\ \nu \models \bigwedge_{k=1}^{n} I_k(\ell_k)\}$,
- $C_{ini} = \{\langle \vec{\ell}_{ini}, \nu_{ini} \rangle\} \cap \mathit{Conf}$,

and the transition relation consists of transitions of the following three types.

Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.

- $\eta ::= v \mid f(v_1, \dots, v_n)$
  $v \in V$, $f \in \mathcal{F}$
- $\mathcal{D}(v) \subseteq \mathbb{Z}$
- $\mathcal{I} : \mathcal{F} \to \bigcup_{n \in \mathbb{N}} (\mathbb{Z}^n \to \mathbb{Z})$
- $\nu : V \to \mathcal{D}(v)$
- $\nu(f(v_1 \dots v_n)) := \mathcal{I}(f)(\nu(v_1), \dots, \nu(v_n))$

Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.
- Extended **timeshift** $\nu + t$, $t \in \mathsf{Time}$, applies to clocks only:
  - $(\nu + t)(x) := \nu(x) + t$, $x \in X$,
  - $(\nu + t)(v) := \nu(v)$, $v \in V$.
- **Effect of modification** $r \in R(X, V)$ on ν, denoted by $\underset{\sim}{\nu[r]}$:

$$\nu[x := 0](a) := \begin{cases} 0, \text{ if } a = x, \\ \nu(a), \text{ otherwise} \end{cases}$$

$$\nu[v := \psi_{int}](a) := \begin{cases} \nu(\psi_{int}), \text{ if } a = v, \\ \nu(a), \text{ otherwise} \end{cases}$$

- We set $\nu[\langle r_1, \dots, r_n \rangle] := \nu[r_1] \dots [r_n] = (((\underset{\sim}{\nu[r_1]})[r_2]) \dots)[r_n]$.

Operational Semantics of Networks: Internal Transitions

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \dots, n\}$ such that

  nr. of automata

  - there is a τ-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell_i') \in E_i$,
  - $\nu \models \varphi$, location of the i-th automaton in $\vec{\ell}$
  - $\vec{\ell}' = \vec{\ell}[\underbrace{\ell_i := \ell_i'}]$, modification of i-th position
  - $\nu' = \nu[\vec{r}]$,
  - $\nu' \models I_i(\ell_i')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$.

    committed locations

Operational Semantics of Networks: Synchronisation Transitio

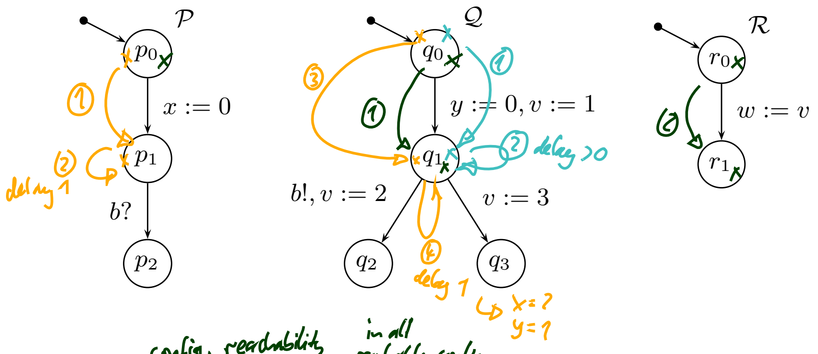
- A **synchronisation transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there are $i, j \in \{1, \dots, n\}$ with $i \neq j$ such that
  - there are edges $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - $\nu \models \varphi_i \wedge \varphi_j$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i'][\ell_j := \ell_j']$,
  - $\nu' = \nu[\vec{r}_i][\vec{r}_j]$, "sender updates are applied first"
  - $\nu' \models I_i(\ell_i') \wedge I_j(\ell_j')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \dots, n\}$ then $\ell_i \in C_i$ or $\ell_j \in C_j$.

Operational Semantics of Networks: Delay Transitions

- A **delay transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{t} \langle \vec{\ell}, \nu + t \rangle$ occurs if
  - $\nu + t \models \bigwedge_{k=1}^{n} I_k(\ell_k)$,
  - (♣) there are no $i, j \in \{1, \dots, n\}$ and $b \in U$ with $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - (♣) there is no $i \in \{1, \dots, n\}$ such that $\ell_i \in C_i$.

urgent channels

Restricting Non-determinism: Example

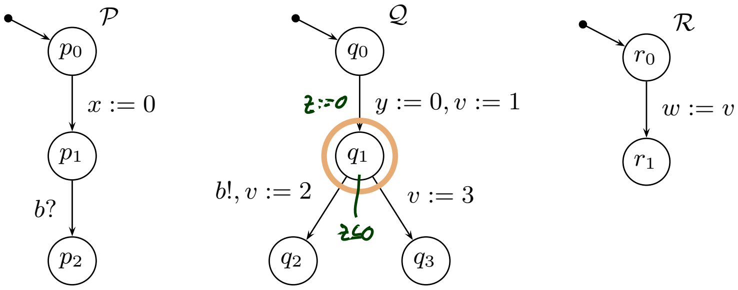


config. reachability

in all reachable config

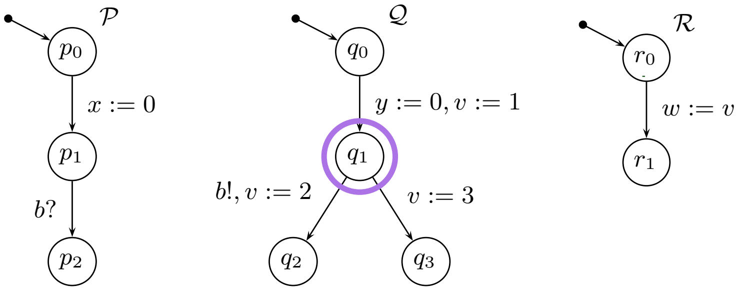
	Property 1 $\exists\Diamond\, w = 1$	Property 2 $\forall\Box(\mathcal{Q}.q_1 \implies y \leq 0)$	Property 3 $\forall\Box(\mathcal{P}.p_1 \land \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N} := \mathcal{P} \Vert \mathcal{Q} \Vert \mathcal{R}$	✓	✗	✗
$\mathcal{N}$, q_1 urgent			
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Location



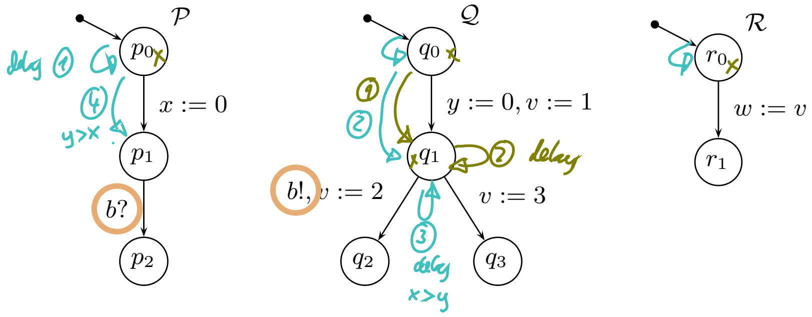
	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✓	✗	✗
$\mathcal{N}$, q_1 urgent	✓	✓	✓
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Committed Location



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✓	✗	✗
$\mathcal{N}$, q_1 urgent	✓	✓	✓
$\mathcal{N}$, q_1 comm.	✗	✓	✓
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Channel



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✔	✔	✔
$\mathcal{N}$, q_1 comm.	✘	✔	✔
$\mathcal{N}$, b urgent	✓	✗	✓

Extended vs. Pure Timed Automata

Extended vs. Pure Timed Automata

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$$

vs.

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$

$$(\ell, \alpha, \varphi, Y, \ell') \in E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$$

- $\mathcal{A}_e$ is in fact (or specialises to) a **pure** timed automaton if
  - $C = \emptyset$,
  - $U = \emptyset$,
  - $V = \emptyset$,
  - for each $\vec{r} = \langle r_1, \dots, r_n \rangle$, every r_i is of the form $x := 0$ with $x \in X$.

  - $I(\ell), \varphi \in \Phi(X)$ is then a consequence of $V = \emptyset$.

Operational Semantics of Extended TA

Theorem 4.41. If $\mathcal{A}_1, \dots, \mathcal{A}_n$ **specialise to pure** timed automata, then the operational semantics of

$$\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$$

and

$$\text{chan}\, b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n),$$

where $\{b_1, \dots, b_m\} = \bigcup_{i=1}^{n} B_i$, **coincide**, i.e.

$$\underbrace{\mathcal{T}_e(\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n))}_{\text{today}} = \underbrace{\mathcal{T}(\text{chan}\, b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n))}_{\text{pure TA}}.$$

Reachability Problems for Extended Timed Automata

Recall

Theorem 4.33. [*Location Reachability*] The location reachability problem for **pure** timed automata is **decidable**.

Theorem 4.34. [*Constraint Reachability*] The constraint reachability problem for **pure** timed automata is **decidable**.

- And what about tea `^W` **extended** timed automata?

What About Extended Timed Automata?

Extended Timed Automata add the following features:

- **Data-Variables**
  - As long as the domains of all variables in V are finite, adding data variables doesn't hurt.
  - If they're infinite, we've got a problem (encode two-counter machine).
- **Structuring Facilities**
  - Don't hurt — they're merely abbreviations.
- **Restricting Non-determinism**
  - Restricting non-determinism doesn't affect the configuration space.
  - Restricting non-determinism only **removes** certain transitions, so makes region automaton even smaller.

The Logic of Uppaal

The Uppaal Fragment of Timed Computation Tree Logic

Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ over data variables V.

- **basic formula**:

  $$atom ::= \mathcal{A}_i.\ell \mid \varphi$$

  where $\ell \in L_i$ is a location and φ a constraint over X_i and V.
- **configuration formulae**:

  $$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$

- **existential path formulae**: (“**exists finally**”, “**exists globally**”)

  $$e\text{-}formula ::= \exists\Diamond\, term \mid \exists\Box\, term$$

- **universal path formulae**: (“**always finally**”, “**always globally**”, “**leads to**”)

  $$a\text{-}formula ::= \forall\Diamond\, term \mid \forall\Box\, term \mid term_1 \longrightarrow term_2$$

- **formulae**:

  $$F ::= e\text{-}formula \mid a\text{-}formula$$

Configurations at Time t

- Recall: **computation path** (or path) **starting in** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$:

  $$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, \underset{0}{t_0} \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, \underset{3.0}{t_1} \xrightarrow{\overset{\tau}{\lambda_2}} \langle \vec{\ell}_2, \nu_2 \rangle, \underset{3.0}{t_2} \xrightarrow{\overset{\in \text{Time} \setminus \{0\}}{\lambda_3}} \dots$$

  which is **infinite or maximally finite**.

- Given ξ and $t \in$ Time, we use $\xi(t)$ to denote the set

  $$\{ \langle \vec{\ell}, \nu \rangle \mid \exists\, i \in \mathbb{N}_0 : t_i \le t \le t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i \}.$$

  of **configurations at time** t.

  - Why is it a set?
  - Can it be empty?

$\xi(0) = \{ \langle \vec{\ell}_0, \nu_0 \rangle \}$

$\xi(0.27) = \{ \langle \vec{\ell}_0, \nu_0 + 0.27 \rangle \}$

$\xi(3.0) = \{ \langle \vec{\ell}_1, \nu_1 \rangle, \langle \vec{\ell}_2, \nu_2 \rangle \}$

Satisfaction of Uppaal-Logic by Configurations

- We define a **satisfaction relation**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models F$$

between **time stamped configurations**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0$$

of a network $\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ and **formulae** F of the Uppaal logic.

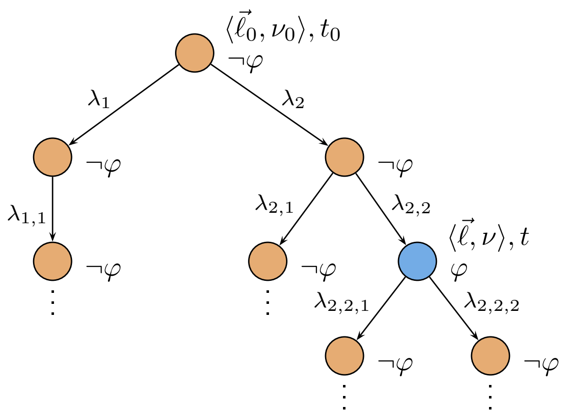
- It is defined inductively as follows: i-th location in $\vec{\ell}_0$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \mathcal{A}_i.\ell$ iff $\ell_{0,i} = \ell$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \varphi$ iff $\nu_0 \models \varphi$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \neg term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models term$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \wedge term_2$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_i$, $i = 1,2$

Satisfaction of Uppaal-Logic by Configurations

Exists finally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Diamond\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\exists t \in \mathsf{Time}, \langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \wedge \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Diamond\, \varphi$



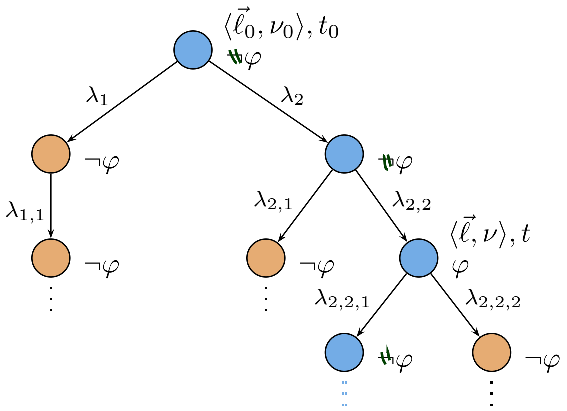
Satisfaction of Uppaal-Logic by Configurations

Exists globally:

note: universally quantifying over all elements in $\xi(t)$

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Box\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in$ Time, $\langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \underline{\xi(t)} \implies \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Box\,\varphi$



Satisfaction of Uppaal-Logic by Configurations

- **Always finally**:

  "∀ path
  ∃ time • term"

  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Diamond\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Box\, \neg term$

- **Always globally**:

  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Box\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Diamond\, \neg term$

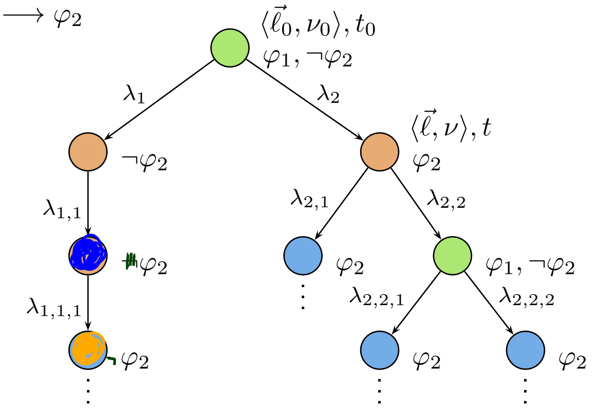
  "∀ path
  ∀ time • term"

Satisfaction of Uppaal-Logic by Configurations

Leads to: (TL: "$AG(term_1 \Rightarrow AF\, term_2)$"

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in \text{Time}, \langle \vec{\ell}, \nu \rangle \in Conf$:
  $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t)$
  $\wedge\, \langle \vec{\ell}, \nu \rangle, t \models term_1$
  implies $\langle \vec{\ell}, \nu \rangle, t \models \forall\Diamond\, term_2$

Example: $\varphi_1 \longrightarrow \varphi_2$

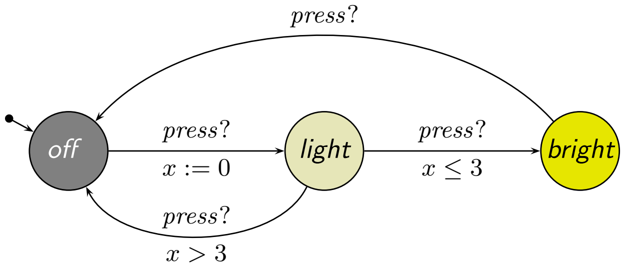


Satisfaction of Uppaal-Logic by Networks

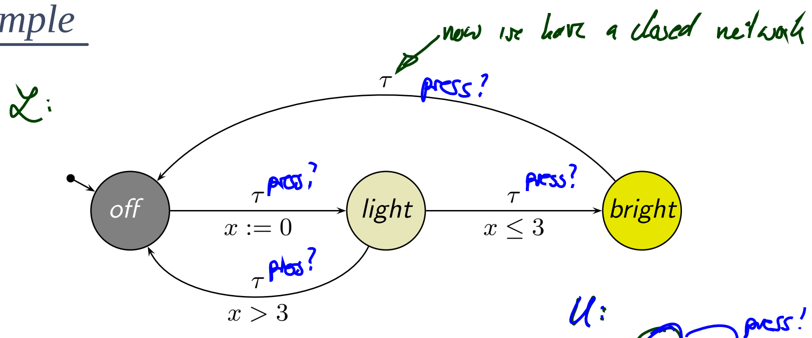
- We write
$$\mathcal{N} \models \textit{e-formula}$$
if and only if
$$\textbf{for some } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models \textit{e-formula}, \qquad (1)$$
and
$$\mathcal{N} \models \textit{a-formula}$$
if and only if
$$\textbf{for all } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models \textit{a-formula}, \qquad (2)$$
where C_{ini} are the initial configurations of $\mathcal{T}_e(\mathcal{N})$.

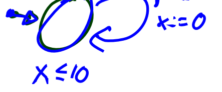
- If $C_{ini} = \emptyset$, (1) is a contradiction and (2) is a tautology.
- If $C_{ini} \neq \emptyset$, then
$$\mathcal{N} \models F \text{ if and only if } \langle \vec{\ell}_{ini}, \nu_{ini} \rangle, 0 \models F.$$

Example



Example





- $\mathcal{N} \models \exists\Diamond\, \mathcal{L}.\mathit{bright}$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.\mathit{bright}$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.\mathit{off}$? ✓
- $\mathcal{N} \models \forall\Diamond\, \mathcal{L}.\mathit{light}$? ✗
- $\mathcal{N} \models \forall\Box (\mathcal{L}.\mathit{bright} \implies x \geq 3)$? ✗ $x \leq 3$ ✗
- $\mathcal{N} \models \mathcal{L}.\mathit{bright} \longrightarrow \mathcal{L}.\mathit{off}$? ✗

- $\mathcal{L} \parallel U \models (\mathcal{L}.\mathit{bright} \longrightarrow \mathcal{L}.\mathit{off})$ ✓

References

References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.