Real-Time Systems

Lecture 14: Extended Timed Automata

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Contents & Goals

Last Lecture:

- Decidability of the location reachability problem: zones.
- Extended TA: data variables.

This Lecture:

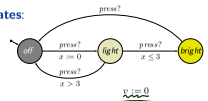
- **Educational Objectives:** Capabilities for following tasks/questions.
  - By what are TA extended? Why is that useful?
  - What's an urgent/committed location? What's the difference?
  - What's an urgent channel?
  - Where has the notion of "input action" and "output action" correspondences in the formal semantics?
- **Content:**
  - Extended TA:
    - Structuring Facilities
    - Restriction of Non-Determinism
  - The Logic of Uppaal

Extended Timed Automata

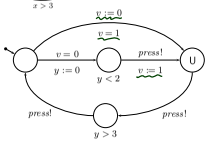
Recall: Example (Partly Already Seen in Uppaal Demo)

Templates:

- $\mathcal{L}$:



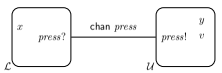
- $\mathcal{U}$:



Extensions:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Location, Urgent Channel

System:



Recall: Data Variables and Expressions

- Let $(v, w \in) V$ be a set of (integer) variables.
  $(\psi_{int} \in) \Psi(V)$: **integer expressions** over V using func. symb. $+, -, \ldots$
  $(\varphi_{int} \in) \Phi(V)$: **integer** (or **data**) **constraints** over V using **integer expressions**, predicate symbols $=, <, \leq, \ldots$, and boolean logical connectives. (w.l. $\vee, \neg, \wedge, \Rightarrow, \Leftrightarrow, \ldots$)
- Recall: $\overline{\mathfrak{B}}(X)$ for clocks X (single clock constraint)
- Let $\langle x, y \in) X$ be a set of clocks.
  $(\varphi \in) \Phi(X, V)$: **(extended) guards**, defined by

$$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

  where $\varphi_{clk} \in \Phi(X)$ is a simple clock constraint (as defined before) and $\varphi_{int} \in \Phi(V)$ an **integer (or data) constraint**.

Examples: Extended guard or not extended guard? Why?

(a) $x < y \wedge v > 2$, (b) $x < y \vee v > 2$, (c) $v < 1 \vee v > 2$, (d) $x < v$

Modification or Reset Operation

- **New**: a **modification** or **reset operation** is

$$x := 0, \qquad x \in X,$$

  or

$$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$

- By $R(X, V)$ we denote the set of all resets.
- By $\vec{r}$ we denote a finite list $\langle r_1, \ldots, r_n \rangle$, $n \in \mathbb{N}_0$, of reset operations $r_i \in R(X, V)$; $\langle\rangle$ is the empty list.
- By $R(X, V)^*$ we denote the set of all such lists of reset operations.

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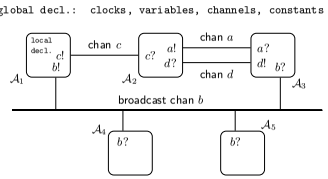
Examples: Modification or not? Why?

(a) $x := y$, (b) $x := v$, (c) $v := x$, (d) $v := w$, (e) $v := 0$

Structuring Facilities



- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels: chan c and broadcast chan b.
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.

Restricting Non-determinism

- **Urgent locations** — enforce local immediate progress.

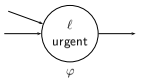
- **Committed locations** — enforce **atomic** immediate progress.

- **Urgent channels** — enforce cooperative immediate progress.

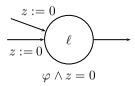
```
urgent chan press;
```


Urgent Locations: Only an Abbreviation...

Replace



with



where z is a fresh clock:

- reset z on all in-going egdes,
- add $z = 0$ to invariant.

Question: How many fresh clocks do we need in the worst case for a network of N extended timed automata?

Extended Timed Automata

Definition 4.39. An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where L, B, X, I, ℓ_{ini} are as in Def. 4.3, except that location invariants in I are **downward closed**, and where

- $C \subseteq L$: **committed locations**,
- $U \subseteq B$: **urgent channels**,
- V: a set of data variables,
- $E \subseteq L \times B_! \times \Phi(X,V) \times R(X,V)^* \times L$: a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \mathsf{chan}(\alpha) \in U \implies \varphi = true.$$

Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a list $\vec{r}$ of **reset operations**.

Operational Semantics of Networks

Definition 4.40. Let $\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$, $1 \leq i \leq n$, be extended timed automata with pairwise disjoint sets of clocks X_i.
The operational semantics of $\mathcal{C}(\mathcal{A}_{e,1}, \ldots, \mathcal{A}_{e,n})$ (closed!) is the labelled transition system

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_{e,1}, \ldots, \mathcal{A}_{e,n})) = (Conf, \mathsf{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} | \ \lambda \in \mathsf{Time} \cup \{\tau\}\}, C_{ini})$$

where

- $X = \bigcup_{i=1}^n X_i$ and $V = \bigcup_{i=1}^n V_i$,
- $Conf = \{\langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : X \cup V \to \mathsf{Time}, \nu \models \bigwedge_{k=1}^n I_k(\ell_k)\}$,
- $C_{ini} = \{\langle \vec{\ell}_{ini}, \nu_{ini} \rangle\} \cap Conf$,

and the transition relation consists of transitions of the following three types.

Helpers: Extended Valuations and Timeshift

- **Now**: $\nu : X \cup V \to \mathsf{Time} \cup \mathcal{D}(V)$
- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.

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Helpers: Extended Valuations and Timeshift

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- Canonically extends to $\nu : \Psi(V) \to \mathcal{D}$ (valuation of expression).
- "$\models$" extends canonically to expressions from $\Phi(X, V)$.
- Extended **timeshift** $\nu + t$, $t \in \mathsf{Time}$, applies to clocks only:
  - $(\nu + t)(x) := \nu(x) + t$, $x \in X$,
  - $(\nu + t)(v) := \nu(v)$, $v \in V$.
- **Effect of modification** $r \in R(X, V)$ on ν, denoted by $\nu[r]$:

$$\nu[x := 0](a) := \begin{cases} 0, \text{ if } a = x, \\ \nu(a), \text{ otherwise} \end{cases}$$

$$\nu[v := \psi_{int}](a) := \begin{cases} \nu(\psi_{int}), \text{ if } a = v, \\ \nu(a), \text{ otherwise} \end{cases}$$

- We set $\nu[\langle r_1, \ldots, r_n \rangle] := \nu[r_1] \ldots [r_n] = (((\nu[r_1])[r_2]) \ldots)[r_n]$.

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Operational Semantics of Networks: Internal Transitions

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \ldots, n\}$ such that
  - there is a τ-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell_i') \in E_i$,
  - $\nu \models \varphi$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i']$,
  - $\nu' = \nu[\vec{r}]$,
  - $\nu' \models I_i(\ell_i')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \ldots, n\}$ then $\ell_i \in C_i$.

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Operational Semantics of Networks: Synchronisation Transitio

- A **synchronisation transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there are $i, j \in \{1, \ldots, n\}$ with $i \neq j$ such that
  - there are edges $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - $\nu \models \varphi_i \wedge \varphi_j$,
  - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i'][\ell_j := \ell_j']$,
  - $\nu' = \nu[\vec{r}_i][\vec{r}_j]$,
  - $\nu' \models I_i(\ell_i') \wedge I_j(\ell_j')$,
  - (♣) if $\ell_k \in C_k$ for some $k \in \{1, \ldots, n\}$ then $\ell_i \in C_i$ or $\ell_j \in C_j$.

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Operational Semantics of Networks: Delay Transitions

- A **delay transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{t} \langle \vec{\ell}, \nu + t \rangle$ occurs if
  - $\nu + t \models \bigwedge_{k=1}^{n} I_k(\ell_k)$,
  - (♣) there are no $i, j \in \{1, \ldots, n\}$ and $b \in U$ with $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell_j') \in E_j$,
  - (♣) there is no $i \in \{1, \ldots, n\}$ such that $\ell_i \in C_i$.

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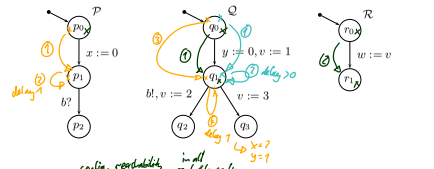
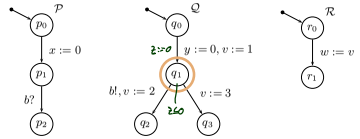
Restricting Non-determinism: Example



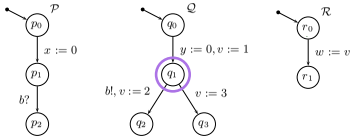
	Property 1	Property 2	Property 3
	$\exists \Diamond w = 1$	$\forall \Box \mathcal{Q}.q_1 \implies y \leq 0)$	$\forall \Box (\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N} := \mathcal{P} \Vert \mathcal{Q} \Vert \mathcal{R}$	✓	✗	✗
$\mathcal{N}$, q_1 urgent			
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Location



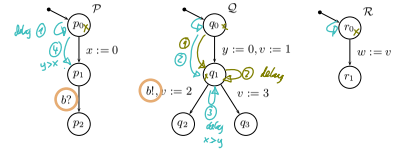
	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✓	✓	✓
$\mathcal{N}$, q_1 comm.			
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Committed Location



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✔	✔	✔
$\mathcal{N}$, q_1 comm.	✘	✓	✓
$\mathcal{N}$, b urgent			

Restricting Non-determinism: Urgent Channel



	Property 1	Property 2	Property 3
	$\exists\Diamond\, w = 1$	$\forall\Box\, \mathcal{Q}.q_1 \implies y \leq 0$	$\forall\Box(\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✔	✘	✘
$\mathcal{N}$, q_1 urgent	✔	✔	✔
$\mathcal{N}$, q_1 comm.	✘	✔	✔
$\mathcal{N}$, b urgent	✓	✗	✓

Extended vs. Pure Timed Automata

Extended vs. Pure Timed Automata

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$$

vs.

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$
$$(\ell, \alpha, \varphi, Y, \ell') \in E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$$

- $\mathcal{A}_e$ is in fact (or specialises to) a **pure** timed automaton if
  - $C = \emptyset$,
  - $U = \emptyset$,
  - $V = \emptyset$,
  - for each $\vec{r} = \langle r_1, \ldots, r_n \rangle$, every r_i is of the form $x := 0$ with $x \in X$.
- $I(\ell), \varphi \in \Phi(X)$ is then a consequence of $V = \emptyset$.

Operational Semantics of Extended TA

Theorem 4.41. If $\mathcal{A}_1, \ldots, \mathcal{A}_n$ **specialise to pure** timed automata, then the operational semantics of

$$\mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)$$

and

$$\mathsf{chan}\, b_1, \ldots, b_m \bullet (\mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_n),$$

where $\{b_1, \ldots, b_m\} = \bigcup_{i=1}^{n} B_i$, **coincide**, i.e.

$$\mathcal{T}_t(\mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)) = \mathcal{T}(\mathsf{chan}\, b_1, \ldots, b_m \bullet (\mathcal{A}_1 \parallel \ldots \parallel \mathcal{A}_n)).$$

Reachability Problems for Extended Timed Automata

Recall

Theorem 4.33. [*Location Reachability*] The location reachability problem for **pure** timed automata is **decidable**.

Theorem 4.34. [*Constraint Reachability*] The constraint reachability problem for **pure** timed automata is **decidable**.

- And what about tea ~~W~~ **extended** timed automata?

What About Extended Timed Automata?

Extended Timed Automata add the following features:

- **Data-Variables**
  - As long as the domains of all variables in V are finite, adding data variables doesn't hurt.
  - If they're infinite, we've got a problem (encode two-counter machine).
- **Structuring Facilities**
  - Don't hurt — they're merely abbreviations.
- **Restricting Non-determinism**
  - Restricting non-determinism doesn't affect the configuration space.
  - Restricting non-determinism only **removes** certain transitions, so makes region automaton even smaller.

The Logic of Uppaal

The Uppaal Fragment of Timed Computation Tree Logic

Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)$ over data variables V.

- **basic formula**:
$$atom ::= \mathcal{A}_i.\ell \mid \varphi$$
where $\ell \in L_i$ is a location and φ a constraint over X_i and V.
- **configuration formulae**:
$$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$
- **existential path formulae**: ("**exists finally**", "**exists globally**")
$$e\text{-}formula ::= \exists\Diamond\, term \mid \exists\Box\, term$$
- **universal path formulae**: ("**always finally**", "**always globally**", "**leads to**")
$$a\text{-}formula ::= \forall\Diamond\, term \mid \forall\Box\, term \mid term_1 \longrightarrow term_2$$
- **formulae**:
$$F ::= e\text{-}formula \mid a\text{-}formula$$

Configurations at Time t

- Recall: **computation path** (or path) **starting in** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$:
$$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \ldots$$
which is **infinite or maximally finite**.
- Given ξ and $t \in \mathsf{Time}$, we use $\xi(t)$ to denote the set
$$\{\langle \vec{\ell}, \nu \rangle \mid \exists i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i\}.$$
of **configurations at time** t.
  - Why is it a set?
  - Can it be empty?

Satisfaction of Uppaal-Logic by Configurations

- We define a **satisfaction relation**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models F$$

between **time stamped configurations**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0$$

of a network $\mathcal{C}(\mathcal{A}_1, \ldots, \mathcal{A}_n)$ and **formulae** F of the Uppaal logic.

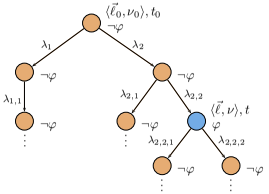
- It is defined inductively as follows:
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \mathcal{A}_i.\ell$ iff $\ell_0 = \ell$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \varphi$ iff $\nu_0 \models \varphi$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \neg term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models term$
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \wedge term_2$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_i, \; i = 1, 2$

Satisfaction of Uppaal-Logic by Configurations

Exists finally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Diamond\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\exists t \in \text{Time}, \langle \vec{\ell}, \nu \rangle \in Conf :$
  $t_0 \le t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \wedge \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Diamond\varphi$

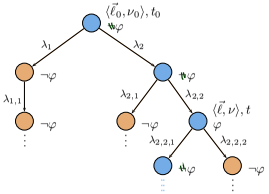


Satisfaction of Uppaal-Logic by Configurations

Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Box\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall t \in \text{Time}, \langle \vec{\ell}, \nu \rangle \in Conf :$
  $t_0 \le t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \implies \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Box\,\varphi$



Satisfaction of Uppaal-Logic by Configurations

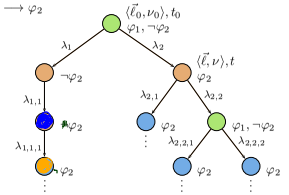
- **Always finally**:
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Diamond\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Box\neg term$
- **Always globally**:
  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall\Box\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists\Diamond\neg term$

Satisfaction of Uppaal-Logic by Configurations

Leads to:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall t \in \text{Time}, \langle \vec{\ell}, \nu \rangle \in Conf :$
  $t_0 \le t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t)$
  $\wedge \langle \vec{\ell}, \nu \rangle, t \models term_1$
  implies $\langle \vec{\ell}, \nu \rangle, t \models \forall\Diamond\, term_2$

Example: $\varphi_1 \longrightarrow \varphi_2$



Satisfaction of Uppaal-Logic by Networks

- We write

$$\mathcal{N} \models e\text{-}formula$$

if and only if

$$\text{for some } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models e\text{-}formula, \qquad (1)$$

and

$$\mathcal{N} \models a\text{-}formula$$

if and only if

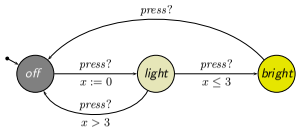
$$\text{for all } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models a\text{-}formula, \qquad (2)$$

where C_{ini} are the initial configurations of $\mathcal{T}_e(\mathcal{N})$.

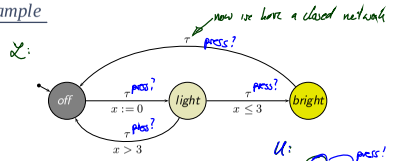
- If $C_{ini} = \emptyset$, (1) is a contradiction and (2) is a tautology.
- If $C_{ini} \neq \emptyset$, then

$$\mathcal{N} \models F \text{ if and only if } \langle \vec{\ell}_{ini}, \nu_{ini} \rangle, 0 \models F.$$

Example



Example



- $\mathcal{N} \models \exists\Diamond\, \mathcal{L}.bright$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.bright$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.off$? ✓
- $\mathcal{N} \models \forall\Diamond\, \mathcal{L}.light$? ✗
- $\mathcal{N} \models \forall\Box(\mathcal{L}.bright \implies x \geq 3)$? ✗
- $\mathcal{N} \models \mathcal{L}.bright \longrightarrow \mathcal{L}.off$? ✗

References

References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.