

# *Real-Time Systems*

## *Lecture 14: Extended Timed Automata*

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# Contents & Goals

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## Last Lecture:

- Decidability of the location reachability problem: zones.
- Extended TA: data variables.

## This Lecture:

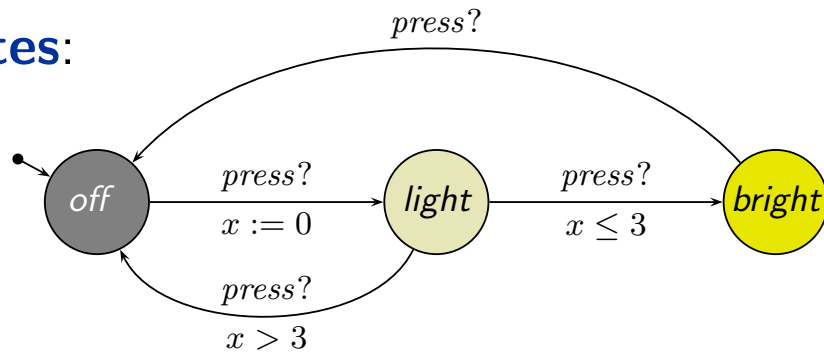
- **Educational Objectives:** Capabilities for following tasks/questions.
  - By what are TA extended? Why is that useful?
  - What's an urgent/committed location? What's the difference?
  - What's an urgent channel?
  - Where has the notion of “input action” and “output action” correspondences in the formal semantics?
- **Content:**
  - Extended TA:
    - Structuring Facilities
    - Restriction of Non-Determinism
  - The Logic of Uppaal

# *Extended Timed Automata*

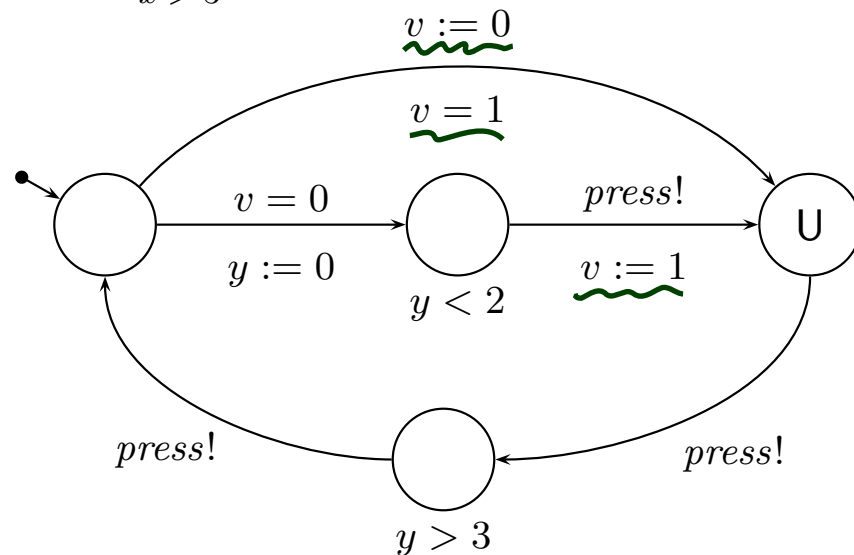
# Recall: Example (Partly Already Seen in Uppaal Demo)

## Templates:

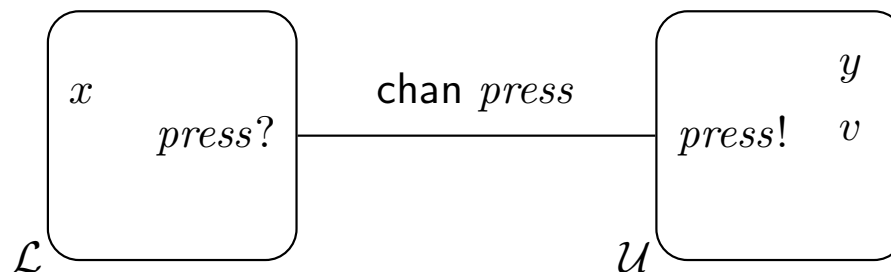
•  $\mathcal{L}$ :



•  $\mathcal{U}$ :



## System:



## Extensions:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Location, Urgent Channel

# Recall: Data Variables and Expressions

- Let  $(v, w \in) V$  be a set of (integer) variables.
  - $(\psi_{int} \in) \Psi(V)$ : **integer expressions** over  $V$  using func. symb.  $+, -, \dots$
  - $(\varphi_{int} \in) \Phi(V)$ : **integer (or data) constraints** over  $V$  using **integer expressions**, predicate symbols  $=, <, \leq, \dots$ , and boolean logical connectives. (incl.  $\vee, \wedge, \Rightarrow, \Leftrightarrow, \neg, \dots$ )
- Recall:  $\Phi(X)$  for clocks  $X$  (simple clock constraint)
- Let  $(x, y \in) X$  be a set of clocks.
  - $(\varphi \in) \Phi(X, V)$ : (**extended**) **guards**, defined by

$$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

where  $\varphi_{clk} \in \Phi(X)$  is a simple clock constraint (as defined before) and  $\varphi_{int} \in \Phi(V)$  an **integer (or data) constraint**.

**Examples:** Extended guard or not extended guard? Why?

- $x - y < 0$
- (a)  $\underbrace{x < y}_{\in \Phi(X)} \wedge \underbrace{v > 2}_{\in \Phi(V)}$  ✓
- (b)  $\underbrace{x < y}_{\in \Phi(X)} \vee \underbrace{v > 2}_{\in \Phi(V)}$  ✗
- (c)  $\underbrace{v < 1 \vee v > 2}_{\in \Phi(V)}$  ✓
- (d)  $\underbrace{x < v}_{\in \Phi(X)} \wedge \underbrace{v > 2}_{\in \Phi(V)}$  ✗

# Modification or Reset Operation

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- **New:** a **modification** or **reset operation** is

$$x := 0, \quad x \in X,$$

or

$$v := \psi_{int}, \quad v \in V, \quad \psi_{int} \in \Psi(V).$$

- By  $R(X, V)$  we denote the set of all resets.
- By  $\vec{r}$  we denote a finite list  $\langle r_1, \dots, r_n \rangle$ ,  $n \in \mathbb{N}_0$ , of reset operations  $r_i \in R(X, V)$ ;  $\langle \rangle$  is the empty list.
- By  $R(X, V)^*$  we denote the set of all such lists of reset operations.

# Modification or Reset Operation

OLD:  
 $(l, \alpha, \varphi, \gamma, l')$   
 NEW:  
 $(l, \alpha, \varphi, \vec{r}, l')$

- **New:** a **modification** or **reset (operation)** is

$$x := 0, \quad x \in X,$$

or

$$v := \psi_{int}, \quad v \in V, \quad \psi_{int} \in \Psi(V).$$

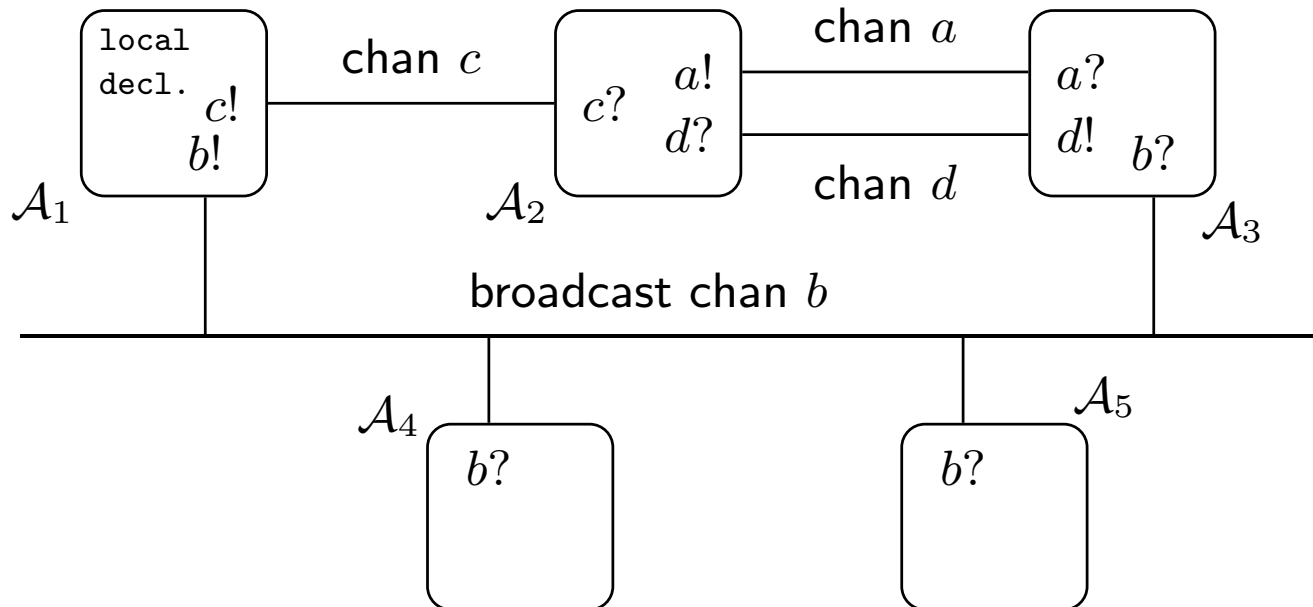
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- By  $R(X, V)^*$  we denote the set of all such lists of reset operations.

**Examples:** Modification or not? Why?

(a) $x := y,$	(b) $x := v,$	(c) $v := x,$	(d) $v := w,$	(e) $v := 0$
clock ↗ not 0 ↗	not 0 ↗	$\notin \Psi(V)$	$\in V$ $\in \Psi(V)$	$\in \Psi(V)$
X	X	X	✓	✓

# Structuring Facilities

global decl.: clocks, variables, channels, constants



- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels:  $\text{chan } c$  and broadcast  $\text{chan } b$ .
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.



# *Restricting Non-determinism*

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- **Urgent locations** — enforce local immediate progress.

U

- **Committed locations** — enforce **atomic** immediate progress.

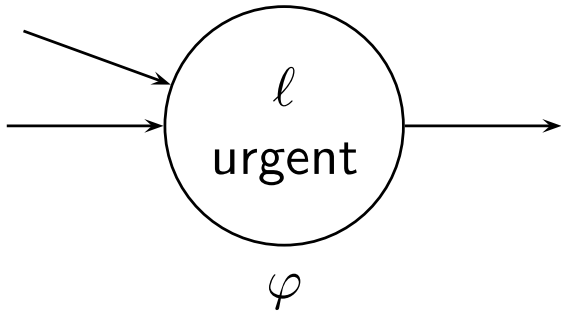
C

- **Urgent channels** — enforce cooperative immediate progress.

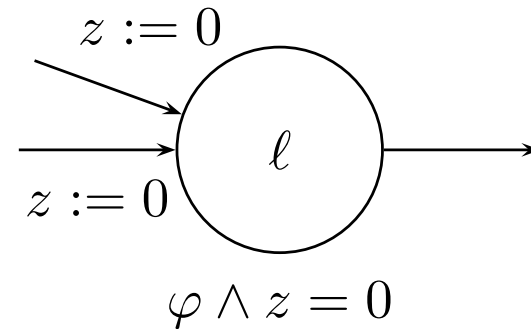
urgent chan press;

# Urgent Locations: Only an Abbreviation...

Replace



with



where  $z$  is a fresh clock:

- reset  $z$  on all in-going edges,
- add  $z = 0$  to invariant.

$N=3$       • 1  
 $|U|=20$     • 3 ✓  
each ant.    •  
at least one • 20

**Question:** How many fresh clocks do we need in the worst case for a network of  $N$  extended timed automata?

**Definition 4.39.** An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where  $L, B, X, I, \ell_{ini}$  are as in Def. 4.3, except that location invariants in  $I$  are **downward closed**, and where

- $C \subseteq L$ : **committed locations**,
- $U \subseteq B$ : **urgent channels**,
- $V$ : a set of data variables,
- $E \subseteq L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$ : a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \text{chan}(\alpha) \in U \implies \varphi = \text{true}.$$

Edges  $(\ell, \alpha, \varphi, \vec{r}, \ell')$  from location  $\ell$  to  $\ell'$  are labelled with an **action**  $\alpha$ , a **guard**  $\varphi$ , and a list  $\vec{r}$  of **reset operations**.

**Definition 4.40.** Let  $\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$ ,  $1 \leq i \leq n$ , be extended timed automata with pairwise disjoint sets of clocks  $X_i$ .

The operational semantics of  $\mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n})$  (closed!) is the labelled transition system

$$\begin{aligned} & \mathcal{T}_e(\mathcal{C}(\mathcal{A}_{e,1}, \dots, \mathcal{A}_{e,n})) \\ &= (\text{Conf}, \text{Time} \cup \{\tau\}, \{\xrightarrow{\lambda} \mid \lambda \in \text{Time} \cup \{\tau\}\}, C_{ini}) \end{aligned}$$

where

- $X = \bigcup_{i=1}^n X_i$  and  $V = \bigcup_{i=1}^n V_i$ ,  $\mathcal{D}(V)$
- $\text{Conf} = \{ \langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : X \cup V \rightarrow \text{Time}, \nu \models \bigwedge_{k=1}^n I_k(\ell_k) \}$ ,
- $C_{ini} = \{ \langle \vec{\ell}_{ini}, \nu_{ini} \rangle \} \cap \text{Conf}$ ,

and the transition relation consists of transitions of the following three types.

# Helpers: Extended Valuations and Timeshift

- **Now:**  $\nu : X \cup V \rightarrow \text{Time} \cup \mathcal{D}(V)$
- Canonically extends to  $\nu : \Psi(V) \rightarrow \mathcal{D}$  (valuation of expression).
- “ $\models$ ” extends canonically to expressions from  $\Phi(X, V)$ .

Handwritten notes in a green box:

- $\eta ::= \nu \mid f(v_1, \dots, v_n)$   
 $\nu \in V, f \in \mathcal{F}$
- $\mathcal{D}(V) \subseteq \mathcal{Z}$
- $\mathcal{I}: \mathcal{F} \rightarrow \bigcup_{n \in \mathbb{N}} (\mathbb{Z}^n \rightarrow \mathcal{Z})$
- $\nu: V \rightarrow \mathcal{D}(V)$
- $\nu(f(v_1, \dots, v_n)) := \mathcal{I}(f)(\nu(v_1), \dots, \nu(v_n))$

# Helpers: Extended Valuations and Timeshift

- **Now:**  $\nu : X \cup V \rightarrow \text{Time} \cup \mathcal{D}(V)$
- Canonically extends to  $\nu : \Psi(V) \rightarrow \mathcal{D}$  (valuation of expression).
- “ $\models$ ” extends canonically to expressions from  $\Phi(X, V)$ .
- Extended **timeshift**  $\nu + t$ ,  $t \in \text{Time}$ , applies to clocks only:
  - $(\nu + t)(x) := \nu(x) + t$ ,  $x \in X$ ,
  - $(\nu + t)(v) := \nu(v)$ ,  $v \in V$ .
- **Effect of modification**  $r \in R(X, V)$  on  $\nu$ , denoted by  $\nu[r]$ :

$$\nu[x := 0](a) := \begin{cases} 0, & \text{if } a = x, \\ \nu(a), & \text{otherwise} \end{cases}$$


$$\nu[v := \psi_{int}](a) := \begin{cases} \nu(\psi_{int}), & \text{if } a = v, \\ \nu(a), & \text{otherwise} \end{cases}$$

- We set  $\nu[\langle r_1, \dots, r_n \rangle] := \nu[r_1] \dots [r_n] = (((\underline{\nu[r_1]})[r_2]) \dots)[r_n]$ .

# Operational Semantics of Networks: Internal Transitions

- An **internal transition**  $\langle \vec{l}, \nu \rangle \xrightarrow{\tau} \langle \vec{l}', \nu' \rangle$  occurs if there is  $i \in \{1, \dots, n\}$  <sup>no. of automata</sup> such that
  - there is a  $\tau$ -edge  $(l_i, \tau, \varphi, \vec{r}, l'_i) \in E_i$ ,
  - $\nu \models \varphi$ ,
  - $\vec{l}' = \vec{l}[l_i := l'_i]$ , <sup>location of the  $i$ -th automaton in  $\vec{l}$</sup>  <sup>modification of  $i$ -th position</sup>
  - $\nu' = \nu[\vec{r}]$ ,
  - $\nu' \models I_i(l'_i)$ ,
  - (**♣**) if  $l_k \in C_k$  for some  $k \in \{1, \dots, n\}$  then  $l_i \in C_i$ . <sup>committed locations</sup>

# Operational Semantics of Networks: Synchronisation Transition

- A **synchronisation transition**  $\langle \vec{l}, \nu \rangle \xrightarrow{\tau} \langle \vec{l}', \nu' \rangle$  occurs if there are  $i, j \in \{1, \dots, n\}$  with  $i \neq j$  such that
  - there are edges  $(l_i, b!, \varphi_i, \vec{r}_i, l'_i) \in E_i$  and  $(l_j, b?, \varphi_j, \vec{r}_j, l'_j) \in E_j$ ,
  - $\nu \models \varphi_i \wedge \varphi_j$ ,
  - $\vec{l}' = \vec{l}[l_i := l'_i][l_j := l'_j]$ ,
  - $\nu' = \nu[\vec{r}_i][\vec{r}_j]$ ,  "sender updates are applied first"
  - $\nu' \models I_i(l'_i) \wedge I_j(l'_j)$ ,
  - (**♣**) if  $l_k \in C_k$  for some  $k \in \{1, \dots, n\}$  then  $l_i \in C_i$  or  $l_j \in C_j$ .



# Operational Semantics of Networks: Delay Transitions

- A **delay transition**  $\langle \vec{l}, \nu \rangle \xrightarrow{t} \langle \vec{l}, \nu + t \rangle$  occurs if

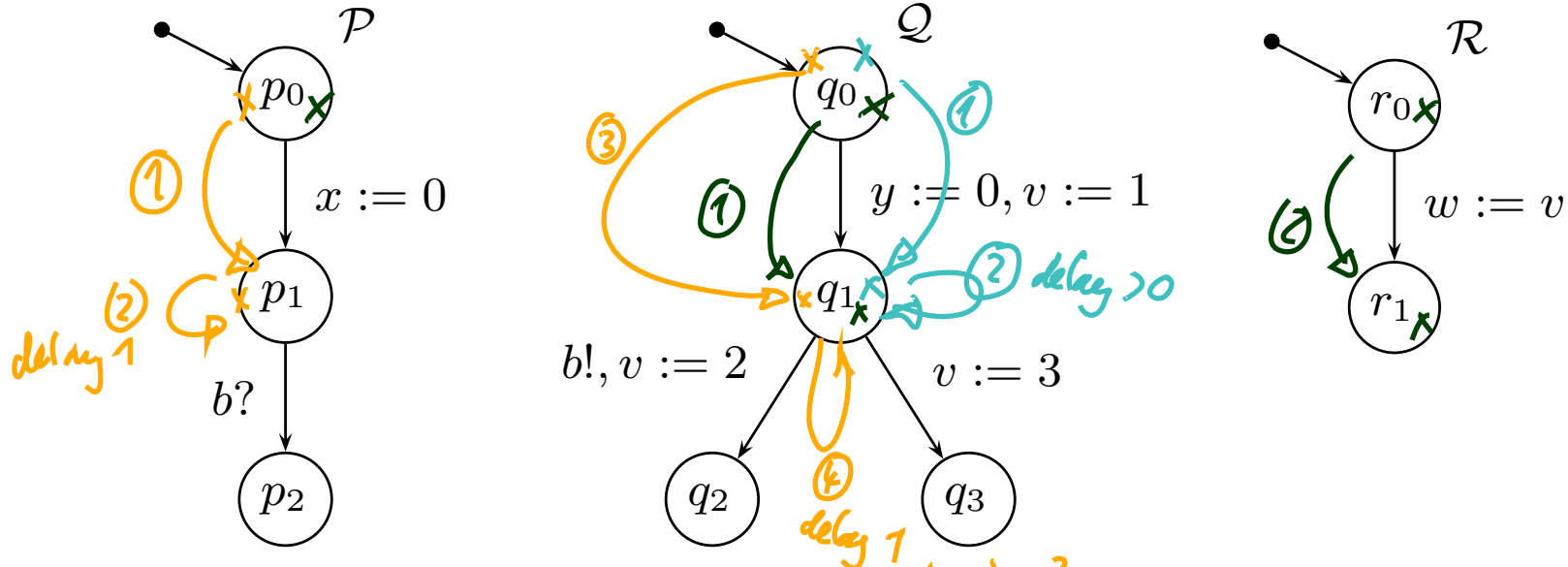
- $\nu + t \models \bigwedge_{k=1}^n I_k(l_k),$

- (♣) there are no  $i, j \in \{1, \dots, n\}$  and  $b \in U$  with  $(l_i, \underline{b!}, \varphi_i, \vec{r}_i, l'_i) \in E_i$  and  $(l_j, \underline{b?}, \varphi_j, \vec{r}_j, l'_j) \in E_j,$

- (♣) there is no  $i \in \{1, \dots, n\}$  such that  $l_i \in C_i.$

urgent channels  
↓

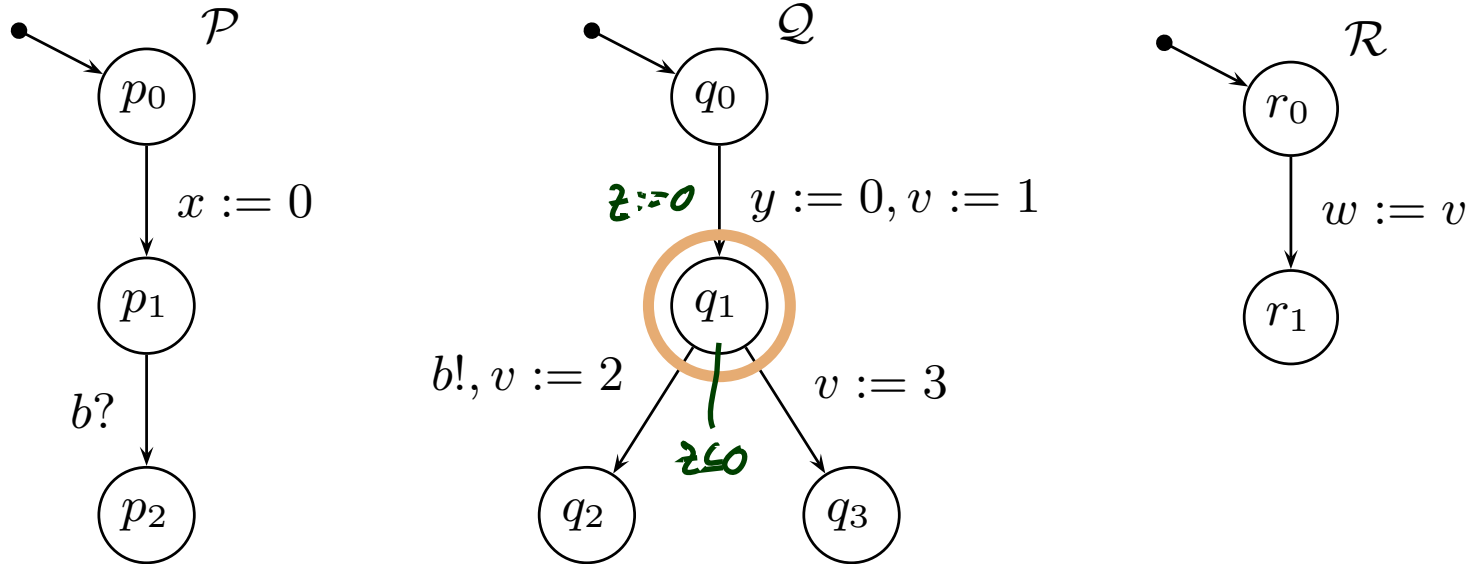
# Restricting Non-determinism: Example



config. readability  
in all reachable config

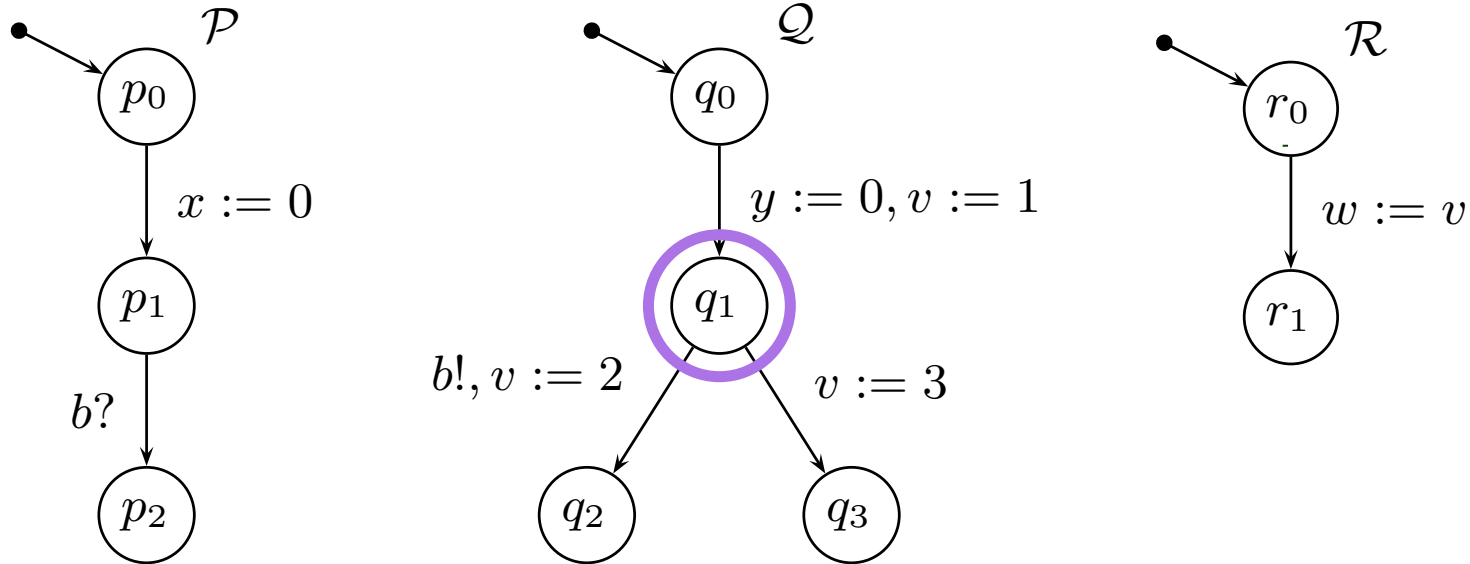
	Property 1 $\exists \diamond w = 1$	Property 2 $\forall \square (Q.q_1 \implies y \leq 0)$	Property 3 $\forall \square (\mathcal{P}.p_1 \wedge Q.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N} := \mathcal{P} \parallel \mathcal{Q} \parallel \mathcal{R}$	✓	✗	✗
$\mathcal{N}, q_1$ urgent			
$\mathcal{N}, q_1$ comm.			
$\mathcal{N}, b$ urgent			

# Restricting Non-determinism: Urgent Location



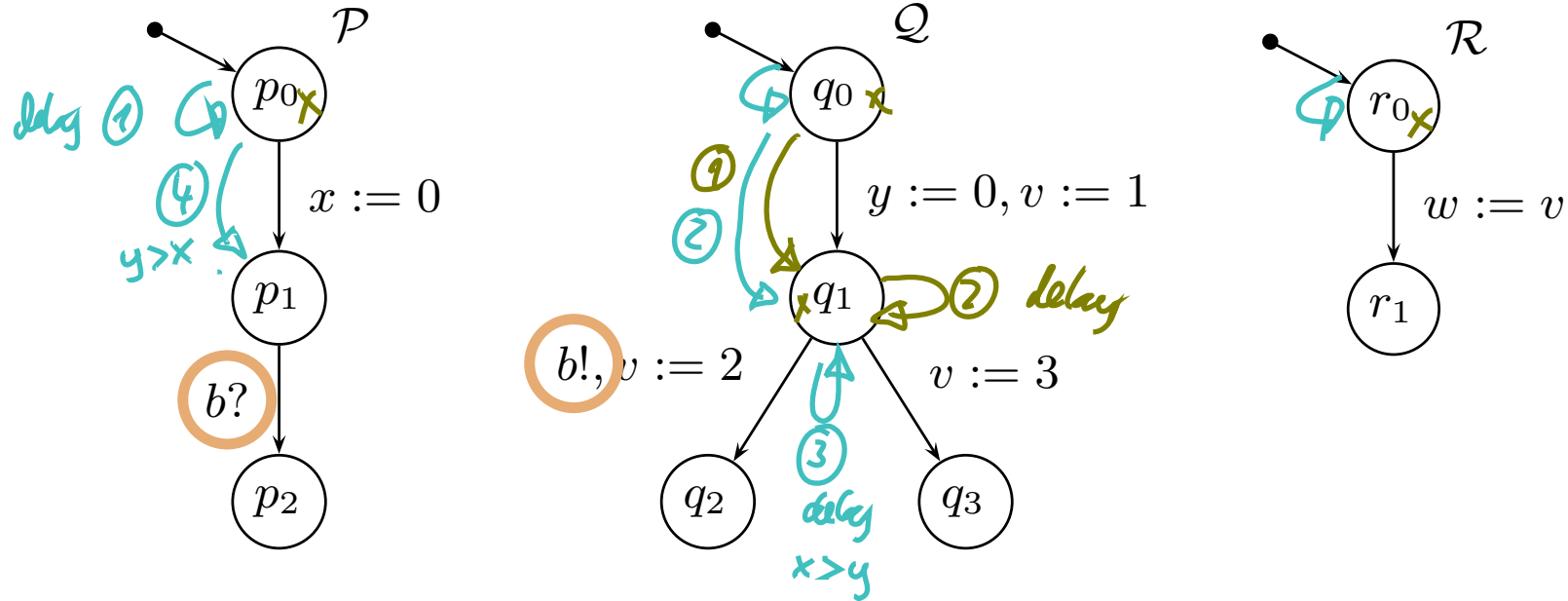
	Property 1	Property 2	Property 3
	$\exists \diamond w = 1$	$\forall \square \mathcal{Q}.q_1 \implies y \leq 0$	$\forall \square (\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✓	✗	✗
$\mathcal{N}, q_1$ urgent	✓	✓	✓
$\mathcal{N}, q_1$ comm.			
$\mathcal{N}, b$ urgent			

# Restricting Non-determinism: Committed Location



	Property 1	Property 2	Property 3
	$\exists \diamond w = 1$	$\forall \square \mathcal{Q}.q_1 \implies y \leq 0$	$\forall \square (\mathcal{P}.p_1 \wedge \mathcal{Q}.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✓	✗	✗
$\mathcal{N}, q_1$ urgent	✓	✓	✓
$\mathcal{N}, q_1$ comm.	✗	✓	✓
$\mathcal{N}, b$ urgent			

# Restricting Non-determinism: Urgent Channel



	Property 1	Property 2	Property 3
	$\exists \diamond w = 1$	$\forall \square Q.q_1 \implies y \leq 0$	$\forall \square (\mathcal{P}.p_1 \wedge Q.q_1 \implies (x \geq y \implies y \leq 0))$
$\mathcal{N}$	✓	✗	✗
$\mathcal{N}, q_1$ urgent	✓	✓	✓
$\mathcal{N}, q_1$ comm.	✗	✓	✓
$\mathcal{N}, b$ urgent	✓	✗	✓

# *Extended vs. Pure Timed Automata*

# Extended vs. Pure Timed Automata

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$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$$

vs.

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$

$$(\ell, \alpha, \varphi, Y, \ell') \in E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$$

- $\mathcal{A}_e$  is in fact (or specialises to) a **pure** timed automaton if
  - $C = \emptyset$ ,
  - $U = \emptyset$ ,
  - $V = \emptyset$ ,
  - for each  $\vec{r} = \langle r_1, \dots, r_n \rangle$ , every  $r_i$  is of the form  $x := 0$  with  $x \in X$ .
- $I(\ell), \varphi \in \Phi(X)$  is then a consequence of  $V = \emptyset$ .

**Theorem 4.41.** If  $\mathcal{A}_1, \dots, \mathcal{A}_n$  specialise to pure timed automata, then the operational semantics of

$$\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$$

and

$$\text{chan } b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n),$$

where  $\{b_1, \dots, b_m\} = \bigcup_{i=1}^n B_i$ , **coincide**, i.e.

$$\mathcal{T}_e(\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)) = \mathcal{T}(\text{chan } b_1, \dots, b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n)).$$

today

per TA



# *Reachability Problems for Extended Timed Automata*

**Theorem 4.33.** [*Location Reachability*] The location reachability problem for **pure** timed automata is **decidable**.

**Theorem 4.34.** [*Constraint Reachability*] The constraint reachability problem for **pure** timed automata is **decidable**.

- And what about  $\hat{W}$  **extended** timed automata?

# What About Extended Timed Automata?

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Extended Timed Automata add the following features:

- **Data-Variables**

- As long as the domains of all variables in  $V$  are finite, adding data variables doesn't hurt.
- If they're infinite, we've got a problem (encode two-counter machine).

- **Structuring Facilities**

- Don't hurt — they're merely abbreviations.

- **Restricting Non-determinism**

- Restricting non-determinism doesn't affect the configuration space.
- Restricting non-determinism only **removes** certain transitions, so makes region automaton even smaller.

# *The Logic of Uppaal*

# The Uppaal Fragment of Timed Computation Tree Logic

Consider  $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$  over data variables  $V$ .

- **basic formula:**

$$atom ::= \mathcal{A}_i.l \mid \varphi$$

where  $l \in L_i$  is a location and  $\varphi$  a constraint over  $X_i$  and  $V$ .

- **configuration formulae:**

$$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$

- **existential path formulae:** (“exists finally”, “exists globally”)

$$e\text{-formula} ::= \exists \diamond term \mid \exists \square term$$

- **universal path formulae:** (“always finally”, “always globally”, “leads to”)

$$a\text{-formula} ::= \forall \diamond term \mid \forall \square term \mid term_1 \longrightarrow term_2$$

- **formulae:**

$$F ::= e\text{-formula} \mid a\text{-formula}$$

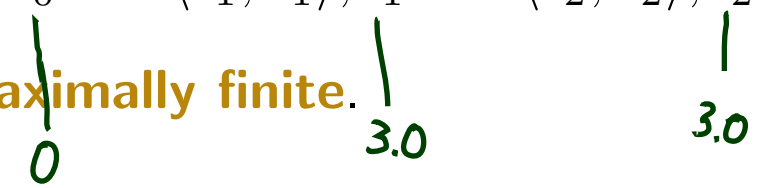
# Configurations at Time $t$

- Recall: **computation path** (or path) **starting in**  $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$ :

$$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \dots$$

$\in \text{Time} \setminus \{0\}$

which is **infinite or maximally finite**.



- Given  $\xi$  and  $t \in \text{Time}$ , we use  $\xi(t)$  to denote the set

$$\{ \langle \vec{\ell}, \nu \rangle \mid \exists i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i \}.$$

of **configurations at time  $t$** .

- Why is it a set?
- Can it be empty?

$$\xi(0) = \{ \langle \vec{\ell}_0, \nu_0 \rangle \}$$

$$\xi(0.27) = \{ \langle \vec{\ell}_0, \nu_0 + 0.27 \rangle \}$$

$$\xi(3.0) = \{ \langle \vec{\ell}_1, \nu_1 \rangle, \langle \vec{\ell}_2, \nu_2 \rangle \}$$

# Satisfaction of Uppaal-Logic by Configurations

- We define a **satisfaction relation**

$$\langle \vec{l}_0, \nu_0 \rangle, t_0 \models F$$

between **time stamped configurations**

$$\langle \vec{l}_0, \nu_0 \rangle, t_0$$

of a network  $\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$  and **formulae**  $F$  of the Uppaal logic.

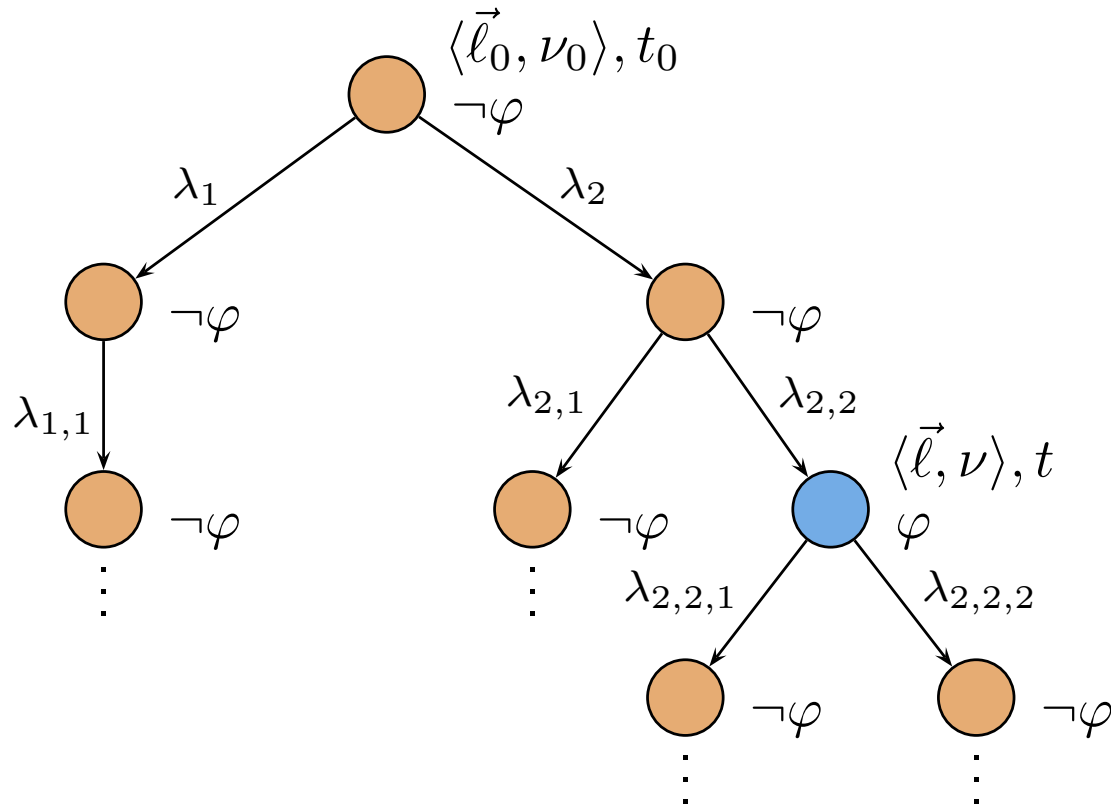
- It is defined inductively as follows:
- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models \mathcal{A}_i.l$  iff  $l_{0,i} = l$   *$i$ -th location in  $\vec{l}_0$*
- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models \varphi$  iff  $\nu_0 \models \varphi$
- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models \neg term$  iff  $\langle \vec{l}_0, \nu_0 \rangle, t_0 \not\models term$
- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models term_1 \wedge term_2$  iff  $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models term_i, i = 1, 2$

# Satisfaction of Uppaal-Logic by Configurations

**Exists finally:**

- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models \exists \diamond term$  iff  $\exists$  path  $\xi$  of  $\mathcal{N}$  starting in  $\langle \vec{l}_0, \nu_0 \rangle, t_0$   
 $\exists t \in \text{Time}, \langle \vec{l}, \nu \rangle \in \text{Conf} :$   
 $t_0 \leq t \wedge \langle \vec{l}, \nu \rangle \in \xi(t) \wedge \langle \vec{l}, \nu \rangle, t \models term$

**Example:**  $\exists \diamond \varphi$



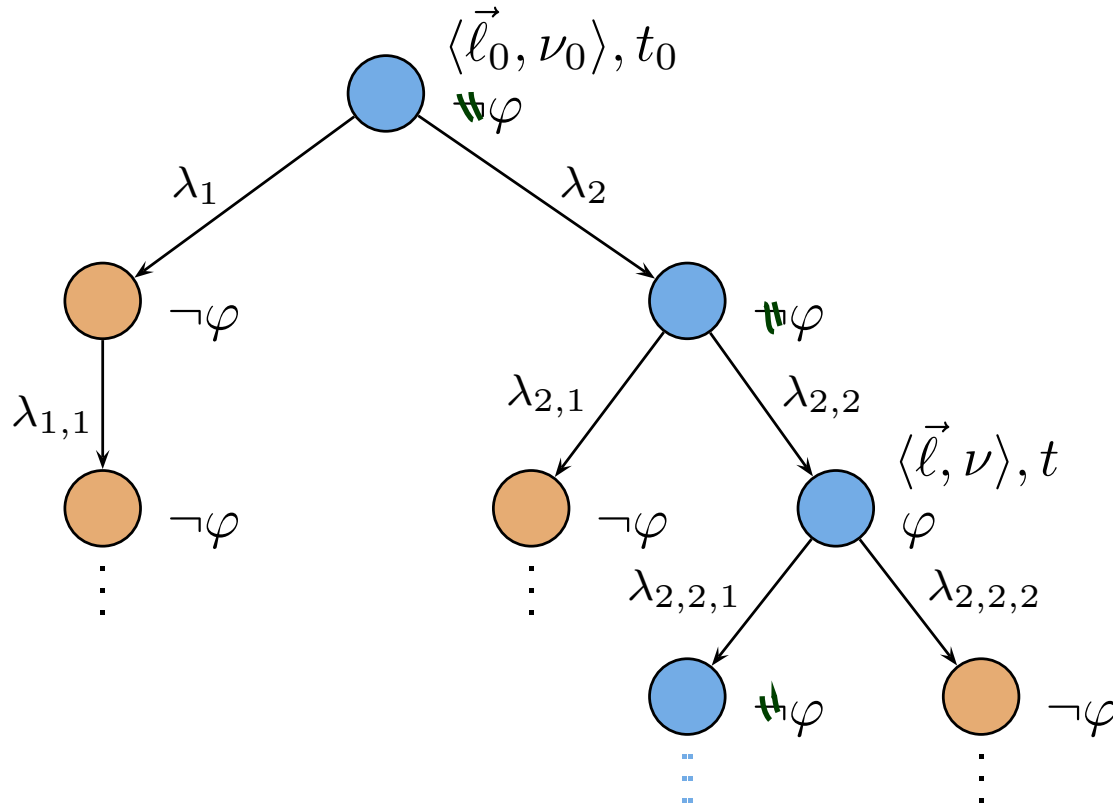


# Satisfaction of Uppaal-Logic by Configurations

## Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists \square \text{ term}$ 
 iff  $\exists$  path  $\xi$  of  $\mathcal{N}$  starting in  $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
- note: universally quantifying over all elements in  $\xi(t)$
- $\forall t \in \text{Time}, \langle \vec{\ell}, \nu \rangle \in \text{Conf} :$   
 $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \implies \langle \vec{\ell}, \nu \rangle, t \models \text{term}$

## Example: $\exists \square \varphi$



# Satisfaction of Uppaal-Logic by Configurations

- **Always finally:**

“ $\forall$  path  
   $\vdash \exists$  time  $\circ$  term”

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall \diamond term$       iff  $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists \square \neg term$

- **Always globally:**

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall \square term$       iff  $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists \diamond \neg term$

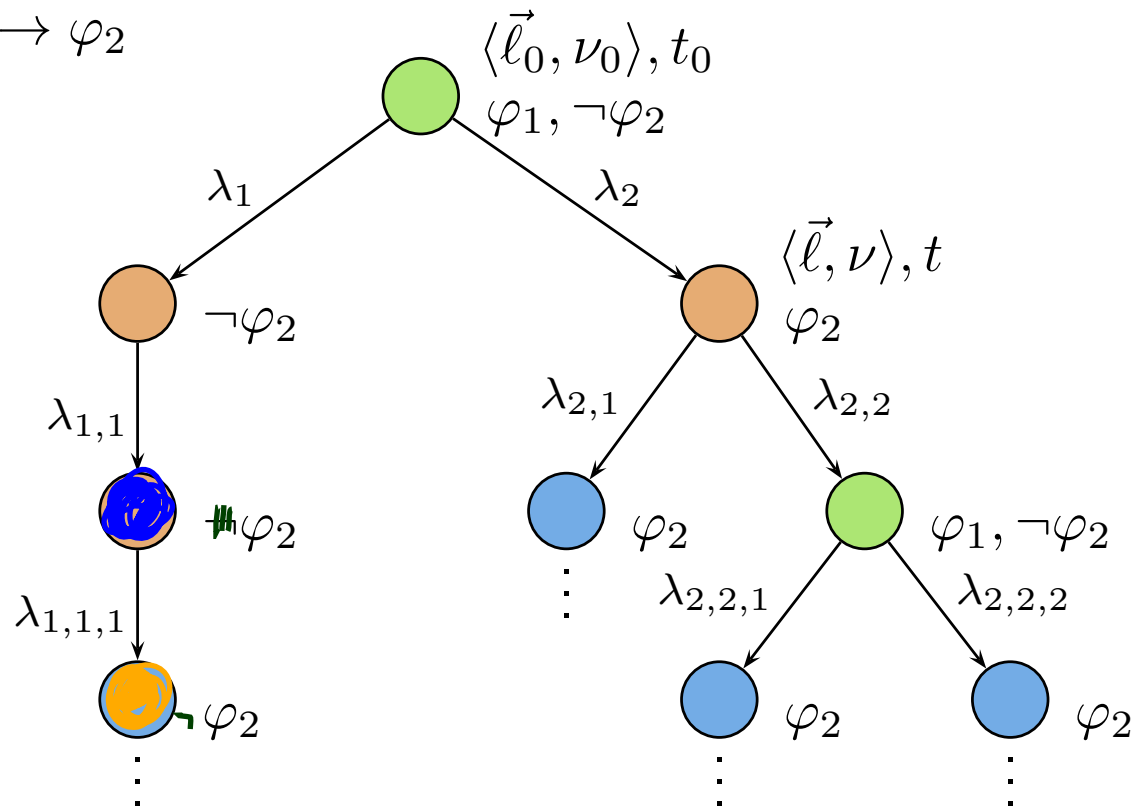
“ $\forall$  path  
   $\forall$  time  $\circ$  term”

# Satisfaction of Uppaal-Logic by Configurations

**Leads to:** (TL: "AG(term<sub>1</sub>) ⇒ AF term<sub>2</sub>")

- $\langle \vec{l}_0, \nu_0 \rangle, t_0 \models \text{term}_1 \longrightarrow \text{term}_2$  iff  $\forall$  path  $\xi$  of  $\mathcal{N}$  starting in  $\langle \vec{l}_0, \nu_0 \rangle, t_0$   
 $\forall t \in \text{Time}, \langle \vec{l}, \nu \rangle \in \text{Conf} :$   
 $t_0 \leq t \wedge \langle \vec{l}, \nu \rangle \in \xi(t)$   
 $\wedge \langle \vec{l}, \nu \rangle, t \models \text{term}_1$   
 implies  $\langle \vec{l}, \nu \rangle, t \models \forall \Diamond \text{term}_2$

**Example:**  $\varphi_1 \longrightarrow \varphi_2$



# Satisfaction of Uppaal-Logic by Networks

- We write

$$\mathcal{N} \models e\text{-formula}$$

if and only if

$$\text{for some } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models e\text{-formula}, \quad (1)$$

and

$$\mathcal{N} \models a\text{-formula}$$

if and only if

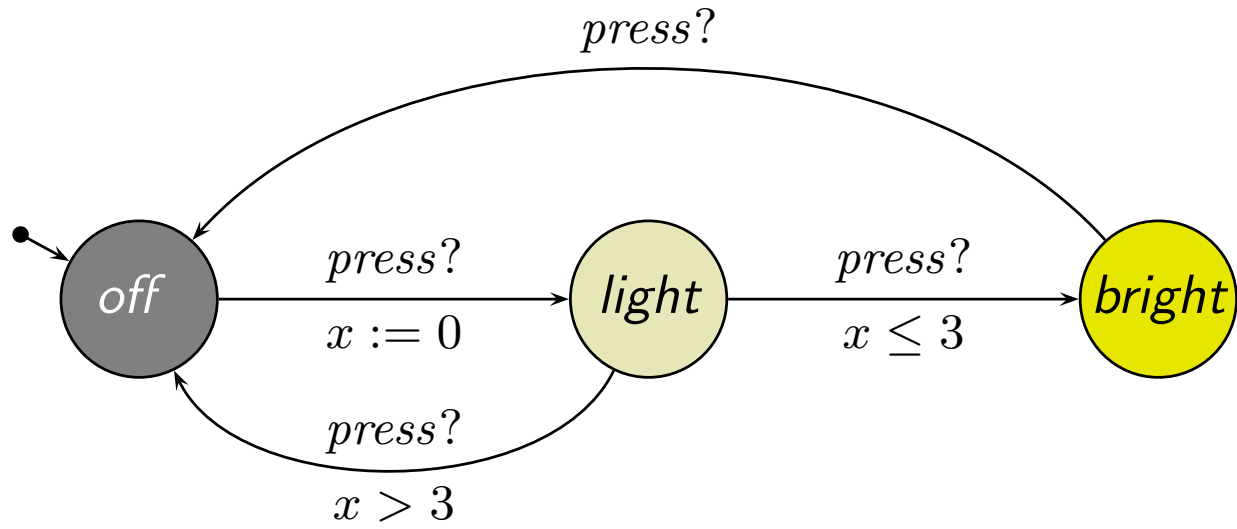
$$\text{for all } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models a\text{-formula}, \quad (2)$$

where  $C_{ini}$  are the initial configurations of  $\mathcal{T}_e(\mathcal{N})$ .

- If  $C_{ini} = \emptyset$ , (1) is a contradiction and (2) is a tautology.
- If  $C_{ini} \neq \emptyset$ , then

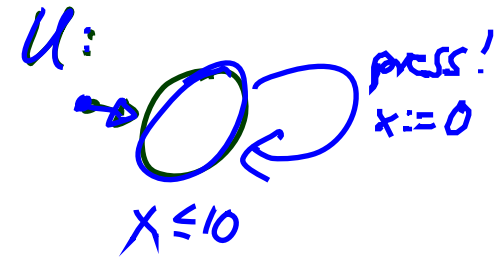
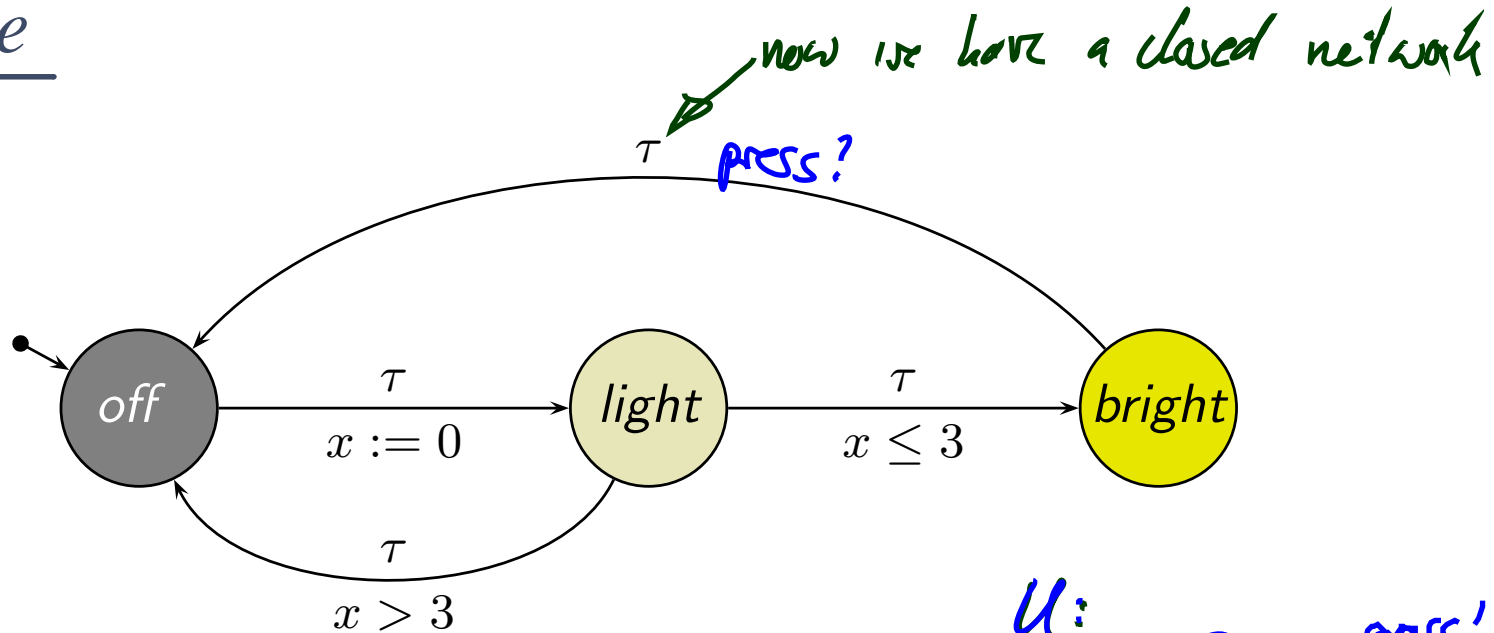
$$\mathcal{N} \models F \text{ if and only if } \langle \vec{\ell}_{ini}, \nu_{ini} \rangle, 0 \models F.$$

# Example



# Example

$\mathcal{L}$ :



- $\mathcal{N} \models \exists \diamond \mathcal{L}. \text{bright?}$  ✓
- $\mathcal{N} \models \exists \square \mathcal{L}. \text{bright?}$  ✓
- $\mathcal{N} \models \exists \square \mathcal{L}. \text{off?}$  ✓
- $\mathcal{N} \models \forall \diamond \mathcal{L}. \text{light?}$  ✗
- $\mathcal{N} \models \forall \square (\mathcal{L}. \text{bright} \implies x \geq 3)?$  ✗
- $\mathcal{N} \models \mathcal{L}. \text{bright} \longrightarrow \mathcal{L}. \text{off?}$  ✗

# *References*

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# References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.