

# *Real-Time Systems*

## *Lecture 15: Automatic Verification of DC Properties for TA*

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### *Contents & Goals*

#### **Last Lecture:**

- Extended Timed Automata
- Uppaal Query Language

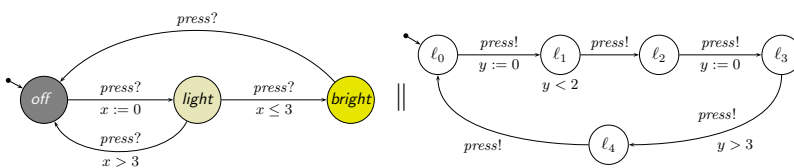
#### **This Lecture:**

- **Educational Objectives:** Capabilities for following tasks/questions.
  - How can we relate TA and DC formulae? What's a bit tricky about that?
  - Can we use Uppaal to check whether a TA satisfies a DC formula?
- **Content:**
  - An evolution-of-observables semantics of TA
  - A satisfaction relation between TA and DC
  - Model-checking DC properties with Uppaal

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## Observer-based Automatic Verification of DC Properties for TA

### Model-Checking DC Properties with Uppaal



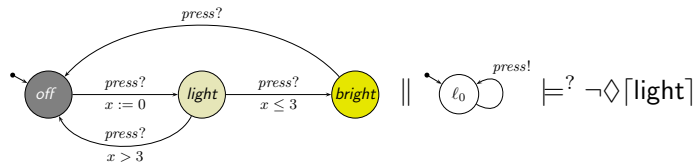
DC formula

↓  
 $\models? F$

- **First Question:** what is the “ $\models$ ” here?
- **Second Question:** what kinds of DC formulae can we check with Uppaal?
  - **Clear:** Not every DC formula. (Otherwise contradicting undecidability results.)
  - **Quite clear:**  $F = \Box[\text{off}]$  or  $F = \neg\Diamond[\text{light}]$ 

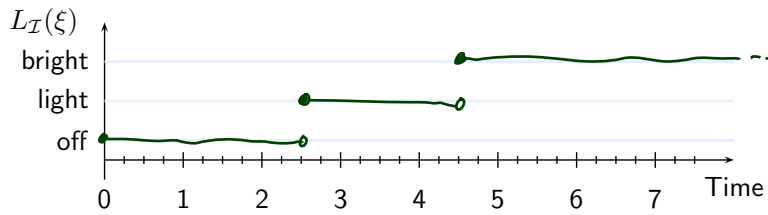
(Use Uppaal's fragment of TCTL, something like  $\forall\Box\text{off}$ , but not exactly (see later).)
  - **Maybe:**  $F = \Box(l > 5 \implies \Diamond[\text{off}]^5)$
  - **Not so clear:**  $F = \neg\Diamond([\text{bright}] ; [\text{light}])$

## Example: Let's Start With Single Runs

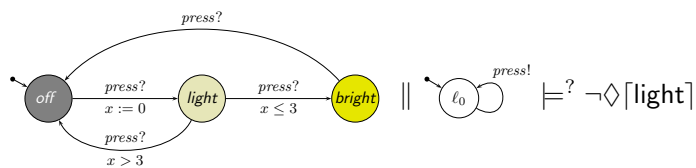


$$\xi = \langle \text{off} \rangle_0, 0 \xrightarrow{2.5} \langle \text{off} \rangle_{2.5}, 2.5 \xrightarrow{\tau} \langle \text{light} \rangle_0, 2.5 \xrightarrow{2.0} \langle \text{light} \rangle_{2.0}, 4.5 \xrightarrow{\tau} \langle \text{bright} \rangle_{2.0}, 4.5 \dots$$

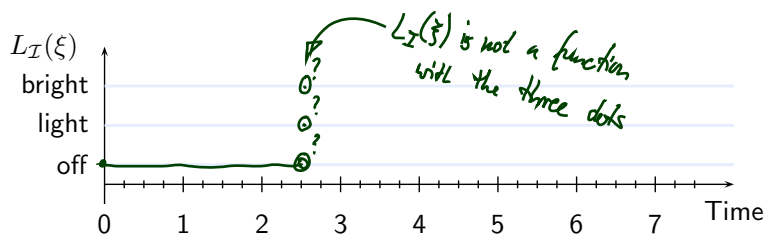
Construct interpretation  $L_{\mathcal{I}}(\xi) : \text{Time} \rightarrow \{\text{off}, \text{light}, \text{bright}\}$ :



## Example 2: Another Single Run



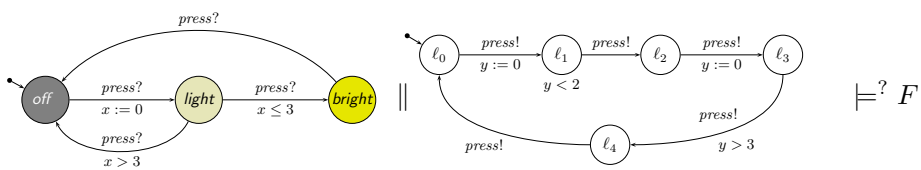
$$\xi = \langle \text{off} \rangle_0, 0 \xrightarrow{2.5} \langle \text{off} \rangle_{2.5}, 2.5 \xrightarrow{\tau} \langle \text{light} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{bright} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{off} \rangle_0, 2.5 \xrightarrow{1.0} \dots$$



We know this problem from the exercises...

## Observing Timed Automata

### DC Properties of Timed Automata



**Wanted:** A satisfaction relation between networks of timed automata and DC formulae, a notion of  $\mathcal{N}$  **satisfies**  $F$ , denoted by  $\mathcal{N} \models F$ .

**Plan:**

- Consider network  $\mathcal{N}$  consisting of TA

$$\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$$

- Define observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$ .
- Define evolution  $\mathcal{I}_\xi$  of  $\text{Obs}(\mathcal{N})$  induced by computation path  $\xi \in \text{CompPaths}(\mathcal{N})$  of  $\mathcal{N}$ ,  
 $\text{CompPaths}(\mathcal{N}) = \{\xi \mid \xi \text{ is a computation path of } \mathcal{N}\}$
- Say  $\mathcal{N} \models F$  if and only if  $\forall \xi \in \text{CompPaths}(\mathcal{N}) : \mathcal{I}_\xi \models_0 F$ .

## Observables of TA Network

Let  $\mathcal{N}$  be a network of  $n$  extended timed automata

$$\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i})$$

**For simplicity:** assume that the  $L_i$  and  $X_i$  are pairwise disjoint and that each  $V_i$  is pairwise disjoint to every  $L_i$  and  $X_i$  (otherwise rename).

- **Definition:** The observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$  are

$$\{\ell_1, \dots, \ell_n\} \cup \bigcup_{1 \leq i \leq n} V_i$$

with

- $\mathcal{D}(\ell_i) = L_i$ ,
- $\mathcal{D}(v)$  as given,  $v \in V_i$ .

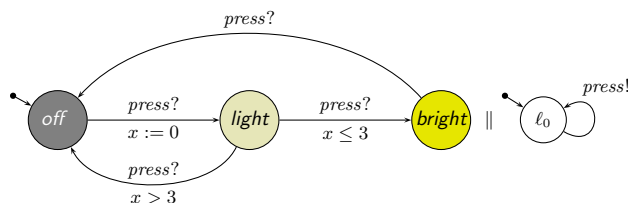
current location of  $\mathcal{A}_{e,i}$   
 (would be less confusing if we used  $\{\odot_1, \dots, \odot_n\}$ )

## Observables of TA Network: Example

$$\mathcal{A}_{e,i} = (L_i, C_i, B_i, U_i, X_i, V_i, I_i, E_i, \ell_{ini,i}).$$

The observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$  are  $\{\ell_1, \dots, \ell_n\} \cup \bigcup_{1 \leq i \leq n} V_i$  with

- $\mathcal{D}(\ell_i) = L_i$ ,
- $\mathcal{D}(v)$  as given,  $v \in V_i$ .



•  $\ell_1$ : off, light, bright  
 •  $\ell_2$ :  $l_0$

## Evolutions of TA Network

**Recall:** computation path

$$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \dots$$

of  $\mathcal{N}$ ,  $\vec{\ell}_k$  denotes a tuple  $\langle \ell_k^1, \dots, \ell_k^n \rangle \in L_1 \times \dots \times L_n$ .

**Recall:** Given  $\xi$  and  $t \in \text{Time}$ , we use  $\xi(t)$  to denote the set

$$\{ \langle \vec{\ell}, \nu \rangle \mid \exists i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i \}.$$

of **configurations at time**  $t$ .

**New:**  $\bar{\xi}(t)$  denotes  $\langle \vec{\ell}_j, \nu_j + t - t_j \rangle$  where  $j = \max\{i \in \mathbb{N}_0 \mid t_i \leq t \wedge \vec{\ell} = \vec{\ell}_i\}$ .

**Our choice:**

- **Ignore** configurations assumed for 0-time only.
- **Extend** finite computation paths to infinite length, staying in last configuration.  
Yet clocks advance – see later.

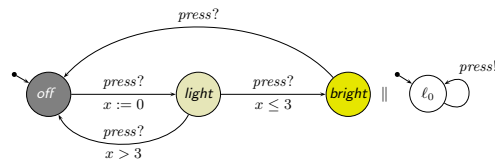
## Evolutions of TA Network: Example

$\bar{\xi}(t)$  denotes  $\langle \vec{\ell}_j, \nu_j + t - t_j \rangle$  where  $j = \max\{i \in \mathbb{N}_0 \mid t_i \leq t \wedge \vec{\ell} = \vec{\ell}_i\}$ .

**Example:**

$$\xi = \langle \text{off}, 0 \rangle, 0 \xrightarrow{2.5} \langle \text{off}, 2.5 \rangle, 2.5 \xrightarrow{\tau} \langle \text{light}, 2.5 \rangle, 2.5 \xrightarrow{\tau} \langle \text{bright}, 2.5 \rangle, 2.5 \xrightarrow{\tau} \langle \text{off}, 2.5 \rangle, 2.5 \xrightarrow{1.0} \langle \text{off}, 3.5 \rangle, 3.5 \xrightarrow{\tau} \dots$$

- $\bar{\xi}(0) = \langle \text{off}, 0 \rangle$
- $\bar{\xi}(1.0) = \langle \text{off}, 1.0 \rangle$
- $\bar{\xi}(2.5) = \langle \text{off}, 2.5 \rangle$



## Evolutions of TA Network Cont'd

$\bar{\xi}$  induces the unique interpretation

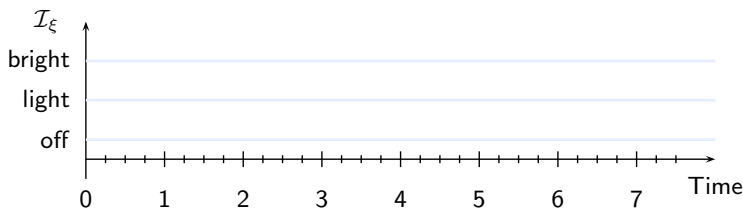
$$\mathcal{I}_{\bar{\xi}} : \text{Obs}(\mathcal{N}) \rightarrow (\text{Time} \rightarrow \mathcal{D})$$

of  $\text{Obs}(\mathcal{N})$  defined pointwise as follows:

$$\mathcal{I}_{\bar{\xi}}(a)(t) = \begin{cases} \ell^i & , \text{ if } a = \ell_i, \bar{\xi}(t) = \langle \langle \ell^1, \dots, \ell^n \rangle, \nu \rangle \\ \nu(a) & , \text{ if } a \in V_i, \bar{\xi}(t) = \langle \vec{\ell}, \nu \rangle \end{cases}$$

**Example:**  $\mathcal{D}(\ell_1) = \{\text{off}, \text{light}, \text{bright}\}$

$$\xi = \langle \text{off} \rangle_0, 0 \xrightarrow{2.5} \langle \text{off} \rangle_{2.5}, 2.5 \xrightarrow{\tau} \langle \text{light} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{bright} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{off} \rangle_0, 2.5 \xrightarrow{1.0} \langle \text{off} \rangle_1, 3.5 \xrightarrow{\tau} \dots$$



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## Evolutions of TA Network Cont'd

$$\xi = \langle \text{off} \rangle_0, 0 \xrightarrow{2.5} \langle \text{off} \rangle_{2.5}, 2.5 \xrightarrow{\tau} \langle \text{light} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{bright} \rangle_0, 2.5 \xrightarrow{\tau} \langle \text{off} \rangle_0, 2.5 \xrightarrow{1.0} \langle \text{off} \rangle_1, 3.5 \xrightarrow{\tau} \dots$$

Abbreviations as usual: *current loc. of 1st automaton*

- $\mathcal{I}_{\bar{\xi}}(\ell_1)(0) = \text{off}$  (by Def.)
- $\mathcal{I}(\ell_1 = \text{off})(0) = \mathcal{I}_z(\ell_1)(0) = \mathcal{I}(\text{off})$
- $\mathcal{I}(\text{off})(1.0) = \mathcal{I}(\ell_1 = \text{off})(1.0)$   
if  $L_i$  pairwise disjoint.

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## Evolutions of TA Network Cont'd

- But **what about clocks**? Why not  $x \in \text{Obs}(\mathcal{N})$  for  $x \in X_i$ ?
- We would know how to define  $\mathcal{I}_\xi(x)(t)$ , namely

$$\mathcal{I}_\xi(x)(t) = \nu_{\xi(t)}(x) + (t - t_{\xi(t)}).$$

- But...  $\mathcal{I}_\xi(x)(t)$  changes too often.

a boolean observable  
 $\downarrow$   
 $\square \langle \Gamma \text{ off } \rangle \Rightarrow x \leq 5$

**Better** (if wanted):

- add  $\Phi(X_1 \cup \dots \cup X_i)$  to  $\text{Obs}(\mathcal{N})$ ,  
with  $\mathcal{D}(\varphi) = \{0, 1\}$  for  $\varphi \in \Phi(X_1 \cup \dots \cup X_i)$ .
- set

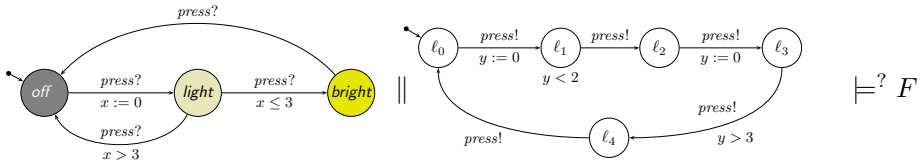
$$\mathcal{I}_\xi(\varphi)(t) = \begin{cases} 1, & \text{if } \nu(x) \models \varphi, \bar{\xi}(t) = \langle \vec{\ell}, \nu \rangle \\ 0, & \text{otherwise} \end{cases}$$

The truth value of constraint  $\varphi$  can endure over non-point intervals.

## Some Checkable Properties



## Model-Checking DC Properties with Uppaal

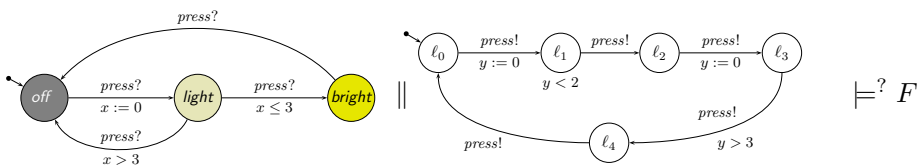


- **First Answer:**  $\mathcal{N} \models F$  if and only if  $\forall \xi \in \text{CompPaths}(\mathcal{N}) : \mathcal{I}_\xi \models_0 F$ .
- **Second Question:** what kinds of DC formulae can we check with Uppaal?
  - **Clear:** Not every DC formula. (Otherwise contradicting undecidability results.)
  - **Quite clear:**  $F = \Box[\text{off}]$  or  $F = \neg\Diamond[\text{light}]$  (Use Uppaal's fragment of TCTL, something like  $\forall\Box\text{off}$ , but not exactly (see later).)
  - **Maybe:**  $F = \Box(\ell > 5 \implies \Diamond[\text{off}]^5)$
  - **Not so clear:**  $F = \neg\Diamond([\text{bright}] ; [\text{light}])$

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## Model-Checking DC Properties with Uppaal



- **Second Question:** what kinds of DC formulae can we check with Uppaal?

### Wanted:

- a function  $f$  mapping DC formulae to Uppaal queries and
- a transformation  $\tilde{\cdot}$  of networks of TA

such that

$$\tilde{\mathcal{N}} \models_{\text{Uppaal}} f(F) \iff \mathcal{N} \models F$$

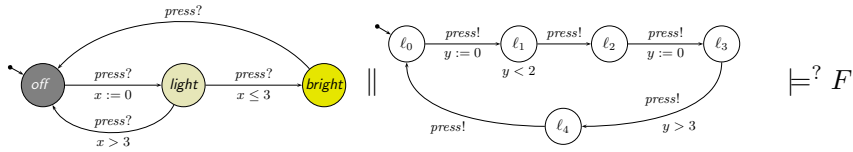
One step more general: an additional **observer** construction  $\mathcal{O}(\cdot)$  such that

$$\tilde{\mathcal{N}} \parallel \mathcal{O}(F) \models_{\text{Uppaal}} f_{\mathcal{O}}(F) \iff \mathcal{N} \models F$$

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## Model-Checking Invariants with Uppaal



- **Quite clear:**  $F = \Box[P]$ .

- Unfortunately, we have

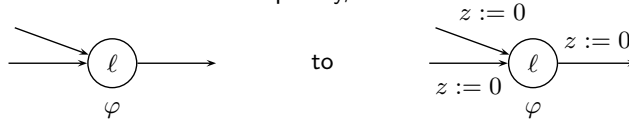
$$\mathcal{N} \models \forall \Box P \implies \mathcal{N} \models \Box[P]$$

but in general not

$$\mathcal{N} \models \Box[P] \implies \mathcal{N} \models \forall \Box P,$$

because Uppaal also considers violations of  $P$  without duration, at points.

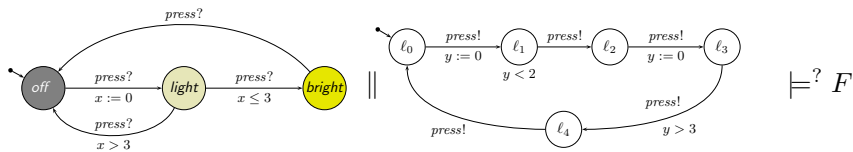
- Possible fix: measure duration explicitly, transform



Then check for  $\mathcal{N} \models \forall \Box(z > 0 \implies P)$ .

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## Model-Checking Certain Durations with Uppaal



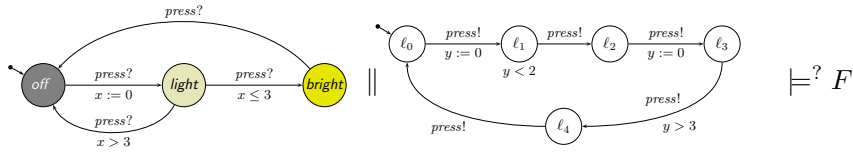
- **Maybe:**  $F = \Box(\ell > t_0 \implies \Diamond[P]_1^{t_1}), t > 0$

Check for

$$\mathcal{N} \models \forall \Diamond(P \wedge z_0 > t_0 \wedge z_1 = t_1).$$

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## Model-Checking Certain Chops with Uppaal



- **Not so clear:**  $F = \neg \diamond ([\text{bright}] ; [\text{light}])$  (Expectation? Holds or not?)

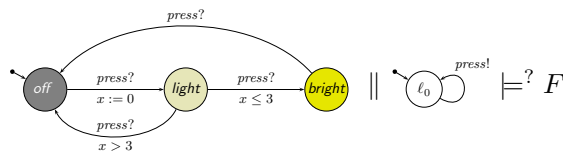
### Off-hand approach:

- Add two auxiliary duration clocks  $z_{\text{light}}$  and  $z_{\text{bright}}$ .
- Add auxiliary variable `prev` with  $\mathcal{D}(\text{prev}) = \{\text{off}, \text{light}, \text{bright}\}$  keeping track of where we came from.
- Observe:  $[\text{bright}] ; [\text{light}]$  means “get from bright directly to light”.
- Check for

$$\mathcal{N} \models \forall \diamond (\mathcal{L}.\text{light} \wedge z_{\text{light}} > 0 \wedge z_{\text{bright}} > 0 \wedge \text{prev} = \text{bright})$$

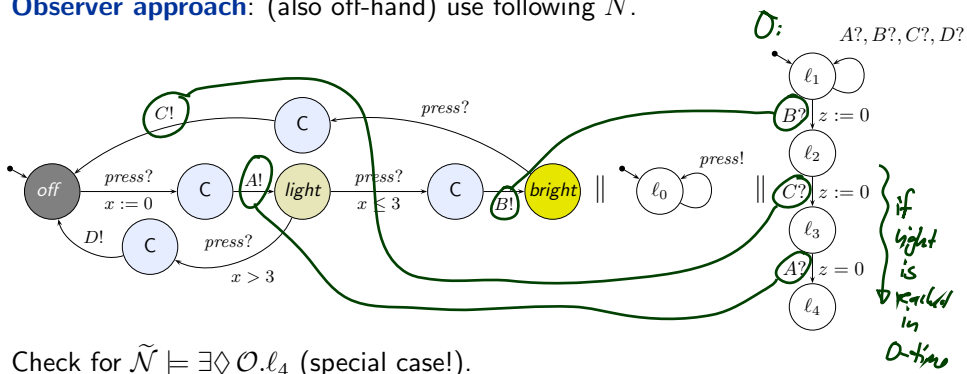
- **Exercise:** Prove  $\mathcal{N} \models F \iff \tilde{\mathcal{N}} \models f(F)$ .

## Model-Checking Certain Chops with Uppaal



- **Not so clear:**  $F = \neg \diamond ([\text{bright}] ; [\text{light}])$  (Expectation? Holds or not?)

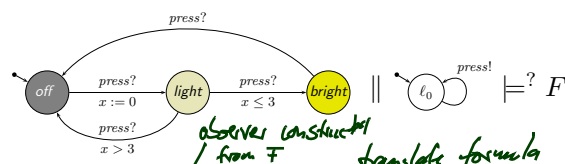
### Observer approach: (also off-hand) use following $\tilde{\mathcal{N}}$ .



Check for  $\tilde{\mathcal{N}} \models \exists \diamond \mathcal{O}.\ell_4$  (special case!).

## Testable DC Properties

### A More Systematic Approach



- We have seen  $f_{\mathcal{O}}$ ,  $\sim$ , and  $\mathcal{O}(\cdot)$  with *modification of  $\mathcal{N}$*   $\rightarrow \tilde{\mathcal{N}} \parallel \mathcal{O}(F) \models_{\text{Uppaal}} f_{\mathcal{O}}(F) \iff \mathcal{N} \models F$  *observe construct from F* *translate formula into query* (\*)

for **some particular**  $F$ . **Tedious**: always have to prove (\*).

- **Better**:
  - characterise a subset (or fragment) of DC,
  - give procedures to construct  $f_{\mathcal{O}}(\cdot)$ ,  $\sim$ , and  $\mathcal{O}(\cdot)$
  - prove once and for all that, if  $F$  is in this fragment, then

$$\tilde{\mathcal{N}} \parallel \mathcal{O}(F) \models_{\text{Uppaal}} f_{\mathcal{O}}(F) \iff \mathcal{N} \models F$$

- **Even better**: exact (syntactic) characterisation of the DC fragment that is testable (not in the lecture).

## Testability

**Definition 6.1.** A DC formula  $F$  is called **testable** if an observer (or test automaton (or monitor))  $\mathcal{A}_F$  exists such that for all networks  $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$  it holds that

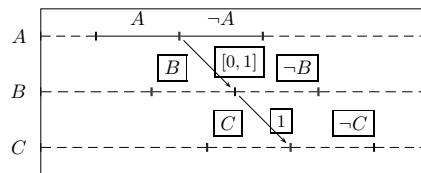
$$\mathcal{N} \models F \quad \text{iff} \quad \mathcal{C}(\mathcal{A}'_1, \dots, \mathcal{A}'_n, \mathcal{A}_F) \models \forall \square \neg(\mathcal{A}_F.q_{bad})$$

Otherwise it's called **untestable**.

**Proposition 6.3.** There exist untestable DC formulae.

**Theorem 6.4.** DC implementables are testable.

## Untestable DC Formulae



"Whenever we observe a change from  $A$  to  $\neg A$  at time  $t_A$ , the system has to produce a change from  $B$  to  $\neg B$  at some time  $t_B \in [t_A, t_A + 1]$  and a change from  $C$  to  $\neg C$  at time  $t_B + 1$ ."

**Sketch of Proof:** Assume there is  $\mathcal{A}_F$  such that, for all networks  $\mathcal{N}$ , we have

$$\mathcal{N} \models F \quad \text{iff} \quad \mathcal{C}(\mathcal{A}'_1, \dots, \mathcal{A}'_n, \mathcal{A}_F) \models \forall \square \neg(\mathcal{A}_F.q_{bad})$$

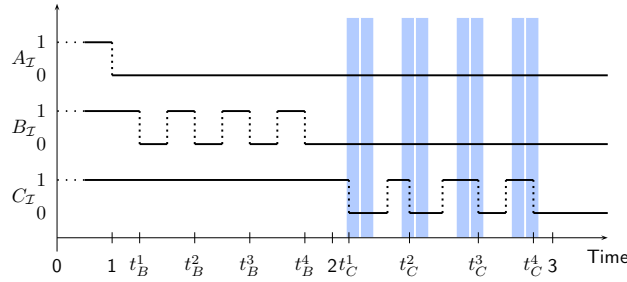
Assume the number of clocks in  $\mathcal{A}_F$  is  $n \in \mathbb{N}_0$ .

## Unstable DC Formulae Cont'd

Consider the following time points:

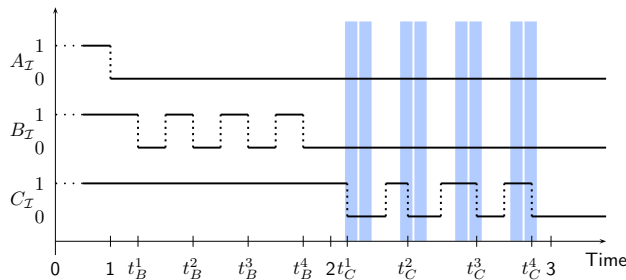
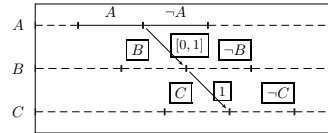
- $t_A := 1$
- $t_B^i := t_A + \frac{2i-1}{2(n+1)}$  for  $i = 1, \dots, n+1$
- $t_C^i \in ]t_B^i + 1 - \frac{1}{4(n+1)}, t_B^i + 1 + \frac{1}{4(n+1)}[$  for  $i = 1, \dots, n+1$   
with  $t_C^i - t_B^i \neq 1$  for  $1 \leq i \leq n+1$ .

**Example:**  $n = 3$



## Unstable DC Formulae Cont'd

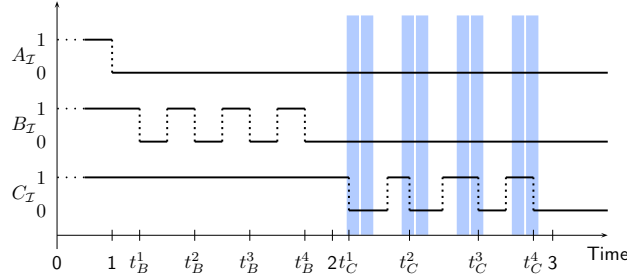
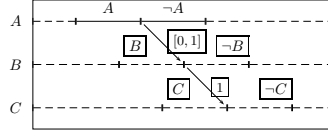
**Example:**  $n = 3$



- The shown interpretation  $\mathcal{I}$  satisfies **assumption** of property.
- It has  $n+1$  candidates to satisfy **commitment**.
- By choice of  $t_C^i$ , the commitment is not satisfied; so  $F$  not satisfied.
- Because  $\mathcal{A}_F$  is a test automaton for  $F$ , it has a computation path to  $q_{bad}$ .
- Because  $n = 3$ ,  $\mathcal{A}_F$  can not save all  $n+1$  time points  $t_B^i$ .
- Thus there is  $1 \leq i_0 \leq n$  such that all clocks of  $\mathcal{A}_F$  have a valuation which is not in  $2 - t_B^{i_0} + (-\frac{1}{4(n+1)}, \frac{1}{4(n+1)})$

## Untestable DC Formulae Cont'd

**Example:**  $n = 3$



- Because  $\mathcal{A}_F$  is a test automaton for  $F$ , it has a computation path to  $q_{bad}$ .
- Thus there is  $1 \leq i_0 \leq n$  such that all clocks of  $\mathcal{A}_F$  have a valuation which is not in  $2 - t_B^{i_0} + (-\frac{1}{4(n+1)}, \frac{1}{4(n+1)})$
- Modify the computation to  $\mathcal{I}'$  such that  $t_C^{i_0} := t_B^{i_0} + 1$ .
- Then  $\mathcal{I}' \models F$ , but  $\mathcal{A}_F$  reaches  $q_{bad}$  via the same path.
- That is:  $\mathcal{A}_F$  claims  $\mathcal{I}' \not\models F$ .
- Thus  $\mathcal{A}_F$  is not a test automaton. **Contradiction.**

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## Testable DC Formulae

**Theorem 6.4.** DC implementables are testable.

- **Initialisation:**  $[\ ] \vee [\pi]; true$
- **Sequencing:**  $[\pi] \longrightarrow [\pi \vee \pi_1 \vee \dots \vee \pi_n]$
- **Progress:**  $[\pi] \xrightarrow{\theta} [\neg\pi]$
- **Synchronisation:**  $[\pi \wedge \varphi] \xrightarrow{\theta} [\neg\pi]$
- **Bounded Stability:**  $[\neg\pi]; [\pi \wedge \varphi] \xrightarrow{\leq \theta} [\pi \vee \pi_1 \vee \dots \vee \pi_n]$
- **Unbounded Stability:**  $[\neg\pi]; [\pi \wedge \varphi] \longrightarrow [\pi \vee \pi_1 \vee \dots \vee \pi_n]$
- **Bounded initial stability:**  $[\pi \wedge \varphi] \xrightarrow{\leq \theta}_0 [\pi \vee \pi_1 \vee \dots \vee \pi_n]$
- **Unbounded initial stability:**  $[\pi \wedge \varphi] \longrightarrow_0 [\pi \vee \pi_1 \vee \dots \vee \pi_n]$

### Proof Sketch:

- For each implementable  $F$ , construct  $\mathcal{A}_F$ .
- Prove that  $\mathcal{A}_F$  is a test automaton.

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## Proof of Theorem 6.4: Preliminaries

- **Note:** DC does not refer to communication between TA in the network, but only to data variables and locations (and clock constraints, if added).

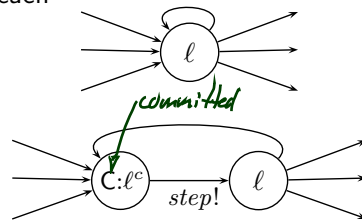
**Example:**

$$\diamond([v = 0] ; [v = 1])$$

- **Recall:** transitions of TA are only triggered by synchronisation, not by changes of data-variables.
- **Approach:** have auxiliary *step* action.

Technically, replace each

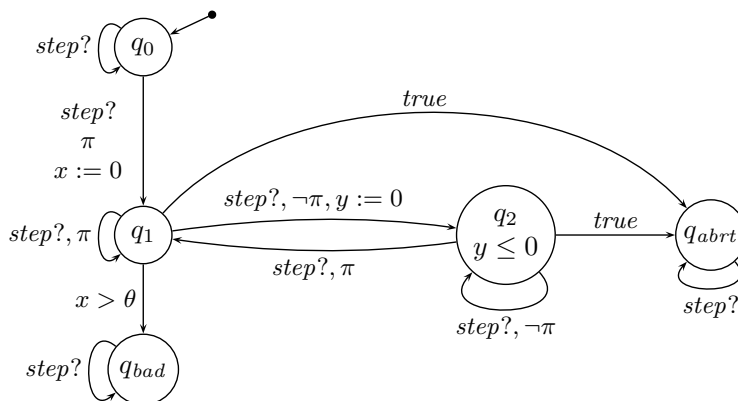
by



Note: the observer sees the data variables **after** the update.

## Proof of Theorem 6.4: Sketch

- Example:  $[\pi] \xrightarrow{\theta} [\neg\pi]$





## Counterexample Formulae

### Definition 6.5.

- A **counterexample formula** (CE for short) is a DC formula of the form:

$$true ; ([\pi_1] \wedge \ell \in I_1) ; \dots ; ([\pi_k] \wedge \ell \in I_k) ; true$$

where for  $1 \leq i \leq k$ ,

- $\pi_i$  are state assertions,
- $I_i$  are non-empty, and open, half-open, or closed time intervals of the form
  - $(b, e)$  or  $[b, e)$  with  $b \in \mathbb{Q}_0^+$  and  $e \in \mathbb{Q}_0^+ \dot{\cup} \{\infty\}$ ,
  - $(b, e]$  or  $[b, e]$  with  $b, e \in \mathbb{Q}_0^+$ . $(b, \infty)$  and  $[b, \infty)$  denote unbounded sets.
- Let  $F$  be a DC formula. A DC formula  $F_{CE}$  is called **counterexample formula for  $F$**  if  $\models F \iff \neg(F_{CE})$  holds.

**Theorem 6.7.** CE formulae are testable.

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## References

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## References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.