

### Real-Time Systems

#### Lecture 15: Automatic Verification of DC Properties for TA

2012-07-12

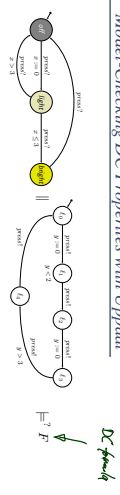
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### Contents & Goals

- Last Lecture:**
  - Extended Timed Automata
  - Uppaal Query Language
- This Lecture:**
  - Educational Objectives:** Capabilities for following tasks/questions:
    - How can we relate TA and DC formulae? What's a bit tricky about that?
    - Can we use Uppaal to check whether a TA satisfies a DC formula?
  - Content:**
    - An evolution-of-observables semantics of TA
    - A satisfaction relation between TA and DC
    - Model-checking DC properties with Uppaal

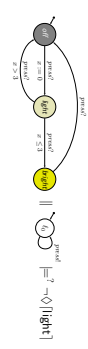
### Observer-based Automatic Verification of DC Properties for TA

### Model-Checking DC Properties with Uppaal



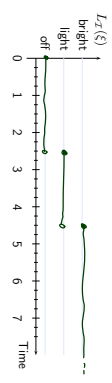
- First Question:** what is the "=" here?
- Second Question:** what kinds of DC formulae can we check with Uppaal?
  - Clear: Not every DC formula (Otherwise contradicting undecidability results)
  - Quite clear:  $F = \Box[\text{off}]$  or  $F = \Diamond[\text{light}]$  (Use Uppaal's fragment of TCTL, something like  $\Box\text{off}$ , but not exactly (see later))
  - Misjoc:  $F = \Box(\langle 5 \rangle \Rightarrow \Diamond[\text{off}]?)$
  - Not so clear:  $F = \langle \Box[\text{bright}] ; \text{light} \rangle$

### Example: Let's Start With Single Runs

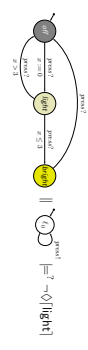


$\xi = \langle \text{off} \rangle, 0 \stackrel{2.5}{\pm} \langle \text{off} \rangle, 2.5 \stackrel{\pm}{\pm} \langle \text{light} \rangle, 3.5 \stackrel{2.0}{\pm} \langle \text{light} \rangle, 4.5 \stackrel{\pm}{\pm} \langle \text{bright} \rangle, 4.5 \dots$

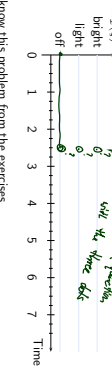
Construct interpretation  $L_T(\xi) : \text{Time} \rightarrow \{\text{off} | \text{light} | \text{bright}\}$ :



### Example 2: Another Single Run



$\xi = \langle \text{off} \rangle, 0 \stackrel{2.5}{\pm} \langle \text{off} \rangle, 2.5 \stackrel{\pm}{\pm} \langle \text{light} \rangle, 3.5 \stackrel{\pm}{\pm} \langle \text{bright} \rangle, 2.5 \stackrel{\pm}{\pm} \langle \text{off} \rangle, 2.5 \stackrel{1.0}{\pm} \dots$



We know this problem from the exercises...

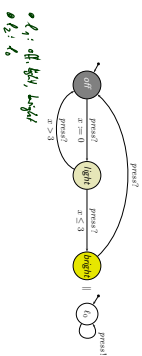
### Observing Timed Automata

**Observables of TA Network: Example**

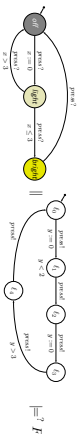
$\mathcal{A}_{t,t} = (L_t, C_t, B_t, U_t, X_t, V_t, I_t, E_t, f_{t,m,t})$

The observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$  are  $\{L_1, \dots, L_n\} \cup \bigcup_{t \in \mathbb{S}^m} V_t$  with

- $D_t(t) = L_t$ ,
- $D_t(v)$  as given,  $v \in V_t$ .



### DC Properties of Timed Automata



**Wanted:** A satisfaction relation between networks of timed automata and DC formulae, a notion of  $\mathcal{N}$  satisfies  $F_t$ , denoted by  $\mathcal{N} \models F_t$ .

- Plan:**
- Consider network  $\mathcal{N}$  consisting of TA
  - Define observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$ .
  - Define evolution  $\mathcal{Z}_t$  of  $\text{Obs}(\mathcal{N})$  induced by computation path  $\xi \in \text{Comp}(\text{paths}(\mathcal{N}))$  of  $\mathcal{N}$ .
  - $\text{Comp}(\text{paths}(\mathcal{N})) = \{\xi \mid \xi \text{ is a computation path of } \mathcal{N}\}$
  - Say  $\mathcal{N} \models F_t$  if and only if  $\forall \xi \in \text{Comp}(\text{paths}(\mathcal{N})) : \mathcal{Z}_t \models F_t$ .

### Evolutions of TA Network

- Recall: computation path**
- $$\xi = \langle \vec{t}_0, v_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{t}_1, v_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{t}_2, v_2 \rangle, t_2 \xrightarrow{\lambda_3} \dots$$
- of  $\mathcal{N}$ ,  $\vec{t}_k$  denotes a tuple  $\langle t_k^1, \dots, t_k^n \rangle \in \mathbb{R}^+ \times \dots \times \mathbb{R}^+$
- Recall:** Given  $\xi$  and  $t \in \text{Time}$ , we use  $\xi(t)$  to denote the set  $\{\langle \vec{t}, v \rangle \mid \exists t' \in \mathbb{N}_0 : t_k \leq t \leq t_{k+1} \wedge \vec{t} = \vec{t}_k \wedge v = v_k + t - t_k\}$
- New:**  $\hat{\xi}(t)$  denotes  $\{\vec{t}_j, v_j + t - t_j\}$  where  $j = \max\{k \in \mathbb{N}_0 \mid t_k \leq t \wedge \vec{t} = \vec{t}_k\}$ .
- Our choice:**
- Ignore configurations assumed for 0-time only.
  - Extend finite computation paths to infinite length, staying in last configuration.
  - Yet docks advance – see later.

### Observables of TA Network

- Let  $\mathcal{N}$  be a network of  $n$  extended timed automata
- $$\mathcal{A}_{t,t} = (L_t, C_t, B_t, U_t, X_t, V_t, I_t, E_t, f_{t,m,t})$$
- For simplicity:** assume that the  $L_t$  and  $X_t$  are pairwise disjoint and that each  $V_t$  is pairwise disjoint to every  $L_t$  and  $X_t$  (otherwise rename).
- Definition:** The observables  $\text{Obs}(\mathcal{N})$  of  $\mathcal{N}$  are  $\{L_1, \dots, L_n\} \cup \bigcup_{1 \leq t \leq n} V_t$
- with
- $D_t(L_t) = L_t$ ,
  - $D_t(v)$  as given,  $v \in V_t$ .
- current location of clock (could be as anything if we need  $\{c_1, \dots, c_n\}$ )*

### Evolutions of TA Network: Example

$\hat{\xi}(t)$  denotes  $\{\vec{t}_j, v_j + t - t_j\}$  where  $j = \max\{k \in \mathbb{N}_0 \mid t_k \leq t \wedge \vec{t} = \vec{t}_k\}$ .

**Example:**

$$\xi = \langle \vec{t}_0, v_0 \rangle, t_0 \xrightarrow{\text{off}, 0} \langle \vec{t}_1, v_1 \rangle, t_1 \xrightarrow{\text{off}, 2.5} \langle \vec{t}_2, v_2 \rangle, t_2 \xrightarrow{\text{light}, 2.5} \langle \vec{t}_3, v_3 \rangle, t_3 \xrightarrow{\text{light}, 2.5} \langle \vec{t}_4, v_4 \rangle, t_4 \xrightarrow{\text{off}, 2.5} \langle \vec{t}_5, v_5 \rangle, t_5 \xrightarrow{\text{off}, 1.0} \langle \vec{t}_6, v_6 \rangle, t_6 \xrightarrow{\text{off}, 3.5} \dots$$

$\hat{\xi}(0) = \emptyset$

$\hat{\xi}(1.0) = \langle \vec{t}_1 \rangle$

$\hat{\xi}(2.5) = \langle \vec{t}_2 \rangle$

$\xi$  induces the unique interpretation

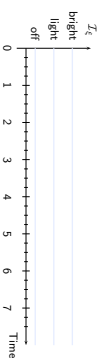
$$\xi_{\mathcal{L}} : \text{Obs}(N) \rightarrow (\text{Time} \rightarrow D)$$

of  $\text{Obs}(N)$  defined pointwise as follows:

$$\xi_{\mathcal{L}}(a)(t) = \begin{cases} a & \text{if } a = \xi(t) \\ \nu(a) & \text{if } a \in V, \xi(t) = \langle \bar{t}, \dots, \nu \rangle \end{cases}$$

**Example:**  $D(\xi) = \{\text{off}, \text{light}, \text{bright}\}$

$$\xi = \langle \text{off}, 0 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{light}, 2.5 \rangle, \langle \text{light}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 3.5 \rangle, \dots$$



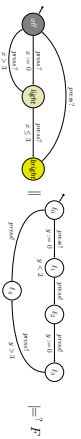
Some Checkable Properties

$$\xi = \langle \text{off}, 0 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{light}, 2.5 \rangle, \langle \text{light}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 2.5 \rangle, \langle \text{off}, 3.5 \rangle, \dots$$

Abbreviations as usual: *convenient like for readability*

- $\xi_{\mathcal{L}}(a)(0) = \xi(\text{In } D_{\mathcal{L}})$
- $\mathcal{I}(a) = \text{off}(0) = \xi_{\mathcal{L}}(a)(0) = \mathcal{I}(\text{off})$
- $\mathcal{I}(\text{off})(1,0) = \mathcal{I}(\xi = \text{off})(2,0)$  if  $L_1$  pairwise disjoint.

Model-Checking DC Properties with Uppaal



- First Answer:**  $N \models F$  if and only if  $\forall \xi \in \text{CompTrajs}(N) : \xi_{\mathcal{L}} \models F$ .
- Second Question:** what kinds of DC formulae can we check with Uppaal?
  - Clear:** Not every DC formula (Otherwise contradicting undecidability results)
  - Quite clear:**  $F = \square[\text{off}]$  or  $F = \neg \diamond[\text{light}]$  (Use Uppaal's fragment of TCTL, something like  $\forall \square \text{off}$  but not exactly (see later))
  - Maybe:**  $F = \square(\ell > 5 \implies \diamond[\text{off}])$
  - Not so clear:**  $F = \neg \diamond(\text{bright} : [\text{light}])$

- But what about clocks? Why not  $x \in \text{Obs}(N)$  for  $x \in X$ ?
- We would know how to define  $\xi_{\mathcal{L}}(x)(t)$ , namely

$$\xi_{\mathcal{L}}(x)(t) = v_x(t(x) + (t - t_0(x)))$$

- But...  $\xi_{\mathcal{L}}(x)(t)$  changes too often.

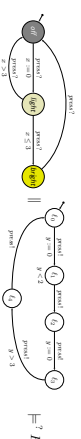
$\square (F \# 1 \Rightarrow x \leq 5)$  *problem disappears*

- Better (if wanted):**
  - add  $\Phi(X_1 \cup \dots \cup X_n)$  to  $\text{Obs}(N)$ ,
  - with  $D(\varphi) = \{0, 1\}$  for  $\varphi \in \Phi(X_1 \cup \dots \cup X_n)$ .
- set

$$\xi_{\mathcal{L}}(\varphi)(t) = \begin{cases} 1, & \text{if } v(\varphi) \models \varphi, \xi(t) = \langle \bar{t}, \nu \rangle \\ 0, & \text{otherwise} \end{cases}$$

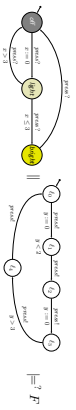
The truth value of constraint  $\varphi$  can endure over non-point intervals.

Model-Checking DC Properties with Uppaal



- Second Question:** what kinds of DC formulae can we check with Uppaal?
  - Wanted:**
    - a function  $f$  mapping DC formulae to Uppaal queries and
    - a transformation  $\tilde{\cdot}$  of networks of TA such that
  - $N \models_{\text{Uppaal}} f(F) \iff N \models F$
- One step more general: an additional observer construction  $O(\cdot)$  such that
 
$$\tilde{N} \models O(F) \models_{\text{Uppaal}} f_O(F) \iff N \models F$$

### Model-Checking Invariants with Uppaal



- Quite clear:  $F = \Box[P]$
- Unfortunately, we have

$$X \models \forall \Delta P \not\Rightarrow X \models \Box P$$

but in general not

$$X \models \Box P \not\Rightarrow X \models \forall \Delta P$$

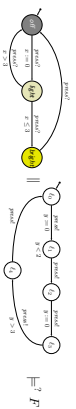
because Uppaal also considers violations of  $P$  without duration, at points.

- Possible fix: measure duration explicitly transform



Then check for  $X \models \forall \Delta (\delta > 0 \Rightarrow P)$ .

### Model-Checking Certain Durations with Uppaal

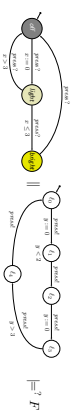


- Maybe:  $F = \Box(t > t_0 \Rightarrow \Diamond[P])$ ,  $t > 0$

Check for

$$X \models \forall \Delta (P \wedge \Delta_0 > t_0 \wedge \Delta_1 = t_1)$$

### Model-Checking Certain Chops with Uppaal



- Not so clear:  $F = \neg \langle [ \text{bright} ] ; [ \text{light} ] \rangle$  (Expectation? Holds or not?)

Off-hand approach:

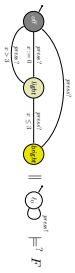
- Add two auxiliary duration clocks  $\Delta_{\text{right}}$  and  $\Delta_{\text{left}}$ .
- Add auxiliary variable  $\text{prev}$  with  $\Delta_{\text{right}} = \text{off} [ \text{right}, \text{bright} ]$  keeping track of where we came from.
- Observe:  $[ \text{bright} ] ; [ \text{light} ]$  means "get from bright directly to light".

Check for

$$X \models \forall \Delta (\Delta_{\text{right}} > 0 \wedge \Delta_{\text{left}} > 0 \wedge \text{prev} = \text{bright})$$

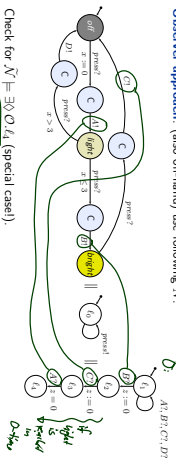
- Exercise: Prove  $X \models F \Leftrightarrow X \models F(F)$ .

### Model-Checking Certain Chops with Uppaal



- Not so clear:  $F = \neg \langle [ \text{bright} ] ; [ \text{light} ] \rangle$  (Expectation? Holds or not?)

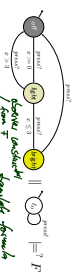
Observer approach: (also off-hand) use following  $X'$



Check for  $X' \models \exists \Delta \exists t_0 t_1$  (special case)

### Testable DC Properties

### A More Systematic Approach



- We have seen  $\text{fo}, \neg$ , and  $\langle \cdot \rangle$  with  $X \models F$

for some particular  $F$ . Tedious: always have to prove (\*)

Better:

- characterize a subset (or fragment) of DC
- give procedures to construct  $F \langle \cdot \rangle$ ,  $\neg$  and  $\langle \cdot \rangle$
- prove once and for all that, if  $F$  is in this fragment, then  $X \models \langle \cdot \rangle (F) \Leftrightarrow \text{Uppaal } \text{fo}(F) \Leftrightarrow X \models F$

- Even better: exact (syntactic) characterization of the DC fragment that is testable (not in the lecture).

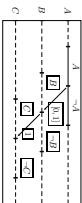
Definition 6.1. A DC formula  $F$  is called **testable** if an observer (or test automaton (or monitor))  $A_F$  exists such that for all networks  $N = (A_1, \dots, A_n)$  it holds that

$$N \models F \quad \text{iff} \quad C(A_1, \dots, A_n, A_F) \models \forall \square \neg (A_F \cdot \text{good})$$

Otherwise it's called **untestable**.

Proposition 6.3. There exist untestable DC formulae.

Theorem 6.4. DC implementables are testable.



Whenever we observe a change from  $A$  to  $\neg A$  at time  $t_A$ , the system has to produce a change from  $B$  to  $\neg B$  at some time  $t_B \in [t_A, t_A + 1]$  and a change from  $C$  to  $\neg C$  at time  $t_B + 1$ .

Sketch of Proof: Assume there is  $A_F$  such that, for all networks  $N$ , we have

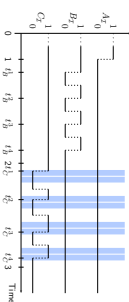
$$N \models F \quad \text{iff} \quad C(A_1, \dots, A_n, A_F) \models \forall \square \neg (A_F \cdot \text{good})$$

Assume the number of clocks in  $A_F$  is  $n \in \mathbb{N}_0$ .

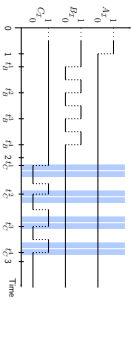
Consider the following time points:

- $t_A := 1$
  - $t_B^i := t_A + \frac{2i-1}{2(n+1)}$  for  $i = 1, \dots, n+1$
  - $t_C^i \in [t_B^i + 1 - \frac{1}{2(n+1)}, t_B^i + 1 + \frac{1}{2(n+1)})$  for  $i = 1, \dots, n+1$
- with  $t_C^i - t_B^i \neq 1$  for  $1 \leq i \leq n+1$ .

Example:  $n = 3$



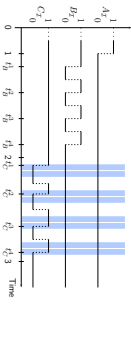
Example:  $n = 3$



- The shown interpretation  $I$  satisfies **assumption of property**.
- It has  $n+1$  candidates to satisfy **commitment**.
- By choice of  $t_C^i$ , the commitment is not satisfied, so  $F$  not satisfied.
- Because  $A_F$  is a test automaton for  $F$ , it has a computation path to  $\text{good}$ .

- Because  $n = 3$ ,  $A_F$  can not save all  $n+1$  time points  $t_B^i$ .
- Thus there is  $1 \leq i_0 \leq n$  such that all clocks of  $A_F$  have a valuation which is not in  $2^{-t_{B}^{i_0}} + (-\frac{1}{2(n+1)}) \cdot \frac{1}{2(n+1)}$ .

Example:  $n = 3$



- Because  $A_F$  is a test automaton for  $F$ , it has a computation path to  $\text{good}$ .
- Thus there is  $1 \leq i_0 \leq n$  such that all clocks of  $A_F$  have a valuation which is not in  $2^{-t_{B}^{i_0}} + (-\frac{1}{2(n+1)}) \cdot \frac{1}{2(n+1)}$ .
- Modify the computation to  $Z$  such that  $t_C^{i_0} = t_B^{i_0} + 1$ .

- Then  $Z \models F$ , but  $A_F$  reaches  $\text{good}$  via the same path.
- That is,  $A_F$  claims  $Z \not\models F$ .
- Thus  $A_F$  is not a test automaton. **Contradiction**.

Theorem 6.4. DC implementables are testable.

- Initialization:**  $\llbracket V \rrbracket \text{true}$
- Sequencing:**  $\llbracket \pi_1 \rrbracket \rightarrow \llbracket \pi_2 \rrbracket$
- Progress:**  $\llbracket \pi \rrbracket \rightarrow \llbracket \pi \rrbracket$
- Synchronization:**  $\llbracket \pi_1 \wedge \pi_2 \rrbracket \rightarrow \llbracket \pi \rrbracket$
- Bounded Stability:**  $\llbracket \pi \rrbracket \rightarrow \llbracket \pi \rrbracket$   $\xrightarrow{\leq \delta} \llbracket \pi \rrbracket$
- Unbounded Stability:**  $\llbracket \pi \rrbracket \rightarrow \llbracket \pi \rrbracket$   $\xrightarrow{\leq \delta} \llbracket \pi \rrbracket$
- Bounded initial stability:**  $\llbracket \pi \wedge \varphi \rrbracket \xrightarrow{\leq \delta} \llbracket \pi \wedge \varphi \rrbracket$
- Unbounded initial stability:**  $\llbracket \pi \wedge \varphi \rrbracket \xrightarrow{\leq \delta} \llbracket \pi \wedge \varphi \rrbracket$

**Proof Sketch:**

- For each implementable  $F$ , construct  $A_F$ .
- Prove that  $A_F$  is a test automaton.

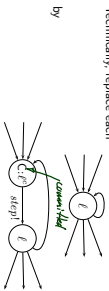
- **Note:** DC does not refer to communication between TA in the network, but only to data variables and locations *and other constraints, if added*.

Example:

$$\diamond [v = 0] ; [v = 1]$$

- **Recall:** transitions of TA are only triggered by synchronisation, not by changes of data-variables.
- **Approach:** have auxiliary *step* action.

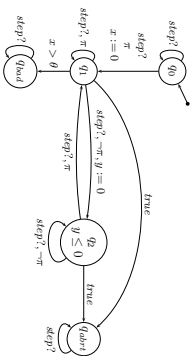
Technically, replace each



by

31.8

- Example:  $[\pi] \xrightarrow{a} [\neg\pi]$



32.8

Definition 6.5

- A **counterexample formula** (CE for short) is a DC formula of the form:

$$true ; ([\pi] \wedge \ell \in I_1) ; \dots ; ([\pi] \wedge \ell \in I_k) ; true$$

where for  $1 \leq i \leq k$ ,

- $\pi_i$  are state assertions,
  - $I_i$  are non-empty, and open, half-open, or closed time intervals of the form
  - $(b_i, e_i)$  or  $[b_i, e_i)$  with  $b_i \in \mathbb{Q}_t^+$  and  $e_i \in \mathbb{Q}_t^+ \cup \{\infty\}$ ,
  - $(b_i, e_i)$  or  $[b_i, e_i)$  with  $b_i, e_i \in \mathbb{Q}_t^+$ ,
  - $(b_i, \infty)$  and  $[b_i, \infty)$  denote unbounded sets.
- Let  $F$  be a DC formula. A DC formula  $F_{CE}$  is called **counterexample formula for  $F$**  if  $F \iff \neg(F_{CE})$  holds.

Theorem 6.7 CE formulae are testable.

33.8

References

[Odlberg and Dierks, 2008] Odlberg, E.-R. and Dierks, H. (2008) *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.