11 Text search

Text search

Different scenarios:

Dynamic texts

- Text editors
- Symbol manipulators

Static texts

- Literature databases
- Library systems
- Gene databases
- World Wide Web

Text search

Data type **string**:

- array of character
- file of character
- list of character

Operations: (Let *T*, *P* be of type **string**)

Length:	length ()
i-**th character**:	*T* [*i*]
concatenation:	cat (*T*, *P*) *T.P*

Problem definition

Input:

Text $t_1\ t_2\ ....\ t_n \quad \in \Sigma$

Pattern $p_1 p_2\ ...\ p_m \quad \in \Sigma$

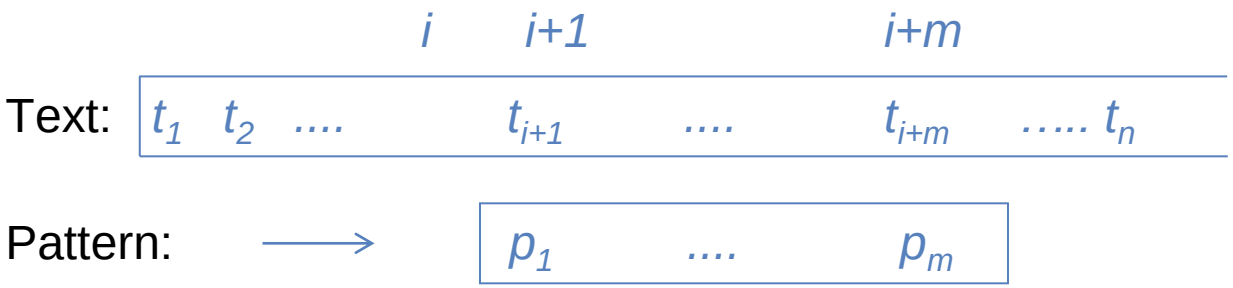
and $m \leq n$

Goal:

Find one or all occurrences of the pattern in the text,
i.e. shifts i $(0 \leq i \leq n - m)$ such that

$$p_1 = t_{i+1}$$
$$p_2 = t_{i+2}$$
$$\vdots$$
$$p_m = t_{i+m}$$

Problem definition



Estimation of cost (time) :

1. # possible shifts: $n - m + 1$ # pattern positions: m
   → $O(n \cdot m)$

2. At least 1 comparison per m consecutive text positions:
   → $\Omega(m + n/m)$

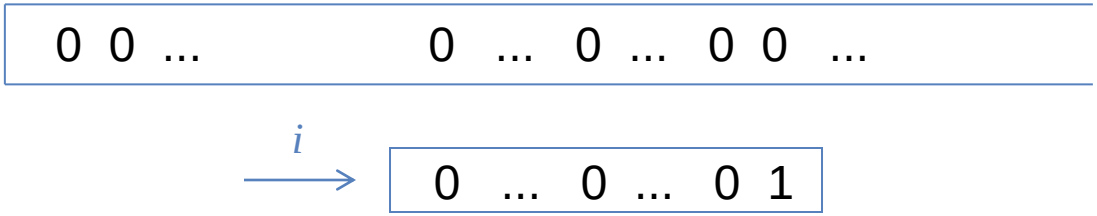
Naïve approach

For each possible shift $0 \leq i \leq n - m$ check at most m pairs of characters. Whenever a mismatch occurs, start with the next shift.

```
textsearchbf  := proc (T : : string, P : : string)
#  Input:   Text T und Pattern P
# Output:  List L of shifts i, at which P occurs in T
     n := length (T); m := length (P);
     L := [];
     for i from 0 to n-m {
              j := 1;
              while j ≤ m and T[i+j] = P[j]
                        do j := j+1 od;
              if j = m+1 then L := [L [] , i] fi;
     }
     RETURN (L)
end;
```

Naïve approach

Cost estimation (time):



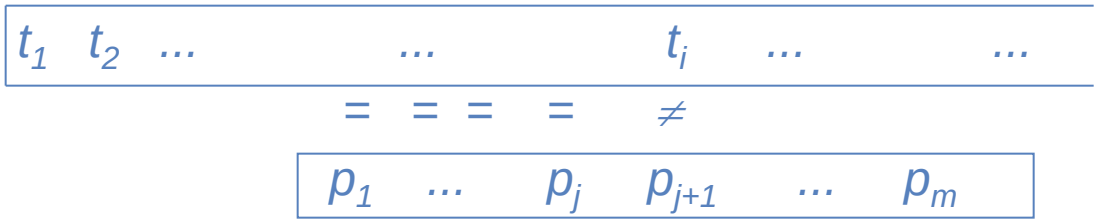
Worst Case: $\Omega(m \cdot n)$

In practice: mismatch often occurs very early

→ running time ~ $c \cdot n$

Method of Knuth-Morris-Pratt (KMP)

Let t_i and p_{j+1} be the characters to be compared:



If, at a shift, the first mismatch occurs at

t_i and p_{j+1}, then:

- The last j characters inspected in T equal the first j characters in P.
- $t_i \neq p_{j+1}$

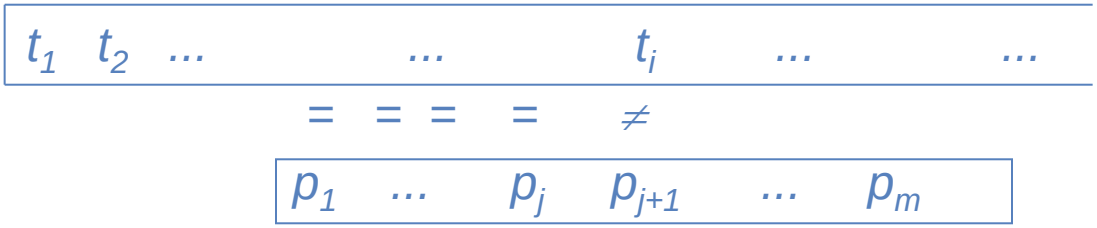
Method of Knuth-Morris-Pratt (KMP)

Idea:

Determine $j' = next[j] < j$ such that t_i can then be compared with $p_{j'+1}$.

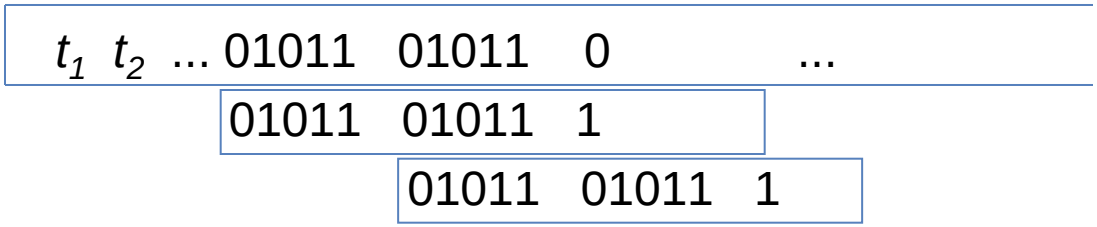
Determine $j' < j$ such that $P_{1...j'} = P_{j-j'+1...j}$.

Find the longest prefix of P that is a proper suffix of $P_{1...j}$.



Method of Knuth-Morris-Pratt (KMP)

Example for determining *next*[*j*]:



next[*j*] = length of the longest prefix of P that is a proper suffix of $P_{1 \ldots j}$.

Method of Knuth-Morris-Pratt (KMP)

UNI FREIBURG

⇒for P = 0101101011, *next* = [0,0,1,2,0,1,2,3,4,5] :

1	2	3	4	5	6	7	8	9	10
0	1	0	1	1	0	1	0	1	1
		0							
		0	1						
					0				
					0	1			
					0	1	0		
					0	1	0	1	
					0	1	0	1	1

Method of Knuth-Morris-Pratt (KMP)

```
KMP := proc (T : : string, P : : string)
# Input: text T and pattern P
# Output: list L of shifts i at which P occurs in T
     n := length (T); m := length(P);
     L :=  []; next := KMPnext(P);
     j  :=  0;
     for i from 1 to n do
              while j>0 and T[i] <> P[j+1] do j := next [j] od;
              if T[i]  = P[j+1]  then j := j+1 fi;
              if j = m then L := [L[] , i-m] ;
                                  j  := next [j]
              fi;
      od;
      RETURN (L);
end;
```

Method of Knuth-Morris-Pratt (KMP)

Pattern: abracadabra, *next* = [0,0,0,1,0,1,0,1,2,3,4]

a b r a c a d a b r a b r a b a b r a c ...
| | | | | | | | | | |
a b r a c a d a b r a

next[11] = 4

a b r a c a d a b r a b r a b a b r a c ...
- - - - ┼
a b r a c

next[4] = 1

Method of Knuth-Morris-Pratt (KMP)

a b r a c a d a b r a b r a b a b r a c ...

- | | | ≠

a b r a c

next [4] = 1

a b r a c a d a b r a b r a b a b r a c ...

- | ≠

a b r a c

next [2] = 0

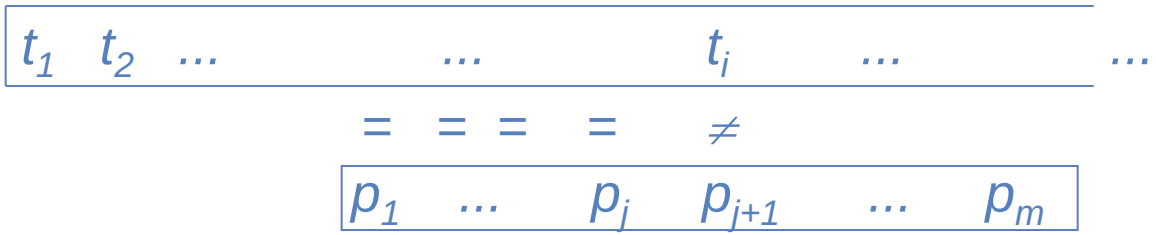
a b r a c a d a b r a b r a b a b r a c ...

| | | | |

a b r a c

Method of Knuth-Morris-Pratt (KMP)

Correctness:



Situation at start of the for-loop:

$P_{1...j} = T_{i-j...i-1}$ and $j \neq m$

if $j = 0$: we are at the first character of P

if $j \neq 0$: P can be shifted while $j > 0$ and $t_i \neq p_{j+1}$

Method of Knuth-Morris-Pratt (KMP)

UNI FREIBURG

If $T[i] = P[j+1]$, j and i can be increased (at the end of the loop).

When P has been compared completely ($j = m$), a position was found, and we can shift.

Method of Knuth-Morris-Pratt (KMP)

UNI FREIBURG

Time complexity:

- Text pointer *i* is never reset
- Text pointer *i* and pattern pointer *j* are always incremented together
- Always: *next*[j] < *j*;
  j can be decreased only as many times as it has been increased.

The KMP algorithm can be carried out in time $O(n)$,
if the *next*-array is known.

Computing the *next*-array

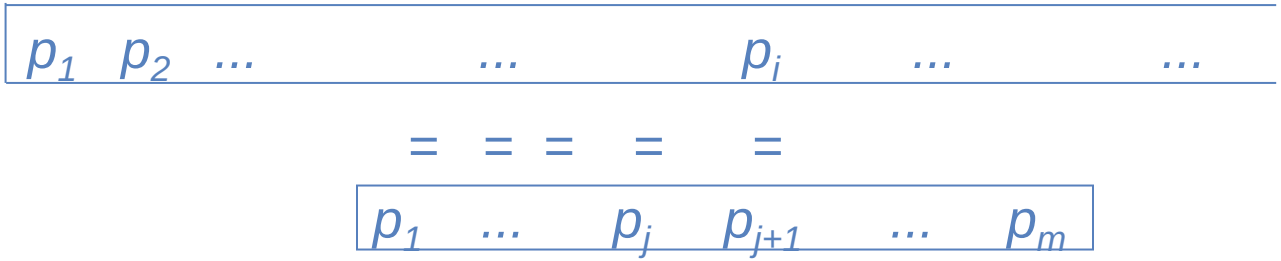
next[i] = length of the longest prefix of *P* that is a proper suffix of $P_1 \ldots_i$.

next[1] = 0

Let *next*[*i*-1] = *j*:

Consider two cases:

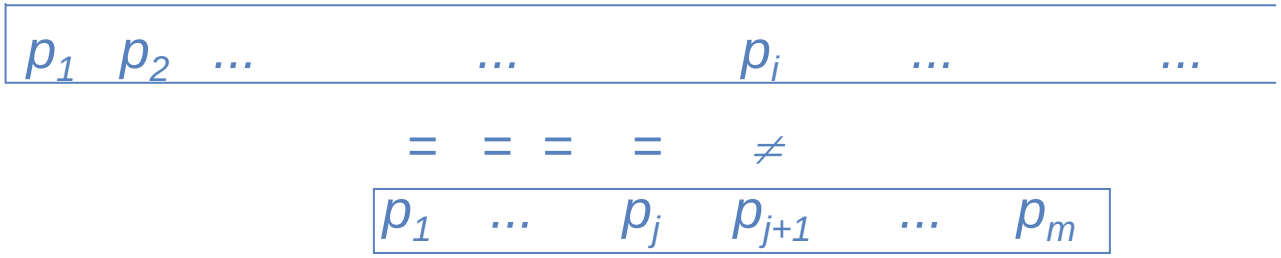
1) $p_i = p_{j+1}$ → *next*[*i*] = *j* + 1



Computing the *next*-array

2) $p_i \neq p_{j+1}$ → replace j by $next[\, j \,]$, until $p_i = p_{j+1}$ or $j = 0$.

If $p_i = p_{j+1}$, we can set $next[i] = j + 1$,
otherwise $next[i] = 0$.



Computing the *next*-array

```
KMPnext := proc (P : : string)
#Input    :  pattern P
#Output :   next-Array for P
    m := length (P);
    next := array (1..m);
    next [1] := 0;
    j := 0;
    for i from 2 to m do
        while j > 0 and P[i] <> P[j+1]
            do j := next [j] od;
        if P[i] = P[j+1] then j := j+1 fi;
        next [i] := j
    od;
    RETURN (next);
end;
```

Running time of KMP

UNI FREIBURG

The KMP algorithm can be carried out in time O($n + m$).