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Theory I, Sheet 3

- The solutions should be submitted in English.
- JUST FOR FUN exercises are not mandatory.
- Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (May 14, 4:00 p.m.).
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 3.1 - AVL trees

Consider an empty AVL tree, then:

1. Insert the keys: 20, 30, 40, 50, 60, 70.
2. Delete the nodes: 70, 60.

For every insertion draw the resulting AVL tree as well as the intermediate AVL tree. Indicate the corresponding operations (rotations, *upin, opout* procedure calls).

Exercise 3.2 - AVL trees

You may recall from the slides of AVL trees (04_Balanced_Tree_AVL.pdf), that the procedure $upin(p)$ was detailed when p was the left child of φp. Namely,

1. p is the left child of φp:
   (a) $bal(\varphi p) = +1 \rightarrow bal(\varphi p) = 0$, done.
   (b) $bal(\varphi p) = 0 \rightarrow bal(\varphi p) = -1$, $upin(\varphi p)$.
   (c) i. $bal(\varphi p) = -1 \rightarrow bal(\varphi p) = -1$ right rotation, done.
   ii. $bal(\varphi p) = -1 \rightarrow bal(\varphi p) = +1$ double rotation left-right, done.
2. p is the right child of φp:

Complete the steps to follow when p is the right child of φp.

Exercise 3.3 - AVL trees

JUST FOR FUN. Show that in an AVL tree t with F_k leaves (where F_k is a Fibonacci number) and $k \geq 7$ the *internal path length* $l(t) \leq F_k \cdot (k-4)$. Proceed as follows:

1. Determine the maximum height h of an AVL tree with F_k leaves. Explain informally why in this case an AVL tree with maximum height also has the maxi-mun internal path length.
2. Using induction show the above inequality.

   Hint: Be sure you start with the right base case(s). For the induction step use the definition of the internal path length and your knowledge about the number of internal nodes in the tree.
3. What does this tell us about the average search path length $D(t)$ in such a tree (in terms of its height)?