• The solutions should be submitted in English.
• JUST FOR FUN exercises are not mandatory.
• Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (June 4, 4:00 p.m.).
• You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

Exercise 5.1 - Dynamic Tables
Suppose that instead of contracting a table by halving its size when its load factor drops below 1/4, we contract it by multiplying its size by 2/3 when its load factor drops below 1/3. Using the potential function
\[ \phi(T) = |2 \cdot \text{num} - \text{size}| \]
show that the amortized cost of a Table-Delete that uses this strategy is bounded by a constant.

Exercise 5.2 - Dynamic Tables
You may recall from the lecture of dynamic tables the following potential function \( \phi \):
\[
\phi(T) = \begin{cases} 
2k - s, & \text{if } \alpha \geq 1/2, \\
s/2 - k, & \text{if } \alpha < 1/2.
\end{cases}
\]
Show that the amortized cost \( a_i \) is an upper bound of the cost \( t_i \) of the \( i \)-th delete operation, i.e.,
\[
\sum_{i=1}^{m} a_i \geq \sum_{i=1}^{m} t_i.
\]
Show that mentioned upper bound holds when the load factor at the \( i - 1 \) operation is greater or equal than 1/2, i.e., \( \alpha_{i-1} \geq 1/2 \), and after performing a delete operation without contracting the hash table, the load factor at the \( i \) operation:
1. is greater or equal than 1/2, i.e., \( \alpha_i \geq 1/2 \).
2. is less than 1/2, i.e., \( \alpha_i < 1/2 \).

Exercise 5.3 - Randomized Quicksort
JUST FOR FUN. Show that
\[
\sum_{k=2}^{n-1} k \lg k \leq \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2
\]
**Hint:** Split the summation into two parts, one for \( k = 2, 3, \ldots, \lfloor n/2 \rfloor - 1 \) and one for \( k = \lfloor n/2 \rfloor, \ldots, n - 1 \).