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## Theory I, Sheet 9

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- The solutions should be submitted in English.
- JUST FOR FUN exercises are not mandatory.
- Your solutions should be delivered to the lockbox in building 051 floor 00, or right before the start of the tutorial (July 2, 4:00 p.m.).
- You are allowed to discuss your solutions with each other. Nevertheless, you are required to write down the answers in your own words.

### Exercise 9.1 - $\Sigma$ -Algebras

1. Explain why  $\widehat{\mathcal{J}}$  is required to be a total function.
2. Define the ADT `bool` for the boolean algebra with the function symbols `true`, `false`, `not`, `and`, `or`, including the usual identities.
3. Give an implementation for the ADT `bool`.

### Exercise 9.2 - Stack ADT

Specify an ADT `Stack(A)` for stacks. The operations available for this ADT should be as follows:

- `new`: Constructs a new empty stack.
- `push`: Adds an element to the top of the given stack.
- `pop`: Removes the top element of the given stack.
- `top`: Returns the top element of the given stack.
- `isEmpty`: Checks whether a given stack is empty.

Specify the signatures for these operations and define sensible identities for them. What are the constructors of the stack ADT?

### Exercise 9.3 - Term induction

JUST FOR FUN. Let  $s, t$  be terms and  $p, q$  be strings over the natural numbers. Prove the following propositions:

1. If  $pq \in \mathcal{Pos}(s)$ , then  $p \in \mathcal{Pos}(s)$ .
2. If  $pq \in \mathcal{Pos}(s)$ , then  $s|_{pq} = (s|_p)|_q$ .
3. If  $p \in \mathcal{Pos}(s)$  and  $q \in \mathcal{Pos}(t)$ , then  $(s[t]_p)|_{pq} = t|_q$ .