• The solutions should be submitted in English.
• JUST FOR FUN exercises are not mandatory.
• Your solutions should be delivered to the lockbox in building 051 floor 00, or right before
  the start of the tutorial (July 2, 4:00 p.m.).
• You are allowed to discuss your solutions with each other. Nevertheless, you are required to
  write down the answers in your own words.

Exercise 9.1 - $\Sigma$-Algebras

1. Explain why $\hat{J}$ is required to be a total function.

2. Define the ADT $\text{bool}$ for the boolean algebra with the function symbols true, false, not, and, or,
   including the usual identities.

3. Give an implementation for the ADT $\text{bool}$.

Exercise 9.2 - Stack ADT

Specify an ADT $\text{Stack}(A)$ for stacks. The operations available for this ADT should be as follows:

- $\text{new}$: Constructs a new empty stack.
- $\text{push}$: Adds an element to the top of the given stack.
- $\text{pop}$: Removes the top element of the given stack.
- $\text{top}$: Returns the top element of the given stack.
- $\text{isEmpty}$: Checks whether a given stack is empty.

Specify the signatures for these operations and define sensible identities for them. What are the
constructors of the stack ADT?

Exercise 9.3 - Term induction

JUST FOR FUN. Let $s, t$ be terms and $p, q$ be strings over the natural numbers. Prove the
following propositions:

1. If $pq \in \mathcal{P}os(s)$, then $p \in \mathcal{P}os(s)$.

2. If $pq \in \mathcal{P}os(s)$, then $s|_{pq} = (s|_p)|_q$.

3. If $p \in \mathcal{P}os(s)$ and $q \in \mathcal{P}os(t)$, then $(s[t|_p]|_{pq} = t|_q)$. 