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Tutorials for Decision Procedures
Exercise sheet 10

Exercise 1: Decision Procedure for T_{A}

Apply the decision procedure for arrays to check *validity* of the following T_{A}-formulae. In the last step of the algorithm, where you have to use the decision procedure for the quantifier-free fragment, you do not need to follow the corresponding algorithm but may argue intuitively.

(a) $(\forall i.a[i] = b[i]) \;\rightarrow\; (\forall i.\; a\langle j \triangleleft v\rangle[i] = b\langle j \triangleleft v\rangle[i])$

(b) $\exists j.a\langle i \triangleleft v\rangle[j] = v$

(c) $\forall j.a\langle i \triangleleft v\rangle[j] = v$

Exercise 2: Decision Procedure for $T_{\mathsf{A}}^{\mathbb{Z}}$

Check *validity* of the formula

$$sorted(a, \ell, k) \;\wedge\; sorted(a, k, u) \;\rightarrow\; sorted(a, \ell, u)$$

where *sorted* is defined as usual:

$$sorted(a, \ell, u) : \quad \forall i, j.\; \ell \leq i \leq j \leq u \;\rightarrow\; a[i] \leq a[j]$$

Again, in the last step you may argue intuitively talking only about the relevant combinations of indices from the index set.

Exercise 3: Correctness of DP for $T_{\mathsf{A}}^{\mathbb{Z}}$

Let I be an interpretation. Prove for $F[\bar{i}] : expr \leq expr$ that $I \models F[\bar{i}] \;\rightarrow\; F[\bar{t}]$, where $\bar{i} = (i_1, \ldots, i_n)$ and $\bar{t}$ is the vector $\bar{t} = (t_1, \ldots, t_n) \in \mathcal{I}^n$ with $\alpha_I[t_k] = proj_{\mathcal{I}}(\alpha_I[i_k])$ (in the notation of the book $\bar{t} = \mathsf{proj}_I(\bar{i})$). The expression *expr* is either a universal variable i_k or a *pexpr*. Note that $\mathcal{I}$ contains all *pexpr* and that

$$proj_{\mathcal{I}}(v) = \begin{cases} max\{\alpha_I[t] \mid t \in \mathcal{I} \;\wedge\; \alpha_I[t] \leq v\} & \text{if for some } t \in \mathcal{I}\text{: } \alpha_I[t] \leq v \\ min\{\alpha_I[t] \mid t \in \mathcal{I}\} & \text{otherwise} \end{cases}$$