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Tutorials for Decision Procedures
Exercise sheet 3

Exercise 1: Prenex Normal Form

Transform the following formula into prenex normal form:

$$\Big(\forall z.\ \Big(\big(\forall x.\ q(x,z)\big) \to p(x,g(y),z)\Big)\Big) \wedge \neg\big(\forall z.\ \neg(\forall x.\ q(f(x,y),z))\big)$$

Exercise 2: Syntax and Semantics of FOL

(a) Assume that a, b are constant symbols, f resp. g are function symbols of arity one resp. two, p is a predicate symbol of arity two, and x is a variable. For each of the following strings determine whether it is a *term*, an *atom*, a *literal*, or a *formula*. Note that it can be more than one of these, or none if it is syntactically incorrect.

(i) $f(a)$
(ii) $g(f(a), f)$
(iii) $p(f(a), x)$
(iv) $g(x, f(x))$
(v) $\neg p(a,b)$
(vi) $\exists a.p(a,b)$
(vii) $\exists x.p(x, f(a))$
(viii) $p(x, p(x,x))$
(ix) $p(a,b) \vee p(b,a)$
(x) $p \wedge \exists x.p(x,x)$
(xi) $\neg\exists x.p(a,b)$
(xii) $\forall x.\exists x.p(x,x)$

(b) For each of the following formulae give a satisfying interpretation. Assume that *equals* and p are binary predicates, f is a unary function, and *add* is a binary function.

(i) $equals(add(2,2),5)$
(ii) $\forall x.p(x,x)$
(iii) $\exists y.\forall x.p(x,y)$
(iv) $\forall x.(p(x,f(x)) \wedge \neg p(f(x),x))$

Exercise 3: Semantic Tableaux for FOL

Use the semantic tableaux method to prove the validity of the following formulae.

(a) $(\forall x.\ (p(x) \to q(a))) \wedge (\exists x.\ p(x)) \to q(a)$

(b) $(\forall x.\ p(f(x))) \wedge (\forall y.\ (q(y) \to \neg p(f(y)))) \to \neg q(b)$

(c) $(\forall x, y.\ (p(x,y) \vee p(y,x))) \to \forall z.p(z,z)$

(d) $\forall y.\ \exists x.\ (p(x) \to p(y))$

(e) $\exists x.\ \forall y.\ (p(x) \to p(y))$