J. Hoenicke
A. Nutz

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Tutorials for Decision Procedures
Exercise sheet 4

Exercise 1: FOL-Sudoku
Solve exercise 3 from exercise sheet 2 ("Sudoku Generator") again, but this time using first-order logic and the theory of quantifier-free linear integer arithmetic (`(set-logic QF_LIA)`). Using `QF_LIA` means that you can use addition and constant multiplication, as well as (in-/dis-)equalities (i.e. `=, distinct, <, <=`), and integer constants, in their usual meanings.

- As in sheet 2 write a script that generates SMTLIBv2 scripts for a given sudoku size.
- Try to find a constraint set that is fast to solve.
- What do you observe, in terms of speed, compared to the boolean constraints? (You do not need to benchmark, just state what you expected and what you observed and how those two match.)

Exercise 2: Induction in T_{PA}
Prove the T_{PA}-validity of the following formula using the semantic tableaux.

$$\forall x.\ 0 + x = x$$

Write down each proof step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as T_{PA}-valid. Note, that you may *not* assume commutativity, associativity, etc. Only use the Peano-axioms and the axioms from T_{E}. You need the induction axiom.

Exercise 3: Semantic Argument in $T_{\mathbb{R}}$
Show the $T_{\mathbb{R}}$-validty of the following formula using the semantic argument.

$$\forall x.\ x \cdot x \geq 0$$

Write down every step explicitly. Besides introducing axioms, you are allowed to introduce formulae that you have previously proven as $T_{\mathbb{R}}$-valid. Additionally, you may use the following derived facts without proving them:

$$\forall x.\ 0 \geq x \rightarrow -x \geq 0$$
$$\forall x.\ (-x) \cdot (-x) = x \cdot x$$