Exercise 1: Integer Arithmetic
Consider the $T_\mathbb{Z}$-formula $F : \exists x. \forall y. \neg (y + 1 = x)$.

(a) Convert $F$ into an equisatisfiable $T_\mathbb{N}$-formula $G$.

(b) Prove unsatisfiability of $G$ using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

(c) Prove validity of the $T_\mathbb{N}$-formula $\exists x. \forall y. \neg (y + 1 = x)$.

Exercise 2: Quantifier Elimination for $T_\mathbb{Q}$
Apply quantifier elimination to the following $T_\mathbb{Q}$-formulae:

(a) $\exists y. (x = 2y \land y < x)$

(b) $\forall y. (25 < x + 2y \lor x + 2y < 25)$

(c) $\forall x. \exists y. (y > x \land -y < x)$

(d) $\forall x. (x > 0 \iff \exists y. (x > y \land -x < y))$

Exercise 3: Sufficient Set
For $T_\mathbb{Q}$ the algorithm in the lecture examines terms $\frac{s + t}{2}$ for all $s, t \in S$. Suppose we split up $S$ in $S_A, S_B, S_C$ depending on whether the term $t$ comes from an (A) $x < t$, (B) $t < x$, or (C) $x = t$ literal. Based on this distinction, give a smaller set of terms that still is sufficient.