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Tutorials for Decision Procedures
Exercise sheet 5

Exercise 1: Integer Arithmetic
Consider the $T_{\mathbb{Z}}$-formula $F : \exists x. \forall y.\ \neg(y + 1 = x)$.

(a) Convert F into an equisatisfiable $T_{\mathbb{N}}$-formula G.

(b) Prove unsatisfiability of G using the semantic tableaux method. You may assume that associativity and commutativity of addition holds.

(c) Prove validity of the $T_{\mathbb{N}}$-formula $\exists x. \forall y.\ \neg(y + 1 = x)$.

Exercise 2: Quantifier Elimination for $T_{\mathbb{Q}}$
Apply quantifier elimination to the following $T_{\mathbb{Q}}$-formulae:

(a) $\exists y.\ (x = 2y \wedge y < x)$

(b) $\forall y.\ (25 < x + 2y \vee x + 2y < 25)$

(c) $\forall x.\ \exists y.\ (y > x \wedge -y < x)$

(d) $\forall x.\ (x > 0 \iff \exists y.\ (x > y \wedge -x < y))$

Exercise 3: Sufficient Set
For $T_{\mathbb{Q}}$ the algorithm in the lecture examines terms $\frac{s+t}{2}$ for all $s, t \in S$. Suppose we split up S in S_A, S_B, S_C depending on whether the term t comes from an (A) $x < t$, (B) $t < x$, or (C) $x = t$ literal. Based on this distinction, give a smaller set of terms that still is sufficient.