Real-Time Systems

Lecture 02: Timed Behaviour

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Contents & Goals

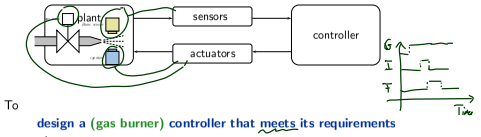
Last Lecture:

- Motivation, Overview

This Lecture:

- **Educational Objectives:**
  - Get acquainted with one (simple but powerful) formal model of timed behaviour.
  - See how first order predicate-logic can be used to state requirements.
- **Content:**
  - Time-dependent State Variables
  - Requirements and System Properities in first order predicate logic
  - Classes of Timed Properties

Recall: Prerequisites



To

design a (gas burner) controller that meets its requirements

we need

- a formal model of behaviour in (quantized) time
- a language to concisely conveniently specify requirements on timed behaviour
- a language to specify behaviour of controller
- a notion of "meet" and a methodology to verify (prove) meeting

Real-Time Behaviour, More Formally...

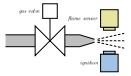
State Variables (or Observables)

- We assume that the real-time systems we consider is characterised by a finite set of **state variables** (or **observables**)

$$obs_1, \ldots, obs_n$$

each equipped with a **domain** $\mathcal{D}(obs_i)$, $1 \leq i \leq n$.

- **Example:** gas burner



- "gas valve open/closed" G, $\mathcal{D}(G) = \{0,1\}$, 0 iff valve closed
- "flame yes/no" F, $\mathcal{D}(F) = \{0,1\}$, 0 iff no flame
- "ignition going on yes/no" I, $\mathcal{D}(I) = \{0,1\}$, 0 iff no ignition
- "heating need yes/no" H, $\mathcal{D}(H) = \{0,1\}$, 0 iff no need

System Evolution over Time

- **One** possible evolution (or **behaviour**) of the considered system over time is represented as a function

$$\pi : \text{Time} \to \mathcal{D}(obs_1) \times \cdots \times \mathcal{D}(obs_n).$$

- If (and only if) observable obs_i has value $d_i \in \mathcal{D}(obs_i)$ at time $t \in \text{Time}$, $1 \leq i \leq n$, we set

$$\pi(t) = (d_1, \ldots, d_n).$$

- For convenience, we use

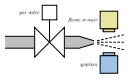
$$obs_i : \text{Time} \to \mathcal{D}(obs_i)$$

to denote the projection of π onto the i-th component.

What's the time?

- There are two main choices for the time domain Time:
  - **discrete time**: Time $= \mathbb{N}_0$, the set of natural numbers.
  - **continuous or dense time**: Time $= \mathbb{R}_0^+$, the set of non-negative real numbers.
- Throughout the lecture we shall use the **continuous** time model and consider **discrete** time as a special case.
  Because
  - plant models usually live in **continuous** time,
  - we avoid too early introduction introduction of hardware considerations,
- Interesting view: continous-time is a well-suited **abstraction** from the discrete-time realms induced by clock-cycles etc.

Example: Gas Burner



One possible evolution of considered system over time is represented as function $\pi : \text{Time} \to \mathcal{D}(obs_1) \times \cdots \times \mathcal{D}(obs_n)$.
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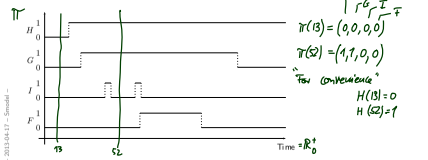


Example: Gas Burner

Levels of Detail

- Note:
  Depending on the **choice of observables** we can describe a real-time system at various levels of detail.

  For instance,
  - if the gas valve has different positions, use
    $G : \text{Time} \to \{0, 1, 2, 3\}$
    (But: $\mathcal{D}(G)$ is never continuous in the lecture, otherwise we had a hybrid system.)
  - if the thermostat and the controller are connected via a bus and exchange messages, use
    $B : \text{Time} \to Msg^*$
    to model the receive buffer as a finite sequence of messages from Msg.
  - etc.

System Properties

Predicate Logic

$$\varphi ::= obs(t) = d \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \implies \varphi_2 \mid \varphi_1 \iff \varphi_2$$
$$\mid \forall t \in \text{Time} \bullet \varphi \mid \forall t \in [t_1 + c_1, t_2 + c_2] \bullet \varphi$$

obs an observable, $d \in \mathcal{D}(obs)$, $t \in$ Var logical variable, $c_1, c_2 \in \mathbb{R}_0^+$ constants.

Example:

$$\forall t \in \text{Time} \bullet \neg G(t) \implies \neg F(t)$$

$$\forall t \in \text{Time} \bullet H(t) \implies \exists t' \in [t, t + 100] \bullet I(t')$$

Predicate Logic

$$\varphi ::= obs(\blacktriangledown) = d \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \implies \varphi_2 \mid \varphi_1 \iff \varphi_2$$
$$\mid \forall \blacktriangledown \in \mathsf{Time} \bullet \varphi \mid \forall t \in [\blacksquare + c_1, \blacksquare + c_2] \bullet \varphi$$

obs an observable, $d \in \mathcal{D}(obs)$, $t \in \mathsf{Var}$ logical variable, $c_1, c_2 \in \mathbb{R}_0^+$ constants.

We assume the **standard semantics** interpreted over system evolutions

$$obs_i : \mathsf{Time} \to \mathcal{D}(obs), 1 \leq i \leq n.$$

That is, given a particular system evolution π and a formula φ, we can tell whether π satisfies φ under a given valuation β, denoted by $\pi, \beta \models \varphi$.

Recall: Predicate Logic, Standard Semantics

Evolution of system over time:	$\pi : \mathsf{Time} \to \mathcal{D}(obs_1) \times \cdots \times \mathcal{D}(obs_n)$.
Iff obs_i has value $d_i \in \mathcal{D}(obs_i)$ at $t \in \mathsf{Time}$, set:	$\pi(t) = (d_1, \ldots, d_n)$.
For convenience: use	$obs_i : \mathsf{Time} \to \mathcal{D}(obs_i)$.

$$\varphi ::= obs(t) = d \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \implies \varphi_2 \mid \varphi_1 \iff \varphi_2$$
$$\mid \forall t \in \mathsf{Time} \bullet \varphi \mid \forall t \in [t_1 + c_1, t_2 + c_2] \bullet \varphi$$

- Let $\beta : \mathsf{Var} \to \mathsf{Time}$ be a **valuation** of the logical variables.
- $\pi, \beta \models obs_i(t) = d$ iff ...
- $\pi, \beta \models \neg\varphi$ iff ...
- $\pi, \beta \models \varphi_1 \vee \varphi_2$ iff ...
- ...
- $\pi, \beta \models \forall t \in \mathsf{Time} \bullet \varphi$ iff ...
- $\pi, \beta \models \forall t \in [t_1 + c_1, t_2 + c_2] \bullet \varphi$ iff ...

Predicate Logic

Note: we can view a closed predicate logic formula φ as a **concise description** of

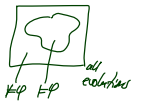
$$\{\pi : \mathsf{Time} \to \mathcal{D}(obs_1) \times \cdots \times \mathcal{D}(obs_n) \mid \pi, \emptyset \models \varphi\}$$

the set of all system evolutions satisfying φ.

For example,

$$\forall t \in \mathsf{Time} \bullet \neg(I(t) \wedge \neg G(t))$$

describes all evolutions where there is no ignition with closed gas valve.



Requirements and System Properties

- So we can use first-order predicate logic to formally specify requirements.

  A **requirement** 'Req' is a set of system behaviours with the pragmatics that, whatever the behaviours of the final **implementation** are, they shall lie within this set.

  For instance,

  $$\mathsf{Req} :\iff \forall t \in \mathsf{Time} \bullet \neg(I(t) \wedge \neg G(t))$$

  says: "an implementation is fine as long as it doesn't ignite without gas in any of its evolutions".

- We can also use first-order predicate logic to formally describe properties of the **implementation** or **design decisions**.

  For instance,

  $$\mathsf{Des} :\iff \forall t \in \mathsf{Time} \bullet I(t) \implies \forall t' \in [t-1, t+1] \bullet G(t'))$$

  says that our controller opens the gas valve at least 1 time unit before ignition and keeps it open.

Correctness

- Let 'Req' be a **requirement**,
- 'Des' be a **design**, and
- 'Impl' be an **implementation**.

Recall: each is a set of evolutions, i.e. a subset of $\left(\mathsf{Time} \to \times_{i=1}^{n} \mathcal{D}(obs_i)\right)$, described in any form.

We say

- 'Des' is a **correct design** (wrt. 'Req') if and only if

  $$\mathsf{Des} \subseteq \mathsf{Req}.$$

- 'Impl' is a **correct implementation** (wrt. 'Des' (or 'Req')) if and only if

  $$\mathsf{Impl} \subseteq \mathsf{Des} \quad (\text{or } \mathsf{Impl} \subseteq \mathsf{Req})$$

If 'Req' and 'Des' are described by formulae of first-oder predicate logic, proving the design correct amounts to proving that 'Des $\implies$ Req' is valid.

Classes of Timed Properties

Safety Properties

- A **safety property** states that
  something bad must never happen [Lamport].
- Example: train inside level crossing with gates open.
- More general, assume observable $C : \text{Time} \to \{0, 1\}$ where $C(t) = 1$ represents a critical system state at time t.

  Then
  $$\forall\, t \in \text{Time} \bullet \neg C(t)$$
  is a safety property.
- In general, a safety property is characterised as a property that can be **falsified** in bounded time.
- But safety is not everything...

Liveness Properties

- The simplest form of a **liveness property** states that
  something good eventually does happen.
- Example: gates open for road traffic.
- More general, assume observable $G : \text{Time} \to \{0, 1\}$ where $G(t) = 1$ represents a good system state at time t.

  Then
  $$\exists t \in \text{Time} \bullet G(t)$$
  is a liveness property.
- Note: not falsified in finite time.
- With real-time, liveness is too weak...

Bounded Response Properties

- A **bounded response property** states that
  the desired reaction on an input occurs in time interval $[b, e]$.
- Example: from request to secure level crossing to gates closed.
- More general, re-consider good thing $G : \text{Time} \to \{0, 1\}$ and request $R : \text{Time} \to \{0, 1\}$.

  Then
  $$\forall\, t_1 \in \text{Time} \bullet (R(t_1) \implies \exists t_2 \in [t_1 + 10, t_1 + 15] \bullet G(t_2))$$
  is a bounded liveness property.
- This property can again be falsified in finite time.
- With gas burners, this is still not everything...

Duration Properties

- A **duration property** states that
  for observation interval $[b, e]$ characterised by a condition $A(b, e)$
  the **accumulated time** in which the system is in a certain critical state has an upper bound $u(b, e)$.
- Example: leakage in gas burner.
- More general, re-consider critical thing $C : \text{Time} \to \{0, 1\}$.

  Then
  $$\forall b, e \in \text{Time} \bullet \left(A(b, e) \implies \int_b^e C(t)\, dt \leq u(b, e) \right)$$
  is a duration property.
- This property can again be falsified in finite time.

References

References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.