Real-Time Systems

Lecture 10: Timed Automata

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Contents & Goals

Last Lecture:

- PLC, PLC automata

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - what's notable about TA syntax? What's simple clock constraint?
  - what's a configuration of a TA? When are two in transition relation?
  - what's the difference between guard and invariant? Why have both?
  - what's a computation path? A run? Zeno behaviour?
- **Content:**
  - Timed automata syntax
  - TA operational semantics

Content

Introduction

- **First-order Logic**
- **Duration Calculus** (DC)
- Semantical Correctness Proofs with DC
- DC Decidability
- DC Implementables
- **PLC-Automata**

obs : Time $\rightarrow \mathscr{D}(obs)$

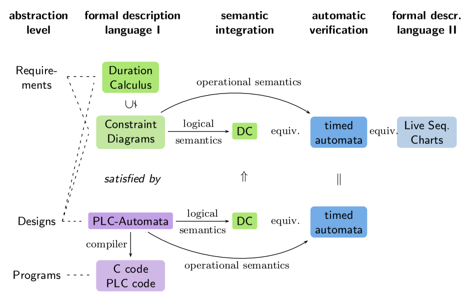
- **Timed Automata** (TA), Uppaal
- Networks of Timed Automata
- Region/Zone-Abstraction
- Extended Timed Automata
- Undecidability Results

$\langle obs_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_0} \langle obs_1, \nu_1 \rangle, t_0 \xrightarrow{\lambda_1} \langle obs_2, \nu_2 \rangle, t_2$

- **Automatic Verification**...
- ...whether TA satisfies DC formula, observer-based

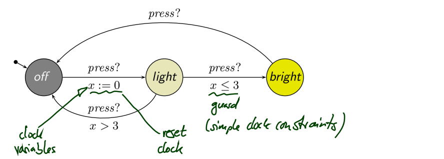
Recap

Recall: Tying It All Together

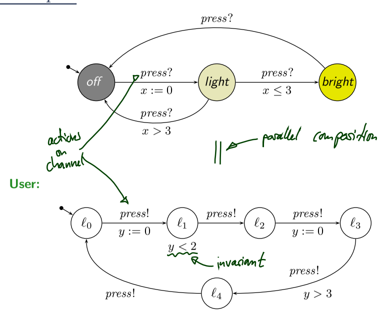


Example: Off/Light/Bright

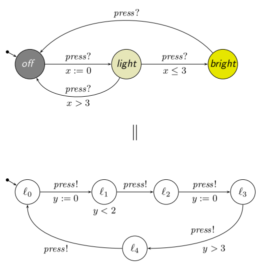
Example



Example



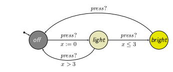
Example Cont'd



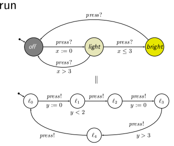
Problems:

- Deadlock freedom [Behrmann et al., 2004]
- Location Reachability ("Is this user able to reach 'bright'?")
- Constraint Reachability ("Can the controller's clock go past 5?")

Plan



- **Pure TA** syntax
  - channels, actions
  - (simple) clock constraints
  - Def. TA
- **Pure TA** operational semantics
  - clock valuation, time shift, modification
  - operational semantics
  - discussion
- Transition sequence, computation path, run
- **Network of TA**
  - parallel composition (syntactical)
  - restriction
  - network of TA semantics
- **Uppaal Demo**
- Region abstraction; zones
- **Extended TA**; Logic of Uppaal



Pure TA Syntax

Channel Names and Actions

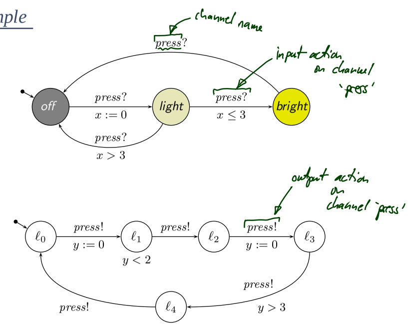
To define timed automata formally, we need the following sets of symbols:

- A set $(a, b \in)$ Chan of **channel names** or **channels**.
- For each channel $a \in$ Chan, two **visible actions**:
  $a?$ and $a!$ denote **input** and **output** on the **channel** ($a?, a! \notin$ Chan).
- $\tau \notin$ Chan represents an **internal action**, not visible from outside.
- $(\alpha, \beta \in)$ $Act := \{a? \mid a \in \text{Chan}\} \cup \{a! \mid a \in \text{Chan}\} \cup \{\tau\}$ is the set of **actions**.
- An **alphabet** B is a set of **channels**, i.e. $B \subseteq$ Chan.
- For each alphabet B, we define the corresponding **action set**

$$B_{?!} := \{a? \mid a \in B\} \cup \{a! \mid a \in B\} \cup \{\tau\}.$$

- Note: $\text{Chan}_{?!} = Act$.

Example



Simple Clock Constraints

- Let $(x, y \in)\ X$ be a set of **clock variables** (or **clocks**).
- The set $(\varphi \in)\ \Phi(X)$ of (**simple**) **clock constraints** (over X) is defined by the following grammar:

$$\varphi ::= x \sim c \mid x - y \sim c \mid \varphi_1 \wedge \varphi_2 \mid true$$

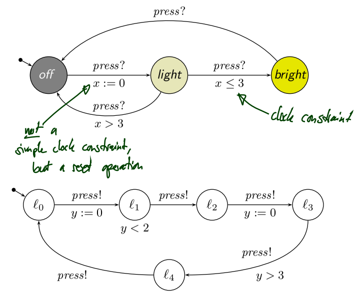
if $X \neq \emptyset$, this can be an abbreviation for $x \geq 0$, $x \in X$.

where
  - $x, y \in X$,
  - $c \in \mathbb{Q}_0^+$, and
  - $\sim \in \{<, >, \leq, \geq\}$.

we may use $x = c$ $(x = y)$ as an abbreviation $x \leq c \wedge x \geq c$ $(x - y \geq 0 \wedge x - y \leq 0)$

- Clock constraints of the form $x - y \sim c$ are called **difference constraints**.

Example



Timed Automaton

Definition 4.3. [*Timed automaton*]
A (pure) **timed automaton** $\mathcal{A}$ is a structure

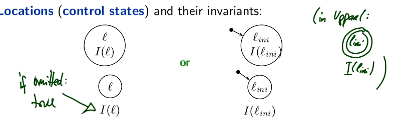
$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$

where
- $(\ell \in)\ L$ is a finite set of **locations** (or **control states**),
- $B \subseteq$ Chan,
- X is a finite set of clocks,
- $I : L \to \Phi(X)$ assigns to each location a clock constraint, its **invariant**,
- $E \subseteq L \times B_{?!} \times \Phi(X) \times 2^X \times L$ a finite set of **directed edges**. Edges $(\ell, \alpha, \varphi, Y, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a set Y of clocks that will be **reset**.
- ℓ_{ini} is the **initial location**.

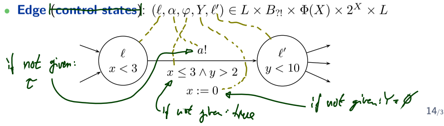
Graphical Representation of Timed Automata

$$\boxed{\mathcal{A} = (L, B, X, I, E, \ell_{ini})}$$

- **Locations** (**control states**) and their invariants:



- **Edge** ~~(control states)~~: $(\ell, \alpha, \varphi, Y, \ell') \in L \times B_{?!} \times \Phi(X) \times 2^X \times L$



Pure TA Operational Semantics

Clock Valuations

- Let X be a set of clocks. A **valuation** ν of **clocks** in X is a mapping

$$\nu : X \to \text{Time} \quad (\mathbb{R}_0^+)$$

assigning each clock $x \in X$ the **current time** $\nu(x)$.

- Let φ be a clock constraint.
The **satisfaction** relation between clock valuations ν and clock constraints φ, denoted by $\nu \models \varphi$, is defined inductively:
  - $\nu \models x \sim c$ iff $\nu(x) \sim c$ — $\nu(x) \hat{\sim} \hat{c}$
  - $\nu \models x - y \sim c$ iff $\nu(x) - \nu(y) \sim c$ — $\nu(x) \hat{-} \nu(y) \hat{\sim} \hat{c}$
  - $\nu \models \varphi_1 \wedge \varphi_2$ iff $\nu \models \varphi_1$ and $\nu \models \varphi_2$ — $\nu \models \varphi_1$ and $\nu \models \varphi_2$

- Two clock constraints φ_1 and φ_2 are called (**logically**) **equivalent** if and only if for all clock valuations ν, we have

$$\nu \models \varphi_1 \text{ if and only if } \nu \models \varphi_2.$$

In that case we write $\models \varphi_1 \iff \varphi_2$.

Operations on Clock Valuations

Let ν be a valuation of clocks in X and $t \in$ Time.

- **Time Shift**

  We write $\underbrace{\nu + t}_{\text{function name}}$ to denote the clock valuation (for X) with

  $$\underbrace{(\nu + t)}(x) = \nu(x) + t.$$

  for all $x \in X$,

- **Modification**

  Let $Y \subseteq X$ be a set of clocks.

  We write $\underbrace{\nu[Y := t]}_{\text{function name}}$ to denote the clock valuation with

  $$\underbrace{(\nu[Y := t])}(x) = \begin{cases} t & \text{, if } x \in Y \\ \nu(x) & \text{, otherwise} \end{cases}$$

  Special case **reset**: $t = 0$.

Operational Semantics of TA

e.g. $\to$ (ℓ_0) $x > 0$

Definition 4.4. The **operational semantics** of a timed automaton

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$

is defined by the (labelled) transition system

$$\mathcal{T}(\mathcal{A}) = (Conf(\mathcal{A}), \text{Time} \cup B_{?!}, \{\xrightarrow{\lambda} \mid \lambda \in \text{Time} \cup B_{?!}\}, C_{ini})$$

the set of labels

a set (!) of transition relations

- can be larger than 1? NO
- always size 1? IN GENERAL NO
- can this be empty? YES, if $\nu_0 \not\models I(\ell_{ini})$

where
- $Conf(\mathcal{A}) = \{\langle \ell, \nu \rangle \mid \ell \in L, \nu : X \to \text{Time}, \ \nu \models I(\ell)\}$
- Time $\cup B_{?!}$ are the transition labels,
- there are **delay transition relations**

  $$\langle \ell, \nu \rangle \xrightarrow{\lambda} \langle \ell', \nu' \rangle, \lambda \in \text{Time}$$

  and **action transition relations**

  $$\langle \ell, \nu \rangle \xrightarrow{\lambda} \langle \ell', \nu' \rangle, \lambda \in B_{?!}. \qquad (\to \text{later slides})$$
- $C_{ini} = \underbrace{\{\langle \ell_{ini}, \nu_0 \rangle\}} \cap Conf(\mathcal{A})$ with $\nu_0(x) = 0$ for all $x \in X$ is the set of **initial configurations**.

Operational Semantics of TA Cont'd

$$\mathcal{A} = (L, B, X, I, E, \ell_{ini})$$

$$\mathcal{T}(\mathcal{A}) = (Conf(\mathcal{A}), \text{Time} \cup B_{?!}, \{\underbrace{\xrightarrow{\lambda}}_{\subseteq Conf(\mathcal{A}) \times Conf(\mathcal{A})} \mid \lambda \in \text{Time} \cup B_{?!}\}, C_{ini})$$

- **Time** or **delay transition**:

  $$\langle \ell, \nu \rangle \xrightarrow{t} \langle \ell, \underbrace{\nu + t} \rangle$$

  if and only if $\forall\, t' \in [0, t] : \nu + t' \models I(\ell)$.

  "Some **time** $t \in$ Time **elapses** respecting invariants, location unchanged."

- **Action** or **discrete transition**:

  $$\langle \ell, \nu \rangle \xrightarrow{\alpha} \langle \ell', \nu' \rangle$$

  if and only if there is $(\ell, \alpha, \varphi, Y, \ell') \in E$ such that

  $$\nu \models \varphi, \quad \nu' = \nu[Y := 0], \quad \text{and } \nu' \models I(\ell').$$

  "An action occurs, location may change, some clocks may be reset, **time does not advance**."

Transition Sequences, Reachability

- A **transition sequence** of $\mathcal{A}$ is any finite or infinite sequence of the form

  $$\langle \ell_0, \nu_0 \rangle \xrightarrow{\lambda_1} \langle \ell_1, \nu_1 \rangle \xrightarrow{\lambda_2} \langle \ell_2, \nu_2 \rangle \xrightarrow{\lambda_3} \dots$$

  with
  - $\langle \ell_0, \nu_0 \rangle \in C_{ini}$,
  - for all $i \in \mathbb{N}$, there is $\xrightarrow{\lambda_{i+1}}$ in $\mathcal{T}(\mathcal{A})$ with $\langle \ell_i, \nu_i \rangle \xrightarrow{\lambda_{i+1}} \langle \ell_{i+1}, \nu_{i+1} \rangle$

Transition Sequences, Reachability

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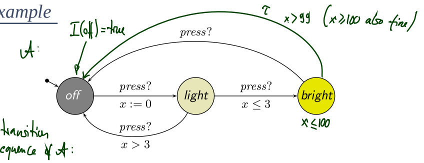
- A **configuration** $\langle \ell, \nu \rangle$ is called **reachable** (in $\mathcal{A}$) if and only if there is a transition sequence of the form

  $$\langle \ell_0, \nu_0 \rangle \xrightarrow{\lambda_1} \langle \ell_1, \nu_1 \rangle \xrightarrow{\lambda_2} \langle \ell_2, \nu_2 \rangle \xrightarrow{\lambda_3} \dots \xrightarrow{\lambda_n} \langle \ell_n, \nu_n \rangle = \langle \ell, \nu \rangle$$

- A **location** ℓ is called **reachable** if and only if **any** configuration $\langle \ell, \nu \rangle$ is reachable, i.e. there exists a valuation ν such that $\langle \ell, \nu \rangle$ is reachable.

Example

$\mathcal{A}$:



a transition sequence of $\mathcal{A}$:

$$\langle \underbrace{off}_{\ell_{ini}}, \underbrace{x = 0}_{\nu_0} \rangle \xrightarrow{2.5} \langle off, x = 2.5 \rangle \xrightarrow{1.7} \langle off, x = 4.2 \rangle \xrightarrow{10} \langle off, x = 14.2 \rangle \xrightarrow{0} \langle off, x = 14.2 \rangle$$

$$\xrightarrow{press?} \langle light, x = 0 \rangle \xrightarrow{2.1} \langle light, x = 2.1 \rangle$$

$$\xrightarrow{press?} \langle bright, x = 2.1 \rangle \xrightarrow{10} \langle bright, x = 12.1 \rangle$$

$$\xrightarrow{press?} \langle off, x = 12.1 \rangle$$

$$\xrightarrow{press?} \langle light, x = 0 \rangle \xrightarrow{0} \langle light, x = 0 \rangle$$

NO

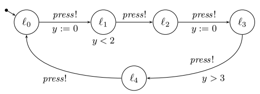
NOT A CONFIGURATION

$$\xrightarrow{press?} \langle bright, x = 0 \rangle \xrightarrow{127} \langle bright, x = 127 \rangle$$

$$\xrightarrow{100} \langle b, x = 100 \rangle \xrightarrow{\tau} \langle off, x = 100 \rangle$$

Discussion: Set of Configurations

Recall the user model for our light controller:



- **"Good" configurations**:

$$\langle \ell_1, y = 0\rangle, \langle \ell_1, y = 1.9\rangle, \quad \langle \ell_2, y = 1000\rangle,$$

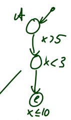
$$\langle \ell_2, y = 0.5\rangle, \quad \langle \ell_3, y = 27\rangle$$

- **"Bad" configurations**: (actually not configs.)

$$\langle \ell_1, y = 2.0\rangle, \langle \ell_1, y = 2.5\rangle$$

Two Approaches to Exclude "Bad" Configurations

- **The approach taken for TA:**
  - Rule out **bad** configurations in the step from $\mathcal{A}$ to $\mathcal{T}(\mathcal{A})$. "Bad" configurations are not even configurations!
  - **Recall Definition 4.4**:
    - $Conf(\mathcal{A}) = \{\langle \ell, \nu\rangle \mid \ell \in L, \nu : X \to \text{Time}, \nu \models I(\ell)\}$
    - $C_{ini} = \{\langle \ell_{ini}, \nu_0\rangle\} \cap Conf(\mathcal{A})$
  - **Note**: Being in $Conf(\mathcal{A})$ doesn't mean to be **reachable**.



$\langle \ell, x=7\rangle \in Conf(\mathcal{A})$, but not reachable in $\mathcal{A}$

- **The approach not taken for TA:**
  - consider every $\langle \ell, \nu\rangle$ to be a configuration, i.e. have

$$Conf(\mathcal{A}) = \{\langle \ell, \nu\rangle \mid \ell \in L, \nu : X \to \text{Time} ~~\nu \models I(\ell)~~\}$$

  - "bad" configurations not in transition relation with others, i.e. have, e.g.,

$$\langle \ell, \nu\rangle \xrightarrow{t} \langle \ell, \nu + t\rangle$$

if and only if $\forall\, t' \in [0, t] : \nu + t' \models I(\ell)$ **and** $\nu + t' \models I(\ell')$.

Computation Path, Run

Computation Paths

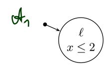
- $\langle \ell, \nu\rangle, t$ is called **time-stamped configuration**, $t \in \text{Time}$
- **time-stamped delay transition**: $\langle \ell, \nu\rangle, t \xrightarrow{t'} \langle \ell, \nu + t'\rangle, t + t'$ iff $t' \in \text{Time}$ and $\langle \ell, \nu\rangle \xrightarrow{t'} \langle \ell, \nu + t'\rangle$.
- **time-stamped action transition**: $\langle \ell, \nu\rangle, t \xrightarrow{\alpha} \langle \ell', \nu'\rangle, t$ iff $\alpha \in B_{?!}$ and $\langle \ell, \nu\rangle \xrightarrow{\alpha} \langle \ell', \nu'\rangle$.
- A sequence of time-stamped configurations

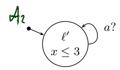
$$\xi = \langle \ell_0, \nu_0\rangle, t_0 \xrightarrow{\lambda_1} \langle \ell_1, \nu_1\rangle, t_1 \xrightarrow{\lambda_2} \langle \ell_2, \nu_2\rangle, t_2 \xrightarrow{\lambda_3} \dots$$

is called **computation path** (or path) of $\mathcal{A}$ **starting in** $\langle \ell_0, \nu_0\rangle, t_0$ if and only if it is either infinite or maximally finite.

- A **computation path** (or path) is a computation path starting at $\langle \ell_0, \nu_0\rangle, 0$ where $\langle \ell_0, \nu_0\rangle \in C_{ini}$.

Timelocks and Zeno Behaviour



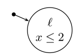


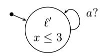
$\langle \ell, x=0\rangle, 0 \xrightarrow{2.0} \langle \ell, x=2.0\rangle, 2 \xrightarrow{0}$

$\langle \ell', x=0\rangle \xrightarrow{3.0} \langle \ell, x=3.0\rangle \xrightarrow{a?} \xrightarrow{0}$

$\langle \ell, x=0\rangle \xrightarrow{1.0} \langle \ell, x=1.0\rangle \xrightarrow{0.5} \langle \ell, x=1.5\rangle \xrightarrow{0.25} \langle \ell, 1.75\rangle \xrightarrow{0.125} \langle \ell, 1.875\rangle \xrightarrow{\frac{1}{16}} \dots$

Timelocks and Zeno Behaviour





- **Timelock**:

$$\langle \ell, x = 0\rangle, 0 \xrightarrow{2} \langle \ell, x = 2\rangle, 2$$

$$\langle \ell', x = 0\rangle, 0 \xrightarrow{3} \langle \ell', x = 3\rangle, 3 \xrightarrow{a?} \langle \ell', x = 3\rangle, 3 \xrightarrow{a?} \dots$$

- **Zeno** behaviour:

$$\langle \ell, x = 0\rangle, 0 \xrightarrow{1/2} \langle \ell, x = 1/2\rangle, \frac{1}{2} \xrightarrow{1/4} \langle \ell, x = 3/4\rangle, \frac{3}{4} \dots$$

$$\xrightarrow{1/2^n} \langle \ell, x = (2^n - 1)/2^n\rangle, \frac{2^n - 1}{2^n} \dots$$

Definition 4.9. An infinite sequence

$$t_0, t_1, t_2, \ldots$$

of values $t_i \in \mathsf{Time}$ for $i \in \mathbb{N}_0$ is called **real-time sequence** if and only if it has the following properties:

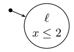
- **Monotonicity**:
$$\forall\, i \in \mathbb{N}_0 : t_i \leq t_{i+1}$$
- **Non-Zeno behaviour** (or **unboundedness** or **progress**):
$$\forall\, t \in \mathsf{Time}\ \exists\, i \in \mathbb{N}_0 : t < t_i$$

Definition 4.10. A **run** of $\mathcal{A}$ **starting** in the time-stamped configuration $\langle \ell_0, \nu_0 \rangle, t_0$ is an infinite computation path of $\mathcal{A}$

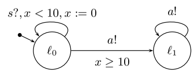
$$\xi = \langle \ell_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \ell_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \ell_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \ldots$$

where $(t_i)_{i \in \mathbb{N}_0}$ is a real-time sequence.
If $\langle \ell_0, \nu_0 \rangle \in C_{ini}$ and $t_0 = 0$, then we call ξ a **run** of $\mathcal{A}$.

Example:



does not have a path



– $\langle \ell_0, 0 \rangle, 0 \xrightarrow{s?} \langle \ell_0, 0 \rangle, 0 \xrightarrow{1.0} \langle \ell_0, 1 \rangle, 1 \xrightarrow{s?} \langle \ell_0, 0 \rangle, 1 \xrightarrow{1.0} \langle \ell_0, 1 \rangle, 2 \xrightarrow{s?} \langle \ell_0, 0 \rangle, 2 \cdots$ RUN

– $\langle \ell_0, 0 \rangle, 0 \xrightarrow{10} \langle \ell_0, 10 \rangle, 10 \xrightarrow{a!} \langle \ell_1, 10 \rangle, 10 \xrightarrow{a!} \langle \ell_1, 10 \rangle, 10 \xrightarrow{a!} \langle \ell_1, 10 \rangle, 10 \xrightarrow{a!} \cdots$ NOT A RUN

$\langle \ell_1, 10 \rangle, 10 \xrightarrow{1.0} \langle \ell_1, 11 \rangle, 11 \xrightarrow{1.0} \langle \ell_1, 12 \rangle, 12 \xrightarrow{1.0} \cdots$ RUN

References

References

[Behrmann et al., 2004] Behrmann, G., David, A., and Larsen, K. G. (2004). A tutorial on uppaal 2004-11-17. Technical report, Aalborg University, Denmark.

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.