Real-Time Systems

Lecture 11: Networks of Timed Automata

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Contents & Goals

Last Lecture:

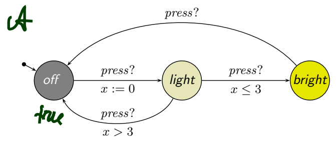
- Timed automata syntax
- TA operational semantics

This Lecture:

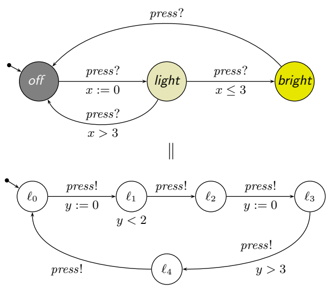
- **Educational Objectives:** Capabilities for following tasks/questions.
  - what's the (syntactical) parallel composition of TA?
- **Content:**
  - parallel composition of TA
  - Uppaal demo

Recall: Plan

- Pure TA syntax
  - channels, actions
  - (simple) clock constraints
  - Def. TA
- Pure TA operational semantics
  - clock valuation, time shift, modification
  - operational semantics
  - discussion
- transition sequence, computation path, run
- network of TA
  - parallel composition (syntactical)
  - restriction
  - network of TA semantics
- Uppaal Demo, part 1
- Extended timed automata



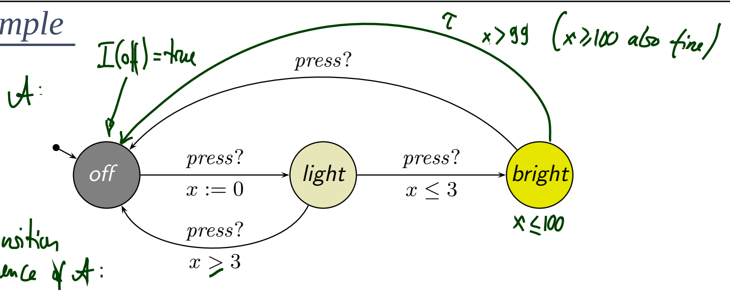
$$\mathcal{T}(\mathcal{A}) = (\underbrace{Conf(\mathcal{A})}_{\langle \ell, \nu \rangle}, B_{?!} \cup Time, \{ \xrightarrow{\lambda} \mid \lambda \in B_{?!} \cup Time \}, C_{ini} \}$$



Network of TA

Recall: Pure Timed Automaton

Example



a transition sequence of A:

$$\langle \underbrace{\textsf{off}}_{\ell_{ini}}, \underbrace{x = 0}_{\nu_0} \rangle \xrightarrow{2.5} \langle \textsf{off}, x = 2.5 \rangle \xrightarrow{1.7} \langle \textsf{off}, x = 4.2 \rangle \xrightarrow{10} \langle \textsf{off}, x = 14.2 \rangle \xrightarrow{0} \langle \textsf{off}, x = 14.2 \rangle$$

$$\xrightarrow{press?} \langle \textsf{light}, x = 0 \rangle \xrightarrow{2.1} \langle \textsf{light}, x = 2.1 \rangle$$

$$\xrightarrow{press?} \langle \textsf{bright}, x = 2.1 \rangle \xrightarrow{10} \langle \textsf{bright}, x = 12.1 \rangle$$

$$\xrightarrow{press?} \langle \textsf{off}, x = 12.1 \rangle$$

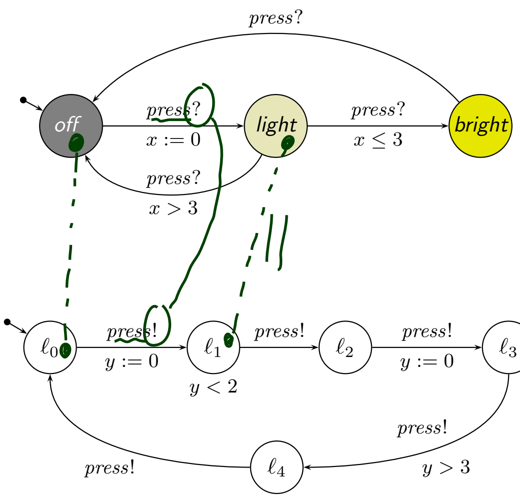
NO NOT A CONFIGURATION

$$\xrightarrow{press?} \langle \textsf{light}, x = 0 \rangle \xrightarrow{0} \langle \textsf{light}, x = 0 \rangle$$

$$\xrightarrow{press?} \langle \textsf{bright}, x = 0 \rangle ~~\xrightarrow{127} \langle \textsf{bright}, x = 127 \rangle~~$$

$$\xrightarrow{100} \langle b, x = 100 \rangle \xrightarrow{\tau} \langle \textsf{off}, x = 100 \rangle$$

Recall: Light Controller and User



Parallel Composition

Definition 4.12.
The **parallel composition** $\mathcal{A}_1 \parallel \mathcal{A}_2$ of two timed automata

$$\mathcal{A}_i = (L_i, B_i, X_i, I_i, E_i, \ell_{ini,i}), \quad i = 1, 2,$$

with **disjoint** sets of clocks X_1 and X_2 yields the timed automaton

$$\mathcal{A} = (L_1 \times L_2, B_1 \cup B_2, X_1 \cup X_2, I, E, (\ell_{ini,1}, \ell_{ini,2}))$$

where

- $I(\ell_1, \ell_2) := I_1(\ell_1) \wedge I_2(\ell_2)$, and
- E consists of **handshake** and **asynchronous communication**. (→ **next slide**)

Helper: Action Complementation

- The **complementation function**

  $$\overline{\cdot} : Act \to Act$$

  is defined pointwise as

  - $\overline{a!} = a?$
  - $\overline{a?} = a!$
  - $\overline{\tau} = \tau$

- **Note**: $\overline{\overline{\alpha}} = \alpha$ for all $\alpha \in Act$.

Parallel Composition: Handshake and Asynchrony

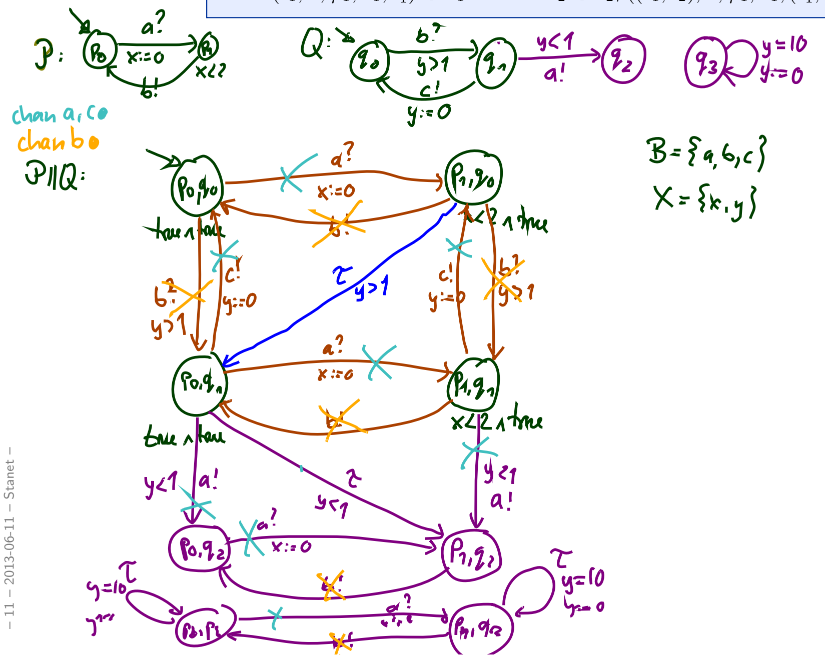
$\mathcal{A}_1 \parallel \mathcal{A}_2 = (L_1 \times L_2, B_1 \cup B_2, X_1 \cup X_2, I, E, (\ell_{ini,1}, \ell_{ini,2}))$ with

- **Handshake**:
  If there is $a \in B_1 \cup B_2$ such that
  $$(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1, \text{ and } (\ell_2, \bar{\alpha}, \varphi_2, Y_2, \ell_2') \in E_2,$$
  and $\{a!, a?\} = \{\alpha, \bar{\alpha}\}$, then
  $$((\ell_1, \ell_2), \tau, \varphi_1 \wedge \varphi_2, Y_1 \cup Y_2, (\ell_1', \ell_2')) \in E.$$
- **Asynchrony**:
  If $(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1$ then for all $\ell_2 \in L_2$,
  $$((\ell_1, \ell_2), \alpha, \varphi_1, Y_1, (\ell_1', \ell_2)) \in E.$$
  If $(\ell_2, \alpha, \varphi_2, Y_2, \ell_2') \in E_2$ then for all $\ell_1 \in L_1$,
  $$((\ell_1, \ell_2), \alpha, \varphi_2, Y_2, (\ell_1, \ell_2')) \in E.$$

Example

$\mathcal{L} \parallel \mathcal{U} = (L_1 \times L_2, B_1 \cup B_2, X_1 \cup X_2, I, E, (\ell_{ini,1}, \ell_{ini,2}))$

- If $a \in B_1 \cup B_2$ s.t. $(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1$, $(\ell_2, \bar{\alpha}, \varphi_2, Y_2, \ell_2') \in E_2$, $\{a!, a?\} = \{\alpha, \bar{\alpha}\}$, then $((\ell_1, \ell_2), \tau, \varphi_1 \wedge \varphi_2, Y_1 \cup Y_2, (\ell_1', \ell_2')) \in E$
- If $(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1$ then f.a. $\ell_2 \in L_2$, $((\ell_1, \ell_2), \alpha, \varphi_1, Y_1, (\ell_1', \ell_2)) \in E$, and conv.



$B = \{a, b, c\}$

$X = \{x, y\}$

Restriction

Definition 4.13.
A **local channel** b is introduced by the **restriction operator** which, for a timed automaton $\mathcal{A} = (L, B, X, I, E, \ell_{ini})$ yields

$$\text{chan}\, b \bullet \mathcal{A} := (L, B \setminus \{b\}, X, I, E', \ell_{ini})$$

where
- $(\ell, \alpha, \varphi, Y, \ell') \in E'$
  if and only if $(\ell, \alpha, \varphi, Y, \ell') \in E$ and $\alpha \notin \{b!, b?\}$.

- **Abbreviation**:

$$\text{chan}\, b_1 \dots b_m \bullet \mathcal{A} := \text{chan}\, b_1 \bullet \dots \text{chan}\, b_m \bullet \mathcal{A}$$

Networks of Timed Automata

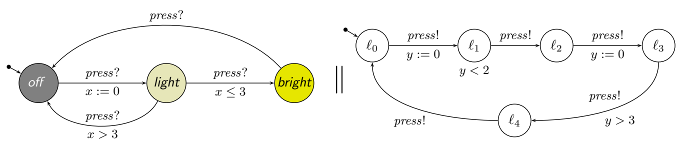
- A timed automaton $\mathcal{N}$ is called **network of timed automata** if and only if it is obtained as

$$\text{chan}\, b_1 \dots b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n)$$

Closed Networks

- A network
$$\mathcal{N} = \mathsf{chan}\, b_1 \dots b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n)$$
is called **closed** if and only if
$$\{b_1, \dots, b_m\} = \bigcup_{i=1}^{n} B_i.$$
- Then, by Lemma 4.16 (later), **local transitions** don't occur (since $B = \emptyset$). Transitions are thus either internal actions τ or delay transitions.

Example:

chan $press \bullet$ 

is closed.

$$\begin{tikzcd}
\mathrm{chan}\, b_1,\dots,b_k \bullet \mathcal{A}_1 \| \dots \| \mathcal{A}_n \arrow[dd, squiggly, "\text{Lemma } 4.16"'] \arrow[dr, squiggly, "\mathrm{chan}\, b \bullet \mathcal{B}", "\mathcal{A}_1 \| \mathcal{A}_2 \quad \text{Def. } 4.12"'] & & \\
 & \mathcal{A} \arrow[d, squiggly, "\text{Def. } 4.4"] & \\
\mathcal{J}(\mathcal{N}) = (Conf(\mathcal{N}), \dots, \{\xrightarrow{\lambda} \mid \dots\}, C_{ini}) & = \mathcal{J}(\mathcal{A}) = (Conf(\mathcal{A}), \dots, \{\xrightarrow{\lambda} \mid \dots\}, C_{ini}) &
\end{tikzcd}$$

Operational Semantics of Networks

Lemma 4.16. Let $\mathcal{A}_i = (L_i, B_i, X_i, I_i, E_i, \ell_{ini,i})$ with $i = 1, \dots, n$ be a set of timed automata with disjoint clocks. Then the operational semantics of the network

$$\mathsf{chan}\, b_1 \dots b_m \bullet (\mathcal{A}_1 \parallel \dots \parallel \mathcal{A}_n)$$

yields the labelled transition system

$$(\mathit{Conf}(\mathcal{N}), \mathsf{Time} \cup B_{?!}, \{\xrightarrow{\lambda} \mid \lambda \in \mathsf{Time} \cup B_{?!}\}, C_{ini})$$

with

- $X = \bigcup_{i=1}^{n} X_i$,
- $B = \bigcup_{i=1}^{n} B_i \setminus \{b_1, \dots, b_m\}$,
- $\mathit{Conf}(\mathcal{N}) = \{\langle \vec{\ell}, \nu \rangle \mid$
  $\vec{\ell} \in L_1 \times \dots \times L_n \wedge \nu : X \to \mathsf{Time} \wedge \nu \models \bigwedge_{k=1}^{n} I_k(\ell_k)\}$,
- $C_{ini} = \{\langle (\ell_{ini,1}, \dots, \ell_{ini,n}), \nu_{ini} \rangle\} \cap \mathit{Conf}(\mathcal{N})$
  where $\nu_{ini}(x) = 0$ for all $x \in X$,
- and three types of transition relations ($\rightarrow$ next slides).

Operational Semantics of Networks: Local Transitions

For each $\lambda \in \mathsf{Time} \cup B_{!?}$ the transition relation $\xrightarrow{\lambda} \subseteq \mathit{Conf}(\mathcal{N}) \times \mathit{Conf}(\mathcal{N})$ has one of the following three types:

(i) **Local transition**:

$$\langle \vec{\ell}, \nu \rangle \xrightarrow{\alpha} \langle \vec{\ell}', \nu' \rangle$$

if there is $i \in \{1, \dots, n\}$ such that

- $(\ell_i, \alpha, \varphi, Y, \ell_i') \in E_i$, $\alpha \in B_{!?}$, (*i*-th automaton has corresp. edge)
- $\nu \models \varphi$, (guard is satisfied)
- $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i']$, (only *i*-th location changes)
- $\nu' = \nu[Y := 0]$, and ($\mathcal{A}_i$'s clocks are reset)
- $\nu' \models I_i(\ell_i')$. (destination invariant holds)

tuple modification

Operational Semantics of Networks: Synchronisation

(ii) **Synchronisation transition**:

$$\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$$

if there are $i, j \in \{1, \dots, n\}$, $i \neq j$, and $b \in B_i \cap B_j$, such that

- $(\ell_i, b!, \varphi_i, Y_i, \ell_i') \in E_i$ and $(\ell_j, b?, \varphi_j, Y_j, \ell_j') \in E_j$,
- $\nu \models \varphi_i \wedge \varphi_j$,
- $\vec{\ell}' = \vec{\ell}[\ell_i := \ell_i'][\ell_j := \ell_j']$,
- $\nu' = \nu[Y_i \cup Y_j := 0]$, and
- $\nu' \models I_i(\ell_i') \wedge I_j(\ell_j')$.

Operational Semantics of Networks: Delay

(iii) **Delay transition**:

$$\langle \vec{\ell}, \nu \rangle \xrightarrow{t} \langle \vec{\ell}, \nu + t \rangle$$

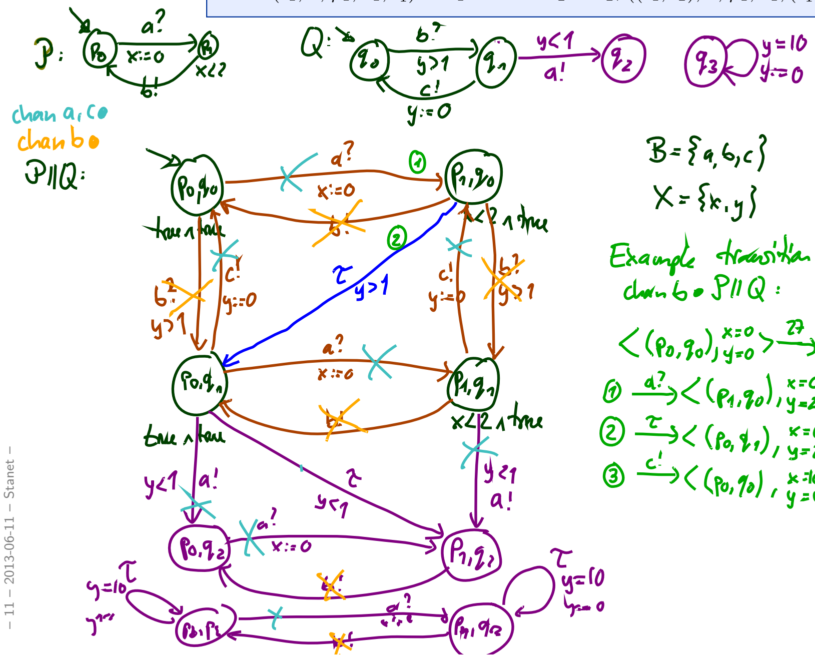
if for all $t' \in [0, t]$,

- $\nu + t' \models \bigwedge_{k=1}^{n} I_k(\ell_k)$.

Example

$\mathcal{L} \parallel \mathcal{U} = (L_1 \times L_2, B_1 \cup B_2, X_1 \cup X_2, I, E, (\ell_{ini,1}, \ell_{ini,2}))$

- If $a \in B_1 \cup B_2$ s.t. $(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1$, $(\ell_2, \bar{\alpha}, \varphi_2, Y_2, \ell_2') \in E_2$, $\{a!, a?\} = \{\alpha, \bar{\alpha}\}$, then $((\ell_1, \ell_2), \tau, \varphi_1 \wedge \varphi_2, Y_1 \cup Y_2, (\ell_1', \ell_2')) \in E$
- If $(\ell_1, \alpha, \varphi_1, Y_1, \ell_1') \in E_1$ then f.a. $\ell_2 \in L_2$, $((\ell_1, \ell_2), \alpha, \varphi_1, Y_1, (\ell_1', \ell_2)) \in E$, and conv.



$B = \{a, b, c\}$

$X = \{x, y\}$

Example transition sequence of chan b• P∥Q:

$\langle (p_0, q_0), \substack{x=0 \\ y=0} \rangle \xrightarrow{27} \langle (p_0, q_0), \substack{x=27 \\ y=27} \rangle$

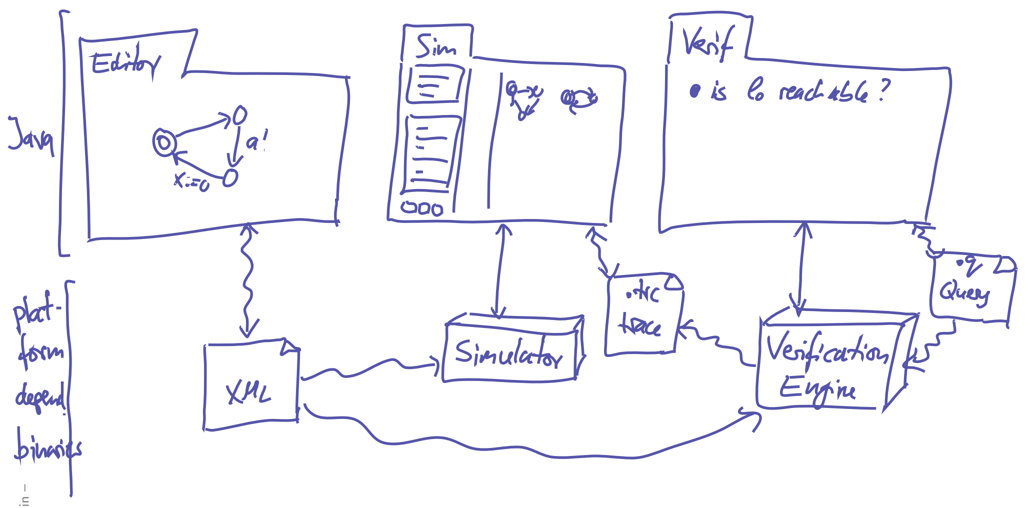
① $\xrightarrow{a?} \langle (p_1, q_0), \substack{x=0 \\ y=27} \rangle$

② $\xrightarrow{\tau} \langle (p_0, q_1), \substack{x=0 \\ y=27} \rangle \xrightarrow{10} \langle (p_0, q_1), \substack{x=10 \\ y=37} \rangle$

③ $\xrightarrow{c!} \langle (p_0, q_0), \substack{x=10 \\ y=0} \rangle$ ...

Uppaal [Larsen et al., 1997, Behrmann et al., 2004] Demo, Vol. 1

Uppaal Architecture



References

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