Real-Time Systems

Lecture 15: The Universality Problem for TBA

2013-06-26

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Contents & Goals

Last Lecture:

- Extended Timed Automata

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What's a TBA and what's the difference to (extended) TA?
  - What's undecidable for timed (Büchi) automata?
  - What's the idea of the proof?

- **Content:**
  - Uppaal Query Language
  - Timed Büchi Automata and timed regular languages [Alur and Dill, 1994].
  - The Universality Problem is undecidable for TBA [Alur and Dill, 1994]
  - Why this is unfortunate.
  - Timed regular languages are not everything.

The Logic of Uppaal

The Uppaal Fragment of Timed Computation Tree Logic

Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ over data variables V.

- **basic formula**:

$$atom ::= \mathcal{A}_i.\ell \mid \varphi$$

  where $\ell \in L_i$ is a location and φ a constraint over X_i and V.

- **configuration formulae**:

$$term ::= atom \mid \neg term \mid term_1 \wedge term_2$$

- **existential path formulae**: ("**exists finally**", "**exists globally**")

$$e\text{-}formula ::= \exists\Diamond\, term \mid \exists\Box\, term$$

- **universal path formulae**: ("**always finally**", "**always globally**", "**leads to**")

$$a\text{-}formula ::= \forall\Diamond\, term \mid \forall\Box\, term \mid term_1 \longrightarrow term_2$$

- **formulae**:

$$F ::= e\text{-}formula \mid a\text{-}formula$$

Configurations at Time t

0 3.0 3.0 τ 3.0 1.0

- Recall: **computation path** (or path) **starting in** $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$:

$$\xi = \langle \vec{\ell}_0, \nu_0 \rangle, t_0 \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1 \rangle, t_1 \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2 \rangle, t_2 \xrightarrow{\lambda_3} \dots$$

  which is **infinite or maximally finite**.

- Given ξ and $t \in \mathrm{Time}$, we use $\xi(t)$ to denote the set

$$\{\langle \vec{\ell}, \nu \rangle \mid \exists\, i \in \mathbb{N}_0 : t_i \leq t \leq t_{i+1} \wedge \vec{\ell} = \vec{\ell}_i \wedge \nu = \nu_i + t - t_i\}.$$

  of **configurations at time** t.

  - Why is it a set?
  - Can it be empty?

$\xi(0) = \{\langle \vec{\ell}_0, \nu_0 \rangle\}$

$\xi(0.27) = \{\langle \vec{\ell}_0, \nu_0 + 0.27 \rangle\}$

$\xi(3.0) = \{\langle \vec{\ell}_1, \nu_1 \rangle, \langle \vec{\ell}_2, \nu_2 \rangle\}$

Satisfaction of Uppaal-Logic by Configurations

- We define a **satisfaction relation**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models F$$

  between **time stamped configurations**

$$\langle \vec{\ell}_0, \nu_0 \rangle, t_0$$

  of a network $\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ and **formulae** F of the Uppaal logic.

- It is defined inductively as follows: i-th location in $\vec{\ell}_0$

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \mathcal{A}_i.\ell$ iff $\ell_{0,i} = \ell$

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \varphi$ iff $\nu_0 \models \varphi$

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \neg term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models term$

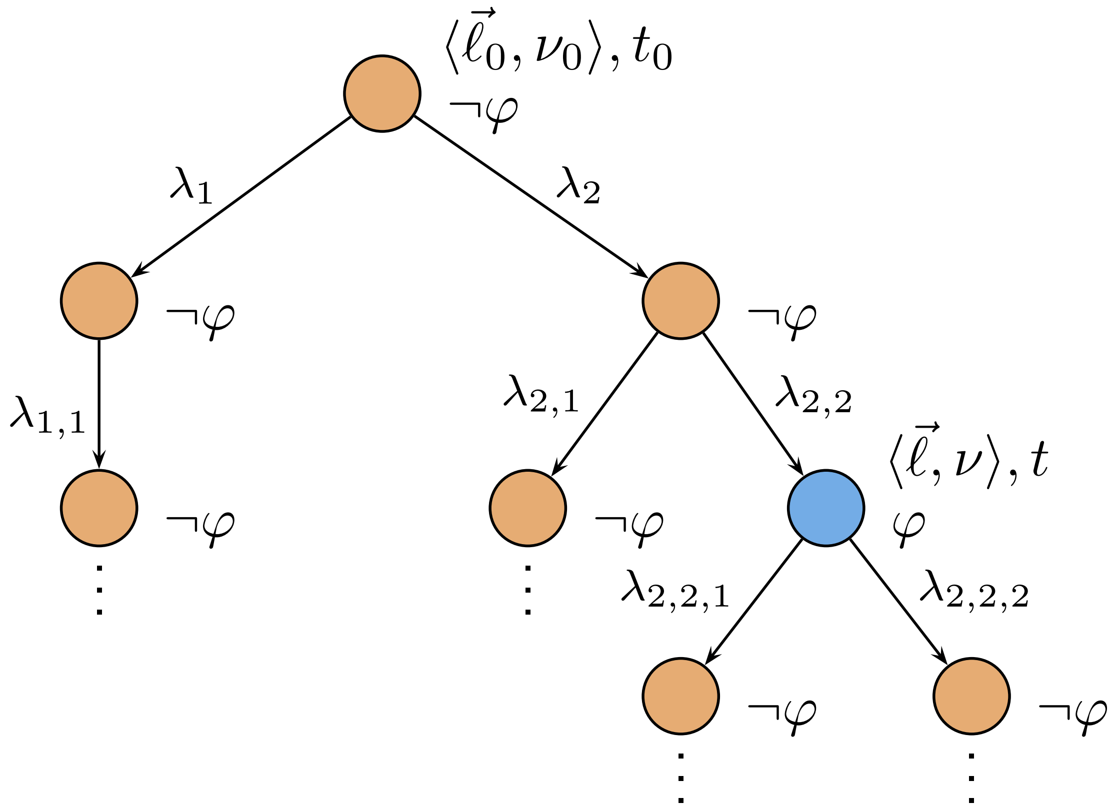
- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \wedge term_2$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_i$, $i = 1, 2$

Satisfaction of Uppaal-Logic by Configurations

Exists finally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Diamond\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\exists t \in \mathsf{Time}, \langle \vec{\ell}, \nu \rangle \in \mathit{Conf}$:
  $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \wedge \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Diamond\, \varphi$



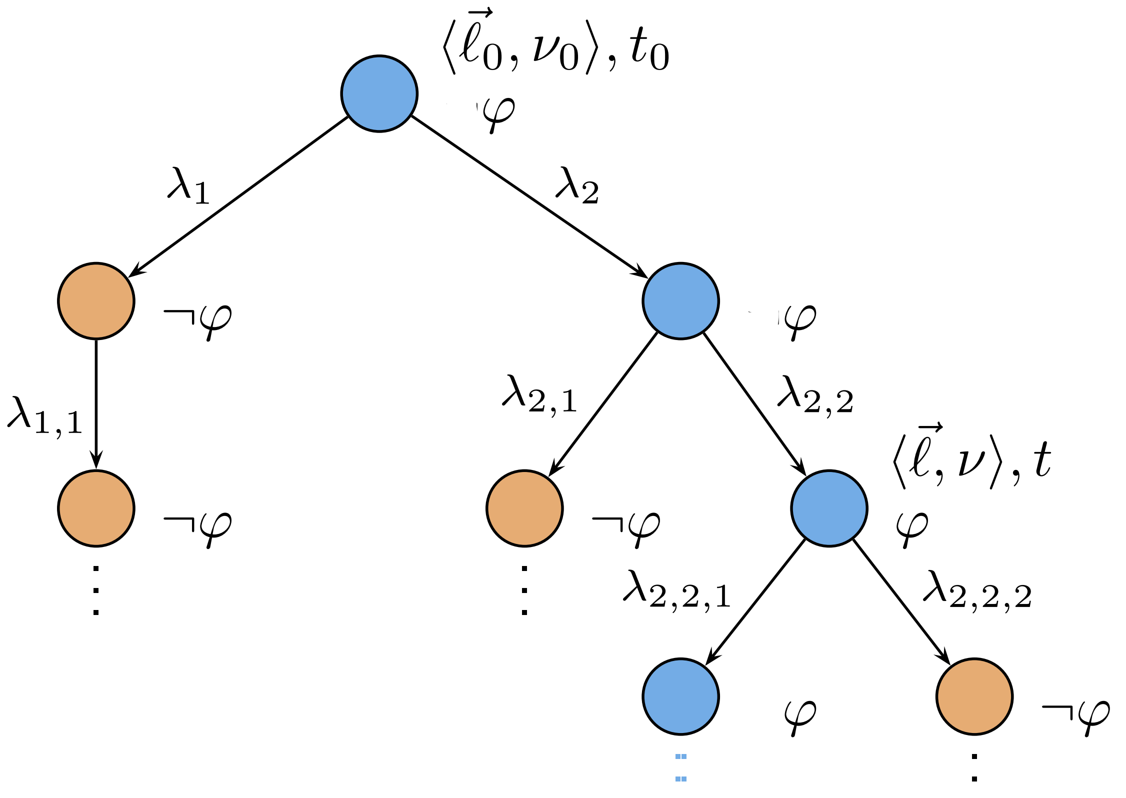
Satisfaction of Uppaal-Logic by Configurations

note: universally quantifying over elements in $\xi(t)$

Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \exists\Box\, term$ iff $\exists$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$ $\forall\, t \in$ Time, $\langle \vec{\ell}, \nu \rangle \in Conf$: $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t) \implies \langle \vec{\ell}, \nu \rangle, t \models term$

Example: $\exists\Box\,\varphi$



Satisfaction of Uppaal-Logic by Configurations

- **Always finally**:

  "∀ path" "∃ t ∈ Time"

  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall \Diamond \, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists \Box \, \neg term$

- **Always globally**:

  - $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models \forall \Box \, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \not\models \exists \Diamond \, \neg term$

  "∀ path" "∀ t ∈ Time"

Satisfaction of Uppaal-Logic by Configurations

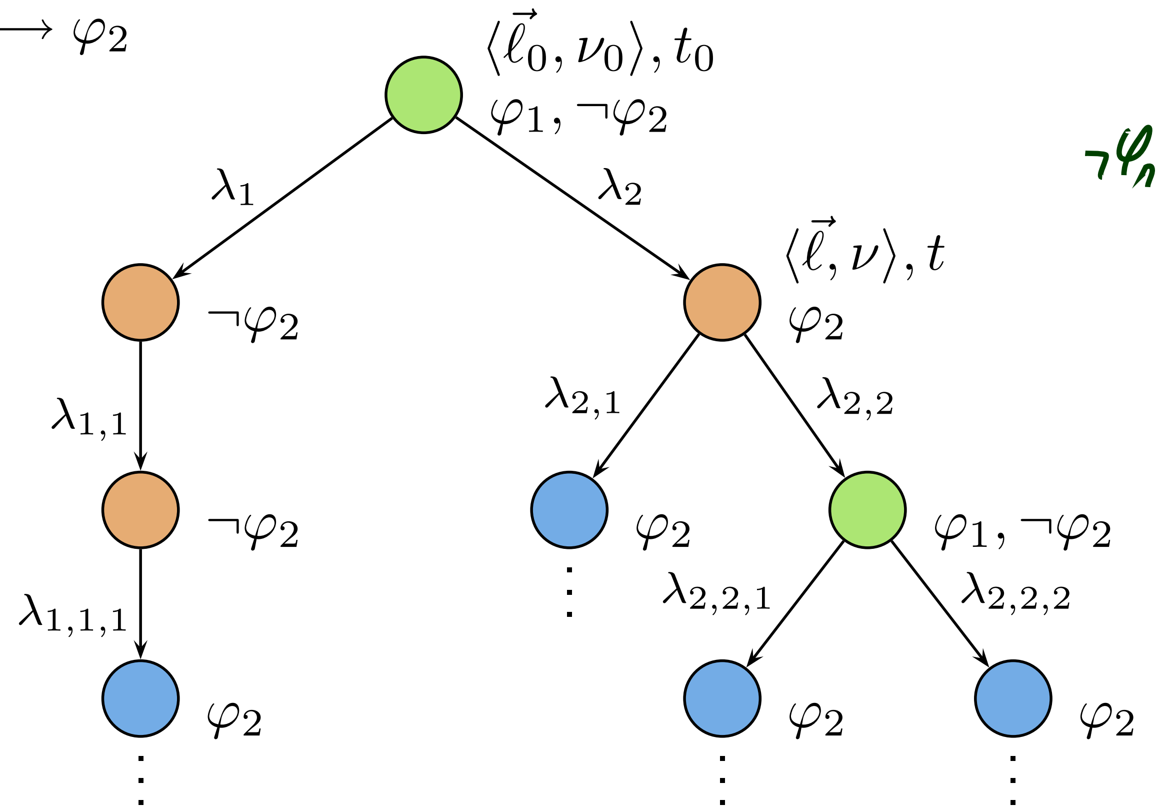
CTL: $AG(term_1 \Rightarrow AF\, term_2)$

Leads to:

- $\langle \vec{\ell}_0, \nu_0 \rangle, t_0 \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle, t_0$
  $\forall\, t \in \mathsf{Time}, \langle \vec{\ell}, \nu \rangle \in \mathit{Conf}$:
  $t_0 \leq t \wedge \langle \vec{\ell}, \nu \rangle \in \xi(t)$
  $\wedge\ \langle \vec{\ell}, \nu \rangle, t \models term_1$
  implies $\langle \vec{\ell}, \nu \rangle, t \models \forall\Diamond\, term_2$

not DC (of course)

Example: $\varphi_1 \longrightarrow \varphi_2$



$\neg\varphi_1$ if not given

Satisfaction of Uppaal-Logic by Networks

- We write

  $$\mathcal{N} \models \textit{e-formula}$$

  if and only if

  $$\textbf{for some } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models \textit{e-formula}, \tag{1}$$

  and

  $$\mathcal{N} \models \textit{a-formula}$$

  if and only if

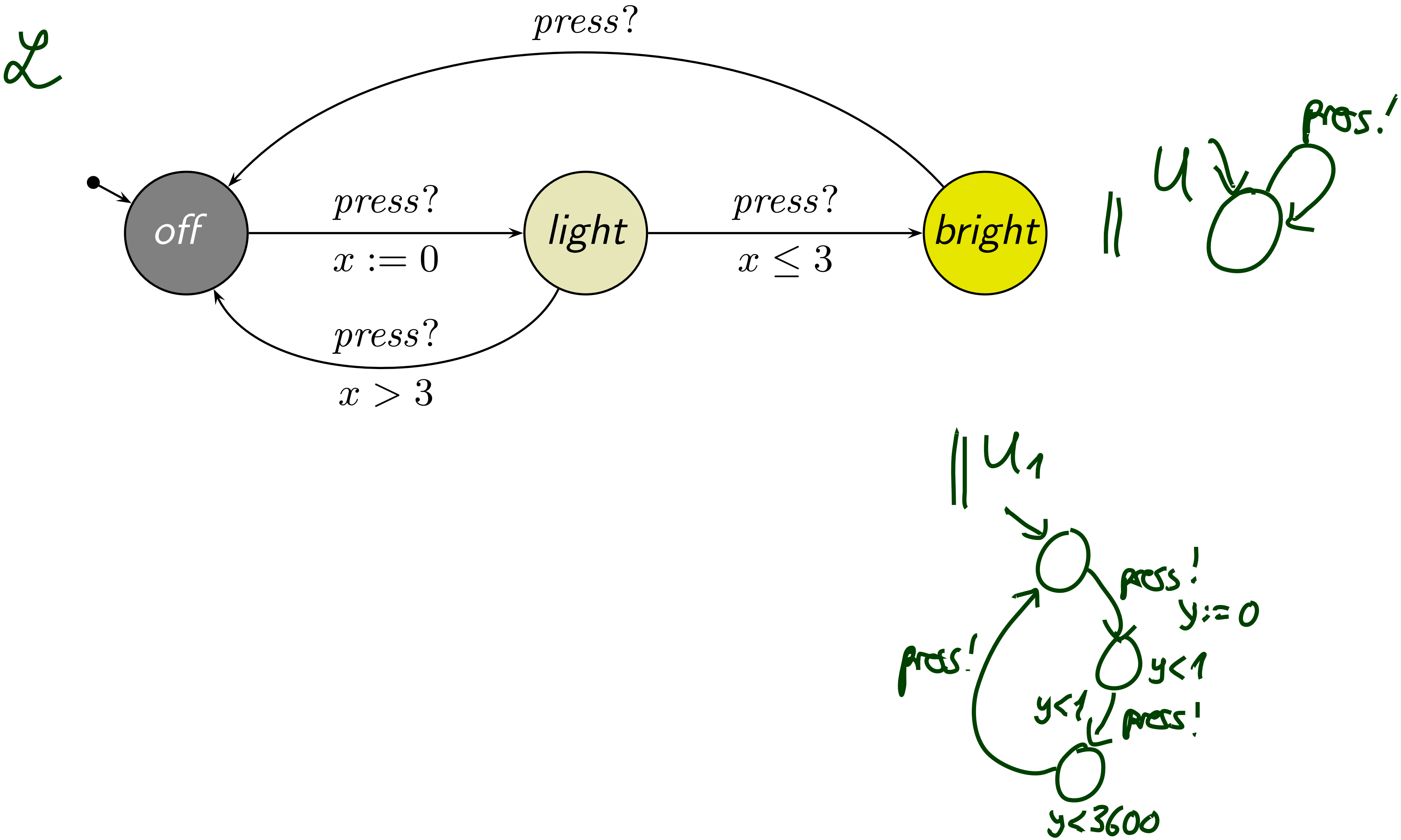
  $$\textbf{for all } \langle \vec{\ell}_0, \nu_0 \rangle \in C_{ini}, \langle \vec{\ell}_0, \nu_0 \rangle, 0 \models \textit{a-formula}, \tag{2}$$

  where C_{ini} are the initial configurations of $\mathcal{T}_e(\mathcal{N})$.

- If $C_{ini} = \emptyset$, (1) is a contradiction and (2) is a tautology.
- If $C_{ini} \neq \emptyset$, then

  $$\mathcal{N} \models F \text{ if and only if } \langle \vec{\ell}_{ini}, \nu_{ini} \rangle, 0 \models F.$$

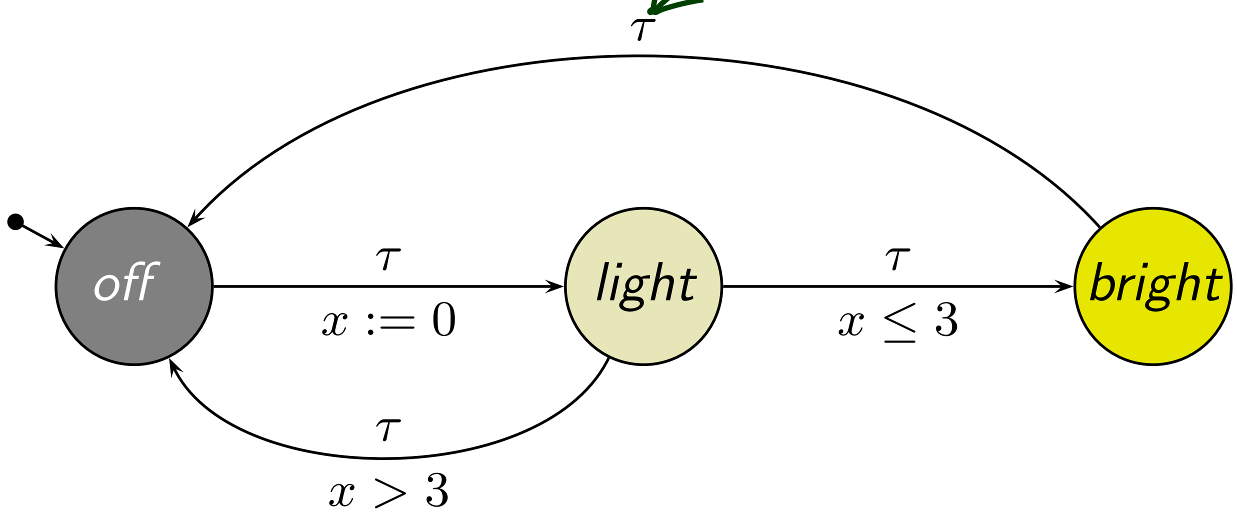
Example



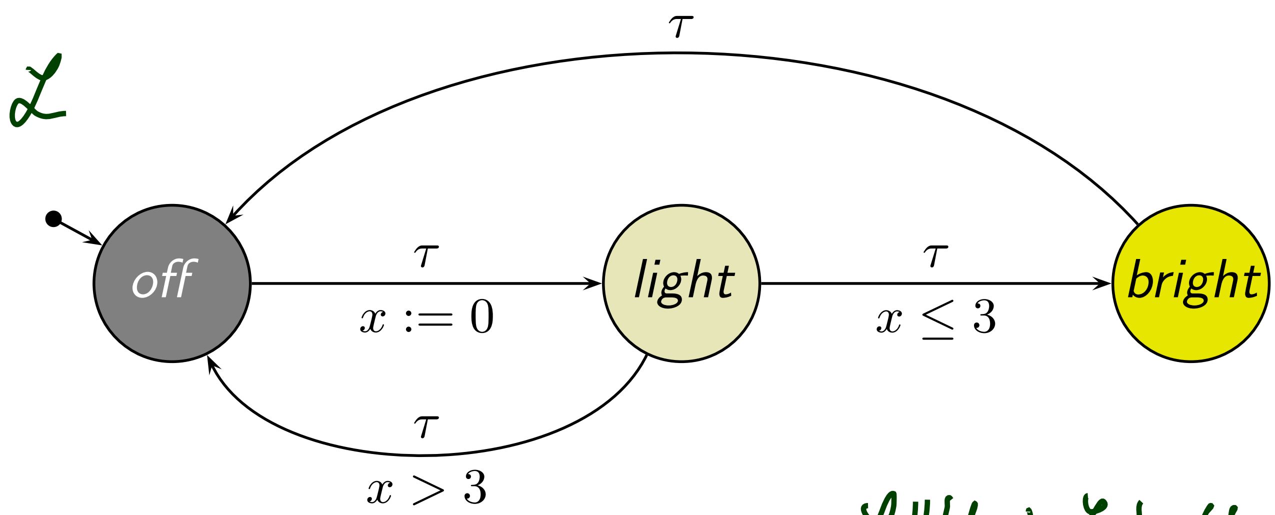
$\mathcal{L}$
press?
off
press?
$x := 0$
light
press?
$x \leq 3$
bright
press?
$x > 3$
$\parallel$
U
press!
$\parallel$ U_1
press!
$y := 0$
$y < 1$
$y < 1$
press!
press!
$y < 3600$

Example

now we have a closed network



Example



$\mathcal{L} \| U_1 \models \mathcal{L}.bright \longrightarrow \mathcal{L}.off$?

because not satisfied in some Zeno paths

X prev. Def

✓ Uppaal (because the tool uses $\stackrel{\alpha}{\Longrightarrow}$ paths, not comp. paths actually)

- $\mathcal{N} \models \exists\Diamond\, \mathcal{L}.bright$? ✓
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.bright$? X (we must be in light before bright)
- $\mathcal{N} \models \exists\Box\, \mathcal{L}.off$? ✓ (*)
- $\mathcal{N} \models \forall\Diamond\, \mathcal{L}.light$? X (because (*))
- $\mathcal{N} \models \forall\Box\, \underbrace{(\mathcal{L}.bright \implies x \geq 3)}_{term}$? X (can have $\mathcal{L}$.bright and $x < 3$)
- $\mathcal{N} \models \mathcal{L}.bright \longrightarrow \mathcal{L}.off$? X

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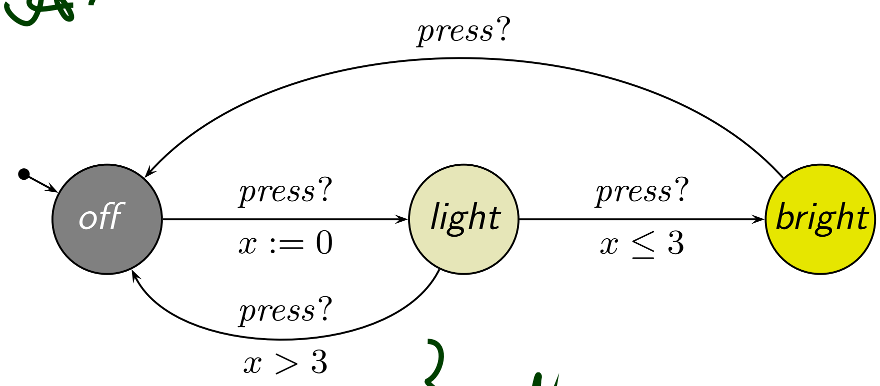
Timed Büchi Automata

[Alur and Dill, 1994]

$\mathcal{A}$:

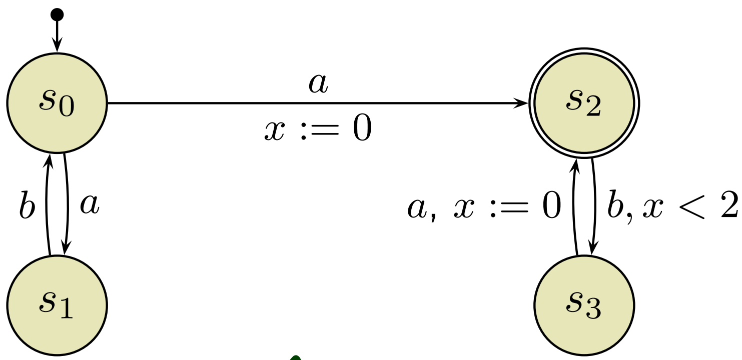


produces

$$\xi = \langle \mathit{off}, 0\rangle, 0 \xrightarrow{1} \langle \mathit{off}, 1\rangle, 1$$
$$\xrightarrow{press?} \langle \mathit{light}, 0\rangle, 1 \xrightarrow{3} \langle \mathit{light}, 3\rangle, 4$$
$$\xrightarrow{press?} \langle \mathit{bright}, 3\rangle, 4 \xrightarrow{..} \dots$$

ξ is a computation path and run of $\mathcal{A}$.

$\mathcal{A}$



accept

New: Given a **timed word**

$$(a, 1), (b, 2), (a, 3), (b, 4), (a, 5), (b, 6), \dots,$$

does $\mathcal{A}$ **accept** it?

New: acceptance criterion is **visiting accepting state infinitely often**.

Definition. A **time sequence** $\tau = \tau_1, \tau_2, \dots$ is an infinite sequence of time values $\tau_i \in \mathbb{R}_0^+$, satisfying the following constraints:

(i) **Monotonicity**:
τ increases **strictly** monotonically, i.e. $\tau_i < \tau_{i+1}$ for all $i \geq 1$.

(ii) **Progress**: For every $t \in \mathbb{R}_0^+$, there is some $i \geq 1$ such that $\tau_i > t$.

set of infinite words over Σ

Definition. A **timed word** over an alphabet Σ is a pair (σ, τ) where

- $\sigma = \sigma_1, \sigma_2, \dots \in \Sigma^\omega$ is an infinite word over Σ, and
- τ is a time sequence.

Definition. A **timed language** over an alphabet Σ is a set of timed words over Σ.

Example: Timed Language

Timed word over alphabet Σ: a pair (σ, τ) where

- $\sigma = \sigma_1, \sigma_2, \ldots$ is an infinite word over Σ, and
- τ is a time sequence (strictly (!) monotonic, non-Zeno).

a could be 'system beeps'
b could be 'system flashes light'

$$L_{crt} = \{((ab)^\omega, \tau) \mid \exists\, i\; \forall\, j \geq i : (\tau_{2j} < \tau_{2j-1} + 2)\}$$

timestamp of a 'b'

timestamp of the 'a' before

1 10.0

a b a b a b ... a b a b a b ...

τ_1 τ_2 τ_3 τ_4 τ_5 τ_6 ... τ_{27} τ_{28} τ_{29} τ_{30} τ_{31} τ_{32}

$\tau_{30} < \tau_{29} + 2$

finite prefix where timestamps don't matter

from here on ...

"b at most (not including) 2 time units after the a before

Timed Büchi Automata

not simple! (negation, but no differences)

Definition. The set $\Phi(X)$ of **clock constraints** over X is defined inductively by

$$\delta ::= x \le c \mid c \le x \mid \neg\delta \mid \delta_1 \wedge \delta_2$$

where $x \in X$ and $c \in \mathbb{Q}$ is a rational constant.

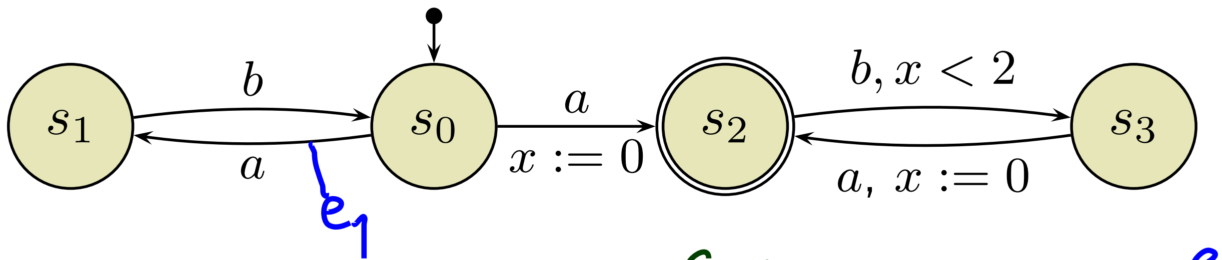
Definition. A **timed Büchi automaton** (TBA) $\mathcal{A}$ is a tuple $(\Sigma, S, S_0, X, E, F)$, where

- Σ is an alphabet,
- S is a finite set of states, $S_0 \subseteq S$ is a set of start states,
- X is a finite set of clocks, and
- $E \subseteq S \times S \times \Sigma \times 2^X \times \Phi(X)$ gives the set of transitions.

  An edge $(s, s', a, \lambda, \delta)$ represents a transition from state s to state s' on input symbol a. The set $\lambda \subseteq X$ gives the clocks to be reset with this transition, and δ is a clock constraint over X.
- $F \subseteq S$ is a set of **accepting states**.

Example: TBA

$$\mathcal{A} = (\Sigma, S, S_0, X, E, F)$$
$$(s, s', a, \lambda, \delta) \in E$$



- $\Sigma = \{a, b\}$
- $S = \{s_1, s_0, s_2, s_3\}$
- $S_0 = \{s_0\}$
- $X = \{x\}$
- $F = \{s_2\}$
- $E = \{ (s_0, s_1, a, \emptyset, true) = e_1,$
  $(s_1, s_0, b, \emptyset, true),$
  $(s_0, s_2, a, \{x\}, true),$
  $(s_2, s_3, b, \emptyset, x<2),$
  $(s_3, s_2, a, \{x\}, true) \}$

References

References

[Alur and Dill, 1994] Alur, R. and Dill, D. L. (1994). A theory of timed automata. *Theoretical Computer Science*, 126(2):183–235.

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.