ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

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1. Exercise Sheet for the Tutorial

Computer Science Theory

Exercise 1: Kleene Closure
Let L be a language such that $\varepsilon \in L$. Show that L^* is the smallest language which contains L and is closed under concatenation. Therefore, you have to show the following:

(a) $L \subseteq L^*$.

(b) $L^* \cdot L^* \subseteq L^*$

(c) For a language L' such that $L \subseteq L'$ and $L' \cdot L' \subseteq L'$, it also holds that $L^* \subseteq L'$.

Exercise 2: Language Operations
Consider the languages $L_1 = \{a, aa, aba\}$ and $L_2 = \{\varepsilon, bc\}$ over the alphabet

$$\Sigma = \{a, b, c, d\}.$$

Determine the following languages explicitly.

(a) $L_1 \cap L_2$

(b) L_1^2

(c) $L_1 \cdot L_2$

(d) $\Sigma^* \cdot L_2 \cdot \Sigma^*$

Exercise 3: Language Properties
Let Σ be an alphabet, and let $L, L' \subseteq \Sigma^*$ be formal languages consisting of finitely many words (i. e., $|L| \in \mathbb{N}$ and $|L'| \in \mathbb{N}$).
Prove or disprove the following statements:

a) $L \cdot L' = L' \cdot L$

b) $|L^n| = |L|^n$

c) $|\Sigma^n| = |\Sigma|^n$

d) $|L \cdot L'| = |L| \cdot |L'|$

e) For all $n \in \mathbb{N}$: $L \cdot L^n = L^n \cdot L$
You may use the fact that concatenation of languages is associative (i. e., for languages $L_1, L_2, L_3 \subseteq \Sigma^*$ it holds that $(L_1 \cdot L_2) \cdot L_3 = L_1 \cdot (L_2 \cdot L_3)$).

Exercise 4: Finite Automata

Construct a deterministic finite automaton over the alphabet $\Sigma = \{0, 1\}$ which accepts the language of all even binary numbers. We allow leading zeros, so for instance 00 is also a binary number. Note that 0 is considered even.

Describe the automaton in two different ways:

- By a finite state diagram.
- By a 5-tuple consisting of an *input alphabet*, a *set of states*, a *transition function*, an *initial state* and a *set of final states*. Use a table to describe the transition function.