ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

Prof. Dr. Andreas Podelski
Matthias Heizmann
Christian Schilling

**3. Exercise Sheet for the Tutorial**
# Computer Science Theory

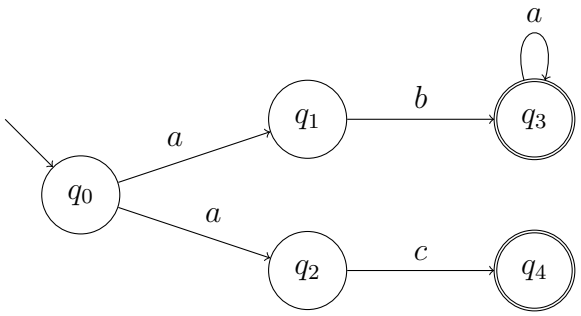
**Exercise 1: Reverse Operator**
The *reverse operator* for strings $w = a_1 a_2 \dots a_n \in \Sigma^*$ is defined as:

$$w^R = a_n a_{n-1} \dots a_1$$

Based upon this definition the *reverse operator* for languages $L \subseteq \Sigma^*$ is defined as:

$$L^R = \{w^R \in \Sigma^* \mid w \in L\}$$

Let $\mathcal{B}$ be the following NFA over the alphabet $\Sigma = \{a, b, c\}$.




(a) Describe the language $\mathcal{L}(\mathcal{B})$ recognized by $\mathcal{B}$ and the corresponding *reverse language* $\mathcal{L}(\mathcal{B})^R$.

(b) Construct an $\varepsilon$-NEA that recognizes the reverse language $\mathcal{L}(\mathcal{B})^R$.

**Exercise 2: Regular Expressions**
Construct regular expressions for the following languages over the alphabet $\Sigma = \{a, b\}$.

(a) $L_1 = \{w \in \Sigma^* \mid$ every $a$ in $w$ is immediately followed by a $b$ $\}$

(b) $L_2 = \{w \in \Sigma^* \mid w$ contains $bb\}$

(c) $L_3 = \{w \in \Sigma^* \mid w$ does not contain $bb\}$

(d) $L_4 = \left\{ w \in \Sigma^* \;\middle|\; \begin{array}{l} w \text{ contains the symbol } a \text{ exactly twice or} \\ w \text{ contains the symbol } b \text{ exactly once} \end{array} \right\}$

(e) Language of all strings ending with an even number of $b$'s:

$$L_5 = \left\{ w \in \Sigma^* \;\middle|\; \begin{array}{l} \text{the length of } w\text{'s longest suffix that contains} \\ \text{only } b\text{'s is even-numbered} \end{array} \right\}$$

**Exercise 3: Pumping Lemma**
Consider the language $L = \{a^k b a^k \mid k \in \mathbb{N}\}$. Use the pumping lemma to show that $L$ is not regular.