3. Exercise Sheet for the Tutorial
Computer Science Theory

Exercise 1: Reverse Operator
The reverse operator for strings \( w = a_1a_2 \ldots a_n \in \Sigma^* \) is defined as:
\[
w^R = a_n a_{n-1} \ldots a_1
\]
Based upon this definition the reverse operator for languages \( L \subseteq \Sigma^* \) is defined as:
\[
L^R = \{ w^R \in \Sigma^* \mid w \in L \}
\]
Let \( \mathcal{B} \) be the following NFA over the alphabet \( \Sigma = \{a, b, c\} \).

(a) Describe the language \( \mathcal{L}(\mathcal{B}) \) recognized by \( \mathcal{B} \) and the corresponding reverse language \( \mathcal{L}(\mathcal{B})^R \).

(b) Construct an \( \varepsilon \)-NEA that recognizes the reverse language \( \mathcal{L}(\mathcal{B})^R \).
Exercise 2: Regular Expressions
Construct regular expressions for the following languages over the alphabet $\Sigma = \{a, b\}$.

(a) $L_1 = \{w \in \Sigma^* \mid \text{every } a \text{ in } w \text{ is immediately followed by a } b \}$

(b) $L_2 = \{w \in \Sigma^* \mid w \text{ contains } bb\}$

(c) $L_3 = \{w \in \Sigma^* \mid w \text{ does not contain } bb\}$

(d) $L_4 = \left\{ w \in \Sigma^* \mid \begin{array}{l} w \text{ contains the symbol } a \text{ exactly twice or } \\
\text{w contains the symbol } b \text{ exactly once} \end{array} \right\}$

(e) Language of all strings ending with an even number of $b$’s:

$L_5 = \left\{ w \in \Sigma^* \mid \begin{array}{l} \text{the length of } w \text{’s longest suffix that contains} \\
\text{only } b \text{’s is even-numbered} \end{array} \right\}$

Exercise 3: Pumping Lemma
Consider the language $L = \{a^kba^k \mid k \in \mathbb{N} \}$. Use the pumping lemma to show that $L$ is not regular.