ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

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7. Exercise Sheet for the Tutorial

Computer Science Theory

Exercise 1: (Un-)Decidable Problems

Let L_1 and L_2 be undecidable languages and let L_3 and L_4 be languages such that $L_3 \subseteq L_1 \subseteq L_4$. Prove or refute the following statements:

(a) $L_1 \cap L_2$ is undecidable.

(b) $L_1 \cup L_2$ is undecidable.

(c) L_3 is undecidable.

(d) L_4 is undecidable.

Exercise 2: Reduction I (Halting on Every Input I)

Prove that the language

$$U = \{bw_\tau \in B^* \mid \tau \text{ halts on every input}\}$$

is undecidable. For this, reduce a problem to U that is known to be undecidable.

Hint: Use H_0.

Exercise 3: (optional*) Halting on Every Input II

*This exercise is not mandatory.

Consider the language

$$U' = \{bw_\tau 00k \in B^* \mid \tau \text{ halts on every input after at most } k \text{ steps}\}$$

Compare U' to U from the preceding exercise. Is U' decidable? Prove your claim.

Exercise 4: (optional*) Reduction II (Writing 23)
*This exercise is not mandatory.

Prove that the set

$$W_{23} = \{bw_\tau 00u \in B^* \mid \tau \text{ applied to } u \text{ eventually writes the binary encoding of 23 on the tape}\}$$

is undecidable. For this, reduce a problem to W_{23} that is known to be undecidable.

Hint: Use H. You may want to change the tape alphabet in a proper way.

Exercise 5: (optional*) Not Halting on Some Input I
*This exercise is not mandatory.

Consider the language

$$\begin{aligned} V = &\{bw_\tau 00k \in B^* \mid \tau \text{ does not halt on some input after at most } k \text{ steps}\} \\ &\cup \{w \in B^* \mid w \text{ is not of the form } bw_\tau 00k \text{ for some Turing machine } \tau \text{ and some } k\}. \end{aligned}$$

Compare V to U' from exercise 3. Is V recursively enumerable? Prove your claim.

Exercise 6: Reduction III (Not Halting on Some Input II)
Let N be the language of all encodings of Turing machines which do not halt on at least one input.

$$N = \{bw_\tau \in B^* \mid \text{there is } u \in B^* \text{ such that } \tau \text{ applied to } u \text{ does not halt}\}$$

Is N decidable? Is N recursively enumerable? Prove your claims.

Hint: Use $\overline{H_0}$ in a reduction.