ALBERT-LUDWIGS-
UNIVERSITÄT FREIBURG

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6. Presence Exercise Sheet for the Lecture
Computer Science Theory
WITH PROPOSALS FOR SOLUTIONS

Exercise 1: Halting Problem for Programs

We consider the alphabet $B = \{0, 1\}$ and programs written in an imperative programming language IMP.

A IMP *program code* is a binary string according to the rules for proper IMP program code (these rules are of no concern for us). It is decidable whether a given binary string is a proper IMP program code or not (by a compiler).

A IMP *program* is the usual interpretation of IMP program code according to the semantic rules of IMP programs (these rules are of no concern for us).

Every IMP program has some IMP program code associated. Also, every IMP program code encodes some IMP program.

To distinguish between programs and their code, we use variable names (e.g., P) for programs and the same variable names with a line above (e.g., $\overline{P}$) for program code.

IMP programs can be executed. They can read an (binary) input word and can produce an (binary) output word. Thus, IMP programs can also read their own encoding as input.

Show that there is no IMP program P_{magic} that executes the following algorithm:

Input: a IMP program code $\overline{P} \in B^*$
Output: 0 or 1, depending on the question: Does P terminate with the input $\overline{P}$?

Hint: Adapt the proof for the special halting problem.

.... Sketch of solution ....

Assume the opposite, so P_{magic} exists. We create a new program P' that first calls P_{magic} and, if the output is 1, goes to an infinite loop. Let `input` be the input word.

$P'(\text{input})$:

```
if  (Pmagic(input) == 0) {
        return 0;
}
else  {  //  Pmagic(input) == 1
        while (true) {
                // do nothing
        }
}
```

We know that there is some IMP program code $\overline{P'}$. Now the following holds:

$$P' \text{ terminates with the input } \overline{P'}$$

$\overset{(1)}{\Leftrightarrow} P_{magic} \text{ terminates with the input } \overline{P'} \text{ with output } 0$

$\overset{(2)}{\Leftrightarrow} P_{magic} \text{ answers the question "Does } P' \text{ terminate with the input } \overline{P'}?" \text{ with "no"}$

$\overset{(3)}{\Leftrightarrow} P' \text{ does not terminate with the input } \overline{P'}$

where

(1) by definition of P'

(2) by definition of P_{magic}

(3) by definition of P_{magic} and the assumption that it works correctly