Exercise Sheet 2

Early submission: Monday, 2014-05-26, 12:00    Regular submission: Tuesday, 2014-05-27, 10:00

Exercise 1  (6/20 Points)

A traffic light for pedestrians is modelled by the observables ‘Light’ of data type \{red, yellow, green\} and ‘Button’ of data type \{press, release\}.
Consider an interpretation $I$ of these observables as given by the timing diagrams in Figure 1.

(i) Calculate the truth value of the following DC formulae:

a) \((true \land \int Light = green = \ell) \land true\)  \hspace{1cm} (1) 

b) \([1] \land \int Light = green = \ell \land [1]\)  \hspace{1cm} (1) 

c) \(\int Button = press \land Light = yellow \leq 1\)  \hspace{1cm} (1) 

in the interval \([0, 4]\). [OD08]

(ii) Is the formula \(\lozenge ([1] \land \int Light = green = \ell \land [1])\)  \hspace{1cm} (1) 
realisable from 0?

(iii) Are the chop points for a) unique?  \hspace{1cm} (1) 

(iv) What is the shortest distance between two chop points with which the formula from a) evaluates to \text{tt}?  \hspace{1cm} (1) 

Figure 1: Interpretation of ‘Light’ and ‘Button’.
Exercise 2 (9/20 Points)

A traffic light for pedestrians is modelled by the observables ‘Light’ of data type \{red, yellow, green\} and ‘Button’ of data type \{press, release\}.

Formalise the following requirements using Duration Calculus:

(i) The button is never pressed when the lights show green. (1)

(ii) The yellow lights is used at least once. (2)

(iii) If the button is pressed, it takes at most 125 time units until green is shown. (1)

(iv) Green phases are at least 25 time units long. (1)

(v) Within 3600 time units, the lights should not show green for more than 1000 time units. (1)

(vi) Pick one task from (i) to (v) and “test” your formula on an evolution, i.e. give at least one positive and one negative example evolution and argue that your formula behaves adequately. (3)

Hint: Explain your understanding of the requirement in natural language as precise as you can. Formalise your understanding. Explain. What is the adequate meaning of “behaves adequately”?

Exercise 3 (5+5/20 Points)

We can abstractly model a rail-road level crossing by the observables

- ‘Track’ with domain \{empty, appr, cross\},
- ‘Gate’ with domain \{open, moving, closed\}.

The track observable represents the presence of the train with two logical regions of the crossing. It is ‘empty’ if there is no train near or on the crossing, it is ‘appr’ if a train is near the crossing, and ‘cross’ if the train is in the area where road and tracks intersect.

The gate can be open, moving (up or down), or closed.

We use the following abbreviations:

\begin{align*}
E & \text{ stands for Track} = \text{empty} \\
A & \text{ stands for Track} = \text{appr} \\
X & \text{ stands for Track} = \text{cross} \\
O & \text{ stands for Gate} = \text{open} \\
C & \text{ stands for Gate} = \text{closed}
\end{align*}

Consider the following DC properties:

\begin{align*}
\Box ([X] \implies [C]) & \quad (\text{‘Safety’}) \\
([E] \land true) \lor [\ ] & \quad (\text{‘Init’}) \\
\Box (([E] ; true ; [X]) \implies \ell \geq \varepsilon) & \quad (\text{‘T-Fast’}) \\
\Box (\lnot [E] \land \ell \geq \varepsilon \implies true ; [C]) & \quad (\text{‘G-Close’})
\end{align*}

(i) Explain informally the meaning of each of these formulae. [OD08] (4/10)

\text{Hint: don’t rust “read them out”; relate them to the behaviour of entities of the real world.}

(ii) We distinguished requirements, design decisions, and assumptions. Which of the formulae serves which purpose here? Briefly explain. (1/10)

(iii) Prove the following implication by using the DC semantics: [OD08] (5 Bonus/10)

\begin{align*}
\text{Init} \land \text{T-Fast} \land \text{G-Close} \implies \text{Safety}
\end{align*}

References