Real-Time Systems

Lecture 04: Duration Calculus II

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Contents & Goals

Last Lecture:

- Started DC Syntax and Semantics: Symbols, State Assertions

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - Read (and at best also write) Duration Calculus terms and formulae.

- **Content:**
  - Duration Calculus Formulae
  - Duration Calculus Abbreviations
  - Satisfiability, Realisability, Validity

Duration Calculus Cont'd

Duration Calculus: Overview

We will introduce three (or five) syntactical "levels":

(i) **Symbols:**

$$f, g, \quad \mathit{true}, \mathit{false}, =, <, >, \leq, \geq, \quad x, y, z, \quad X, Y, Z, \quad d$$

(ii) **State Assertions:**

$$P ::= 0 \mid 1 \mid X = d \mid \neg P_1 \mid P_1 \wedge P_2$$

(iii) **Terms:**

$$\theta ::= x \mid \ell \mid \textstyle\int P \mid f(\theta_1, \dots, \theta_n)$$

(iv) **Formulae:**

$$F ::= p(\theta_1, \dots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall\, x \bullet F_1 \mid F_1 \,;\, F_2$$

(v) **Abbreviations:**

$$\lceil\,\rceil, \quad \lceil P \rceil, \quad \lceil P \rceil^t, \quad \lceil P \rceil^{\leq t}, \quad \Diamond F, \quad \Box F$$

Terms: Remarks

Remark 2.5. The semantics $\mathcal{I}[\![\theta]\!]$ of a term is insensitive against changes of the interpretation $\mathcal{I}$ at individual time points.

Remark 2.6. The semantics $\mathcal{I}[\![\theta]\!](\mathcal{V}, [b, e])$ of a **rigid** term does not depend on the interval $[b, e]$.

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Formulae: Syntax

- The set of **DC formulae** is defined by the following grammar:

$$F ::= p(\theta_1, \ldots, \theta_n) \mid \neg F_1 \mid F_1 \wedge F_2 \mid \forall\, x \bullet F_1 \mid F_1 \,\mathbf{;}\, F_2$$

  where p is a predicate symbol, θ_i a term, x a global variable.

- **chop operator**: '**;**'
- **atomic formula**: $p(\theta_1, \ldots, \theta_n)$
- **rigid formula**: all terms are rigid
- **chop free**: '**;**' doesn't occur
- usual notion of **free** and **bound** (global) variables

- Note: quantification only over (**first-order**) global variables, not over (**second-order**) state variables.

Formulae: Priority Groups

- To avoid parentheses, we define the following five priority groups from highest to lowest priority:

  - $\neg$ **(negation)**
  - **;** **(chop)**
  - $\wedge$, $\vee$ **(and/or)**
  - $\implies$, $\iff$ **(implication/equivalence)**
  - $\exists$, $\forall$ **(quantifiers)**

Examples:

- $\neg F \,;\, F \vee H$

- $\forall x \bullet F \wedge G$

Syntactic Substitution...

...of a term θ for a variable x in a formula F.

- We use

$$F[x := \theta]$$

to denote the formula that results from performing the following steps:

(i) transform F into $\tilde{F}$ by (consistently) renaming bound variables such that no free occurrence of x in $\tilde{F}$ appears within a quantified subformula $\exists\, z \bullet G$ or $\forall\, z \bullet G$ for some z occurring in θ,

(ii) textually replace all free occurrences of x in $\tilde{F}$ by θ.

Syntactic Substitution...

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- We use
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  (ii) textually replace all free occurrences of x in $\tilde{F}$ by θ.

Examples: $F := (x \geq y \implies \exists\, z \bullet z \geq 0 \wedge x = y + z)$, $\quad \theta_1 := \ell$, $\quad \theta_2 := \ell + z$,

- $F[x := \theta_1] = (x \geq y \implies \exists\, z \bullet z \geq 0 \wedge x = y + z)$
- $F[x := \theta_2] = (x \quad \geq y \implies \exists\, z \bullet z \geq 0 \wedge x \quad = y + z)$

Formulae: Semantics

- The **semantics** of a **formula** is a function

$$\mathcal{I}[\![F]\!] : \mathsf{Val} \times \mathsf{Intv} \to \{\mathsf{tt}, \mathsf{ff}\}$$

  i.e. $\mathcal{I}[\![F]\!](\mathcal{V}, [b,e])$ is the truth value of F under interpretation $\mathcal{I}$ and valuation $\mathcal{V}$ in the interval $[b,e]$.

- This value is defined **inductively** on the structure of F:

$$\mathcal{I}[\![p(\theta_1, \ldots, \theta_n)]\!](\mathcal{V}, [b,e]) = \hat{p}(\mathcal{I}[\![\theta_1]\!](\mathcal{V}, [b,e]), \ldots, \mathcal{I}[\![\theta_n]\!](\mathcal{V}, [b,e])),$$

$$\mathcal{I}[\![\neg F_1]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b,e]) = \mathsf{ff},$$

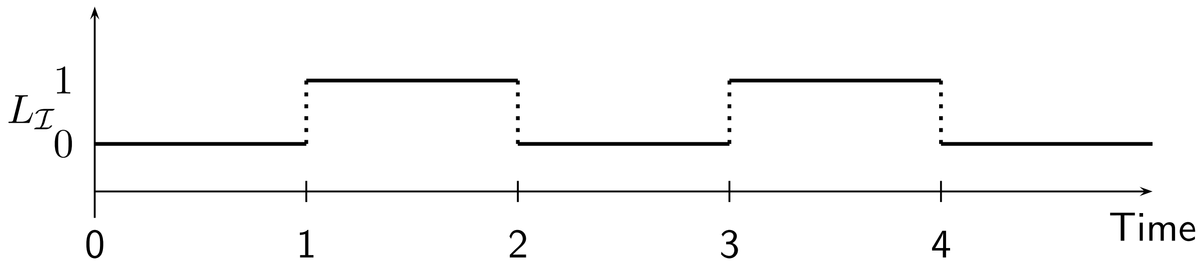
$$\mathcal{I}[\![F_1 \wedge F_2]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b,e]) = \mathcal{I}[\![F_2]\!](\mathcal{V}, [b,e]) = \mathsf{tt},$$

$$\mathcal{I}[\![\forall\, x \bullet F_1]\!](\mathcal{V}, [b,e]) = \mathsf{tt} \text{ iff for all } a \in \mathbb{R}, \quad \mathcal{I}[\![F_1[x := a]]\!](\mathcal{V}, [b,e]) = \mathsf{tt}$$

$$\mathcal{I}[\![F_1 \,;\, F_2]\!](\mathcal{V}, [b,e]) = \text{ iff there is an } m \in [b,e] \text{ such that } \mathcal{I}[\![F_1]\!](\mathcal{V}, [b,m]) = \mathcal{I}[\![F_2]\!](\mathcal{V}, [m,e]) = \mathsf{tt}.$$

Formulae: Example

$$F := \int L = 0 \,;\, \int L = 1$$



- $\mathcal{I}[\![F]\!](\mathcal{V}, [0, 2]) =$

Formulae: Remarks

Remark 2.10. [*Rigid and chop-free*] Let F be a duration formula, $\mathcal{I}$ an interpretation, $\mathcal{V}$ a valuation, and $[b, e] \in \mathsf{Intv}$.

- If F is **rigid**, then

$$\forall\, [b', e'] \in \mathsf{Intv} : \mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \mathcal{I}[\![F]\!](\mathcal{V}, [b', e']).$$

- If F is **chop-free** or θ is **rigid**,
  then in the calculation of the semantics of F,
  every occurrence of θ denotes the same value.

Substitution Lemma

Lemma 2.11. [*Substitution*]
Consider a formula F, a global variable x, and a term θ such that F is **chop-free** or θ is **rigid**.

Then for all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and intervals $[b, e]$,

$$\mathcal{I}[\![F[x := \theta]]\!](\mathcal{V}, [b, e]) = \mathcal{I}[\![F]\!](\mathcal{V}[x := a], [b, e])$$

where $a = \mathcal{I}[\![\theta]\!](\mathcal{V}, [b, e])$.

- $F := \ell = x \,;\, \ell = x \implies \ell = 2 \cdot x, \quad \theta := \ell$

Duration Calculus: Overview

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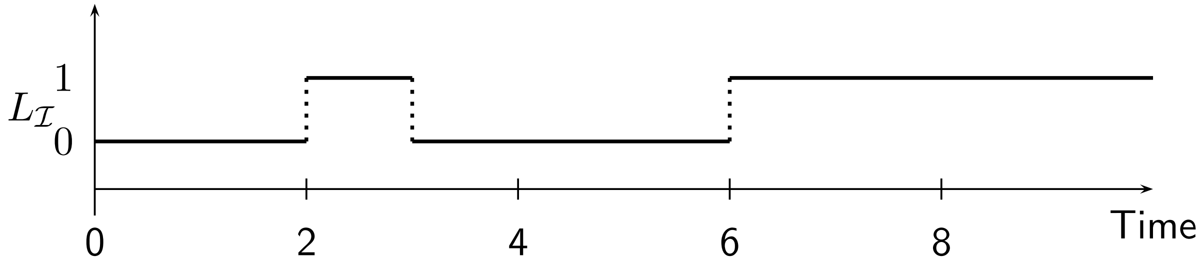
Duration Calculus Abbreviations

Abbreviations

- $\lceil\rceil := \ell = 0$ **(point interval)**
- $\lceil P \rceil := \int P = \ell \wedge \ell > 0$ **(almost everywhere)**
- $\lceil P \rceil^t := \lceil P \rceil \wedge \ell = t$ **(for time t)**
- $\lceil P \rceil^{\leq t} := \lceil P \rceil \wedge \ell \leq t$ **(up to time t)**

- $\Diamond F := \mathit{true} \,;\, F \,;\, \mathit{true}$ **(for some subinterval)**
- $\Box F := \neg \Diamond \neg F$ **(for all subintervals)**

Abbreviations: Examples



$$\mathcal{I}[\![\ \textstyle\int L = 0\ ]\!](\mathcal{V},\ [0,2]\ ) =$$

$$\mathcal{I}[\![\ \textstyle\int L = 1\ ]\!](\mathcal{V},\ [2,6]\ ) =$$

$$\mathcal{I}[\![\ \textstyle\int L = 0\ ;\ \int L = 1\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

$$\mathcal{I}[\![\ \lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,2]\ ) =$$

$$\mathcal{I}[\![\ \lceil L \rceil\ ]\!](\mathcal{V},\ [2,3]\ ) =$$

$$\mathcal{I}[\![\ \lceil \neg L \rceil\ ;\ \lceil L \rceil\ ]\!](\mathcal{V},\ [0,3]\ ) =$$

$$\mathcal{I}[\![\ \lceil \neg L \rceil\ ;\ \lceil L \rceil\ ;\ \lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

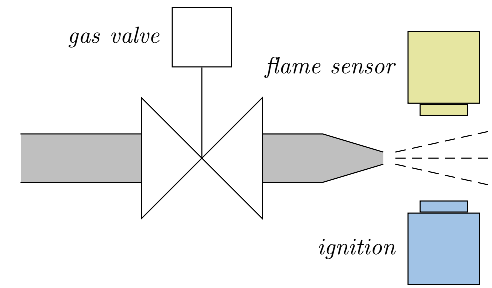
$$\mathcal{I}[\![\ \Diamond \lceil L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

$$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

$$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil^2\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

$$\mathcal{I}[\![\ \Diamond \lceil \neg L \rceil^2\ ;\ \lceil \neg L \rceil^1\ ;\ \lceil \neg L \rceil^3\ ]\!](\mathcal{V},\ [0,6]\ ) =$$

Duration Calculus: Preview



- $G, F, I, H : \{0, 1\}$
- Define $L : \{0, 1\}$ as $G \wedge \neg F$.

- Duration Calculus is an **interval logic**.
- Formulae are evaluated in an (**implicitly given**) interval.

Strangest operators:

- **almost everywhere** — Example: $\lceil G \rceil$

  (Holds in a given interval $[b, e]$ iff the gas valve is open almost everywhere.)

- **chop** — Example: $(\lceil \neg I \rceil \,;\, \lceil I \rceil \,;\, \lceil \neg I \rceil) \implies \ell \geq 1$

  (Ignition phases last at least one time unit.)

- **integral** — Example: $\ell \geq 60 \implies \int L \leq \frac{\ell}{20}$

  (At most 5% leakage time within intervals of at least 60 time units.)

DC Validity, Satisfiability, Realisability

Validity, Satisfiability, Realisability

Let $\mathcal{I}$ be an interpretation, $\mathcal{V}$ a valuation, $[b, e]$ an interval, and F a DC formula.

- $\mathcal{I}, \mathcal{V}, [b, e] \models F$ ("F **holds** in $\mathcal{I}$, $\mathcal{V}$, $[b, e]$") iff $\mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \mathrm{tt}$.

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- F is called **satisfiable** iff it holds in some $\mathcal{I}$, $\mathcal{V}$, $[b,e]$.

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- $\mathcal{I}, \mathcal{V} \models F$ ("$\mathcal{I}$ and $\mathcal{V}$ **realise** F") iff $\forall\, [b, e] \in \mathsf{Intv} : \mathcal{I}, \mathcal{V}, [b, e] \models F$.

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- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ realise F.

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- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ realise F.

- $\mathcal{I} \models F$ ("$\mathcal{I}$ **realises** F") iff $\quad \forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models F$.

Validity, Satisfiability, Realisability

Let $\mathcal{I}$ be an interpretation, $\mathcal{V}$ a valuation, $[b, e]$ an interval, and F a DC formula.

- $\mathcal{I}, \mathcal{V}, [b, e] \models F$ ("F **holds** in $\mathcal{I}$, $\mathcal{V}$, $[b, e]$") iff $\mathcal{I}[\![F]\!](\mathcal{V}, [b, e]) = \mathsf{tt}$.
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- F is called **realisable** iff some $\mathcal{I}$ and $\mathcal{V}$ realise F.

- $\mathcal{I} \models F$ ("$\mathcal{I}$ **realises** F") iff $\forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models F$.

- $\models F$ ("F is **valid**") iff $\forall$ interpretation $\mathcal{I} : \mathcal{I} \models F$.

Validity vs. Satisfiability vs. Realisability

Remark 2.13. For all DC formulae F,

- F is satisfiable iff $\neg F$ is not valid,
  F is valid iff $\neg F$ is not satisfiable.
- If F is valid then F is realisable, but not vice versa.
- If F is realisable then F is satisfiable, but not vice versa.

Examples: Valid? Realisable? Satisfiable?

- $\ell \geq 0$
- $\ell = \int 1$
- $\ell = 30 \iff \ell = 10 \,;\, \ell = 20$
- $((F \,;\, G) \,;\, H) \iff (F \,;\, (G \,;\, H))$
- $\int L \leq x$
- $\ell = 2$

Initial Values

- $\mathcal{I}, \mathcal{V} \models_0 F$ ("$\mathcal{I}$ and $\mathcal{V}$ **realise** F **from** 0") iff

$$\forall\, t \in \mathsf{Time} : \mathcal{I}, \mathcal{V}, [0, t] \models F.$$

- F is called **realisable from** 0 iff some $\mathcal{I}$ and $\mathcal{V}$ realise F from 0.

- Intervals of the form $[0, t]$ are called **initial intervals**.

- $\mathcal{I} \models_0 F$ ("$\mathcal{I}$ **realises** F **from** 0") iff $\forall\, \mathcal{V} \in \mathsf{Val} : \mathcal{I}, \mathcal{V} \models_0 F$.

- $\models_0 F$ ("F is **valid from** 0") iff $\forall$ interpretation $\mathcal{I} : \mathcal{I} \models_0 F$.

Initial or not Initial...

For all interpretations $\mathcal{I}$, valuations $\mathcal{V}$, and DC formulae F,

(i) $\mathcal{I}, \mathcal{V} \models F$ implies $\mathcal{I}, \mathcal{V} \models_0 F$,

(ii) if F is realisable then F is realisable from 0, but not vice versa,

(iii) F is valid iff F is valid from 0.

Specification and Semantics-based Correctness Proofs of Real-Time Systems with DC

Methodology: Ideal World...

(i) Choose a collection of **observables** 'Obs'.

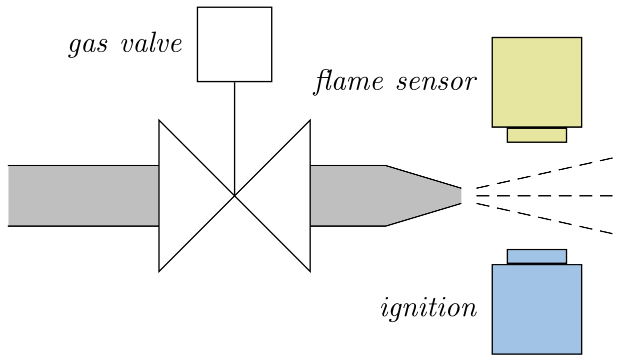
(ii) Provide the **requirement/specification** 'Spec'
as a conjunction of DC formulae (over 'Obs').

(iii) Provide a description 'Ctrl'
of the **controller** in form of a DC formula (over 'Obs').

(iv) We say 'Ctrl' is **correct** (wrt. 'Spec') iff

$$\models_0 \mathsf{Ctrl} \implies \mathsf{Spec}.$$

Gas Burner Revisited



(i) Choose **observables**:

- two boolean observables G and F (i.e. Obs $= \{G, F\}$, $\mathcal{D}(G) = \mathcal{D}(F) = \{0, 1\}$)
- $G = 1$: gas valve open (output)
- $F = 1$: have flame (input)
- define $L := G \wedge \neg F$ (leakage)

(ii) Provide the **requirement**:

$$\text{Req} :\iff \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

Gas Burner Revisited

(iii) Provide a description 'Ctrl'
of the **controller** in form of a DC formula (over 'Obs').
Here, firstly consider a **design**:

- Des-1 : $\iff \Box(\lceil L \rceil \implies \ell \leq 1)$

- Des-2 : $\iff \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$

(iv) Prove **correctness**:

- We want (or do we want $\models_0 \ldots$?):

$$\models (\text{Des-1} \wedge \text{Des-2} \implies \text{Req}) \qquad \text{(Thm. 2.16)}$$

Gas Burner Revisited

(iii) Provide a description 'Ctrl'
of the **controller** in form of a DC formula (over 'Obs').
Here, firstly consider a **design**:

- Des-1 : $\iff \Box(\lceil L \rceil \implies \ell \leq 1)$

- Des-2 : $\iff \Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)$

(iv) Prove **correctness**:

- We want (or do we want $\models_0$...?):

$$\models (\text{Des-1} \land \text{Des-2} \implies \text{Req}) \qquad \text{(Thm. 2.16)}$$

- We do show

$$\models \text{Req-1} \implies \text{Req} \qquad \text{(Lem. 2.17)}$$

with the simplified requirement

$$\text{Req-1} := \Box(\ell \leq 30 \implies \textstyle\int L \leq 1),$$

Gas Burner Revisited: Lemma 2.17

Claim:

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)}_{\text{Req}}$$

Proof:

Gas Burner Revisited: Lemma 2.17

Claim:

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)}_{\text{Req}}$$

Proof:

- Assume 'Req-1'.

Gas Burner Revisited: Lemma 2.17

Claim:

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)}_{\text{Req}}$$

Proof:

- Assume 'Req-1'.
- Let $L_\mathcal{I}$ be any interpretation of L, and $[b, e]$ an interval with $e - b \geq 60$.

Gas Burner Revisited: Lemma 2.17

Claim:

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}} \implies \underbrace{\Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)}_{\text{Req}}$$

Proof:

- Assume 'Req-1'.
- Let $L_\mathcal{I}$ be any interpretation of L, and $[b, e]$ an interval with $e - b \geq 60$.
- Show "$20 \cdot \int L \leq \ell$", i.e.

$$\mathcal{I}[\![20 \cdot \textstyle\int L \leq \ell]\!](\mathcal{V}, [b, e]) = \mathrm{tt}$$

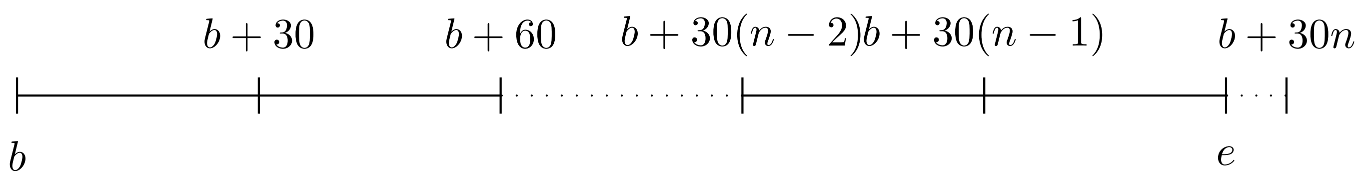
i.e.

$$\hat{20} \,\hat{\cdot} \int_b^e L_\mathcal{I}(t)\, dt \,\hat{\leq}\, (e - b)$$

Gas Burner Revisited: Lemma 2.17

$$\models \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}}$$
$$\implies \Box(\ell \geq 60 \implies 20 \cdot \textstyle\int L \leq \ell)$$

- Set $n := \lceil \frac{e-b}{30} \rceil$, i.e. $n \in \mathbb{N}$ with $n - 1 < \frac{e-b}{30} \leq n$, and split the interval



Some Laws of the DC Integral Operator

Theorem 2.18.
For all state assertions P and all real numbers $r_1, r_2 \in \mathbb{R}$,

(i) $\models \int P \leq \ell$,

(ii) $\models (\int P = r_1)\,;(\int P = r_2) \implies \int P = r_1 + r_2$,

(iii) $\models \lceil \neg P \rceil \implies \int P = 0$,

(iv) $\models \lceil\rceil \implies \int P = 0$.

Gas Burner Revisited: Lemma 2.18

Claim:

$$\models (\underbrace{\Box(\lceil L \rceil \implies \ell \leq 1)}_{\text{Des-1}} \wedge \underbrace{\Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)}_{\text{Des-2}}) \implies \underbrace{\Box(\ell \leq 30 \implies \textstyle\int L \leq 1)}_{\text{Req-1}}$$

Proof:

Gas Burner Revisited: Lemma 2.

(i) $\models \int P \leq \ell$, (iv) $\models \lceil\rceil \implies \int P = 0$
(ii) $\models (\int P = r_1) \,;\, (\int P = r_2) \implies \int P = r_1 + r_2$,
(iii) $\models \lceil \neg P \rceil \implies \int P = 0$,

Claim:

$$\models (\underbrace{\Box(\lceil L \rceil \implies \ell \leq 1)}_{\text{Des-1}} \wedge \underbrace{\Box(\lceil L \rceil \,;\, \lceil \neg L \rceil \,;\, \lceil L \rceil \implies \ell > 30)}_{\text{Des-2}}) \implies \underbrace{\Box(\ell \leq 30 \implies \int L \leq 1)}_{\text{Req-1}}$$

Proof:

Gas Burner Revisited: Lemma 2.18

References

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). *Real-Time Systems - Formal Specification and Automatic Verification*. Cambridge University Press.