Contents & Goals

Last Lecture:
• DC Implementables

This Lecture:
• Educational Objectives:
  - Capabilities for following tasks/questions.
  - Facts: (un)decidability properties of DC in discrete/continuous time.
  - What’s the idea of the considered (un)decidability proofs?

Content:
• DC Implementables Cont’d
• RDC in discrete time
• Satisfiability and realizability from 0 is decidable for RDC in discrete time
• Undecidable problems of DC in continuous time

Recall: DC Implementables

DC Implementables are special patterns of DC standard forms (due to A. P. Ravn).

π, π₁, …, πₙ, n ≥ 0, denote phases of the same state variable Xᵢ.
ϕ denotes a state assertion not depending on Xᵢ.
θ denotes a rigid term.
Initialisation: ⌈⌉ ∨ ⌈π⌉; true
Sequencing: ⌈π⌉ −→ ⌈π ∨ π₁ ∨ · · · ∨ πₙ⌉
Progress: ⌈π⌉ θ −→ ⌈¬π⌉
Synchronisation: ⌈π ∧ ϕ⌉ θ −→ ⌈¬π⌉

Recall: Control Automata

Model of Gas Burner controller as a system of four control automata:
• H: Boolean, representing heat request (input)
• F: Boolean, representing flame (input)
• C with D(C) = {idle, purge, ignite, burn}, representing the controller (local)
• G: Boolean, representing gas valve (output)

Recall: DC Implementables Cont’d

Bounded Stability:
⌈¬π⌉; ⌈π ∧ ϕ⌉ ≤ θ −→ ⌈π ∨ π₁ ∨ · · · ∨ πₙ⌉

Unbounded Stability:
⌈¬π⌉; ⌈π ∧ ϕ⌉ −→ ⌈π ∨ π₁ ∨ · · · ∨ πₙ⌉

Bounded Initial Stability:
⌈π ∧ ϕ⌉ ≤ θ −→ 0 ⌈π ∨ π₁ ∨ · · · ∨ πₙ⌉

Unbounded Initial Stability:
⌈π ∧ ϕ⌉ −→ 0 ⌈π ∨ π₁ ∨ · · · ∨ πₙ⌉
Lemma 3.16 Cont'd

Case 2: $I, V, [b, e] |= \lceil \text{burn} \rceil$; true $\wedge \ell \leq 30 \Rightarrow \lceil \text{burn} \lor \text{idle} \rceil$ (Seq-4)

Case 3: $I, V, [b, e] |= \lceil \text{ignite} \rceil$; true $\wedge \ell \leq 30 \Rightarrow \lceil \text{ignite} \lor \text{burn} \rceil$ (Seq-3)

Case 4: $I, V, [b, e] |= \lceil \text{purge} \rceil$; true $\wedge \ell \leq 30 \Rightarrow \lceil \text{purge} \lor \text{ignite} \rceil$ (Seq-2)

Theorem 3.17. $| = (\text{GB-Ctrl} \wedge \epsilon \leq 112) = \Rightarrow \text{Req}$

Discussion

We used only 'Seq-1', 'Seq-2', 'Seq-3', 'Seq-4', 'Prog-2', 'Syn-2', 'Syn-3', 'Stab-2', 'Stab-5', 'Stab-6'. What about 'Prog-1'?

for example?
\[ F ::= \lfloor P \rfloor \oplus \neg F_1 \oplus F_1 \lor F_2 \] where \( P \) is a state assertion, but with boolean observables only.

Note:

- No global variables, thus don't need \( V \).

DiscreteTimeInterpretations

An interpretation \( I \) is called discrete-time interpretation if and only if, for each state variable \( X \),

\[ X_I : \text{Time} \rightarrow D(X) \] with

- \( \text{Time} = \mathbb{R}^+ \),
- all discontinuities are in \( \mathbb{N}_0 \).

An interval \([b,e]\) \( \subseteq \text{Int} \) is called discrete if and only if \( b,e \in \mathbb{N}_0 \).

We say (for a discrete-time interpretation \( I \) and a discrete interval \([b,e]\)) \( I, [b,e] \models F \) if and only if there exists \( m \in [b,e] \cap \mathbb{N}_0 \) such that \( I, [b,m] \models F_1 \) and \( I, [m,e] \models F_2 \).

Expressiveness of RDC

- \( \ell = 1 \) \( \iff \lfloor 1 \rfloor \land \neg (\lfloor 1 \rfloor ; \lfloor 1 \rfloor) \) (in discrete).

- \( \ell = 0 \) \( \iff \neg \lfloor 1 \rfloor \)

- true \( \iff \ell = 0 \lor \neg (\ell = 0) \)

- \( \int P = 0 \) \( \iff (\int P = 0) ; (\lfloor P \rfloor \land \ell = 1) ; (\int P = 0) \)

- \( \int P = k + 1 \) \( \iff (\int P = k) ; (\int P = 1) \)

- \( \int P \geq k \) \( \iff (\int P = k) \land \text{true} \)

- \( \int P > k \) \( \iff \int P \geq k + 1 \)

- \( \int P < k \) \( \iff \neg (\int P > k) \)

- \( \text{Discrete Time Interpretations} \)
Theorem 3.6. The satisfiability problem for RDC with discrete time is decidable.

Theorem 3.9. The realisability problem for RDC with discrete time is decidable.

References
