Real-Time Systems
Lecture 10: DC Properties IIb

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Dr. Bernd Westphal
Albert-Ludwigs-Universität Freiburg, Germany

Contents & Goals

Last Lecture:
• Satisfiability and realisability from 0 is decidable for RDC in discrete time
• Undecidable problems of DC in continuous time

This Lecture:
• Educational Objectives:
  • Capabilities for following tasks/questions.
  • Facts: (un)decidability properties of DC in discrete/continuous time.
  • What's the idea of the considered (un)decidability proofs?
• Content:
  • Undecidable problems of DC in continuous time cont'd

Variants of) RDC in Continuous Time

Sketch: Proof of Theorem 3.10

Reduce divergence of two-counter machines to realisability from 0:
• Given a two-counter machine $M$ with final state $q_{\text{fin}}$,
• construct a DC formula $F(M) := \text{encoding}(M)$
• such that $M$ diverges if and only if the DC formula $F(M) \land \neg \Diamond \lceil q_{\text{fin}} \rceil$ is realisable from 0.
• If realisability from 0 was (semi-)decidable, divergence of two-counter machines would be (which it isn't).

Reducing Divergence to DC realisability: Idea

• A single configuration $K$ of $M$ can be encoded in an interval of length 4;
  being an encoding interval can be characterised by a DC formula.
• An interpretation on 'Time' encodes the computation of $M$ if
  • each interval $[4n, 4(n+1)]$, $n \in \mathbb{N}_0$ encodes a configuration $K_n$,
  • each two subsequent intervals $[4n, 4(n+1)]$ and $[4(n+1), 4(n+2)]$, $n \in \mathbb{N}_0$ encode configurations $K_n \vdash K_{n+1}$ in transition relation.
• Being encoding of the run can be characterised by DC formula $F(M)$.
• Then $M$ diverges if and only if $F(M) \land \neg \Diamond \lceil q_{\text{fin}} \rceil$ is realisable from 0.

Construction of $F(M)$

In the following, we give DC formulae describing
• the initial configuration,
• the general form of configurations,
• the transitions between configurations,
• the handling of the final state.
$F(M)$ is the conjunction of all these formulae.
Following [Chaochen and Hansen, 2004] we can observe that $M$ halts if and only if the DC formula $F(M) \land \lozenge \lceil q_{\text{fin}} \rceil$ is satisfiable. This yields Theorem 3.11. The satisfiability problem for DC with continuous time is undecidable. (It is semi-decidable.)

Furthermore, by taking the contraposition, we see $M$ diverges if and only if $M$ does not halt if and only if $F(M) \land \neg \lozenge \lceil q_{\text{fin}} \rceil$ is not satisfiable.

Thus whether a DC formula is not satisfiable is not decidable, not even semi-decidable.

By Remark 2.13, $F$ is valid iff $\neg F$ is not satisfiable, so Corollary 3.12. The validity problem for DC with continuous time is undecidable, not even semi-decidable.

Discussion

Note: the DC fragment defined by the following grammar is sufficient for the reduction $F ::= \lceil P \rceil | \neg F | F_1 \lor F_2 | F_1; F_2 | \ell = 1 | \ell = x | \forall x \cdot F_1$, $P$ a state assertion, $x$ a global variable.

Formulae used in the reduction are abbreviations:

- $\ell \geq 4 \iff \ell = 4$
- $\ell = 1$ (true)
- $\ell = x + y + 4 \iff \ell = x ; \ell = y ; \ell = 4$

Length 1 is not necessary — we can use $\ell = z$ instead, with fresh $z$.

This is RDC augmented by "$\ell = x$" and "$\forall x$", which we denote by $\text{RDC}^{\ell = x, \forall x}$. References
