Real-Time Systems

Lecture 14: Regions and Zones

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Contents & Goals

Last Lecture:

- Location reachability decidability

This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What's a zone? In contrast to a region?
  - Motivation for having zones?
  - What's a DBM? Who needs to know DBMs?

- **Content:**
  - Zones
  - Difference Bound Matrices

Zones

(Presentation following [Fränzle, 2007])

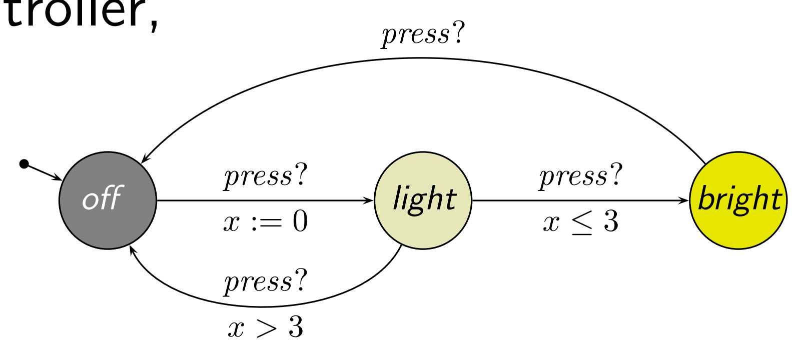
Recall: Number of Regions

Lemma 4.28. Let X be a set of clocks, $c_x \in \mathbb{N}_0$ the maximal constant for each $x \in X$, and $c = \max\{c_x \mid x \in X\}$. Then

$$(2c+2)^{|X|} \cdot (4c+3)^{\frac{1}{2}|X| \cdot (|X|-1)}$$

is an **upper bound** on the **number of regions**.

- In the desk lamp controller,



many regions are reachable in $\mathcal{R}(\mathcal{L})$, but we convinced ourselves that it's **actually** only important whether $\nu(x) \in [0,3]$ or $\nu(x) \in (3,\infty)$.

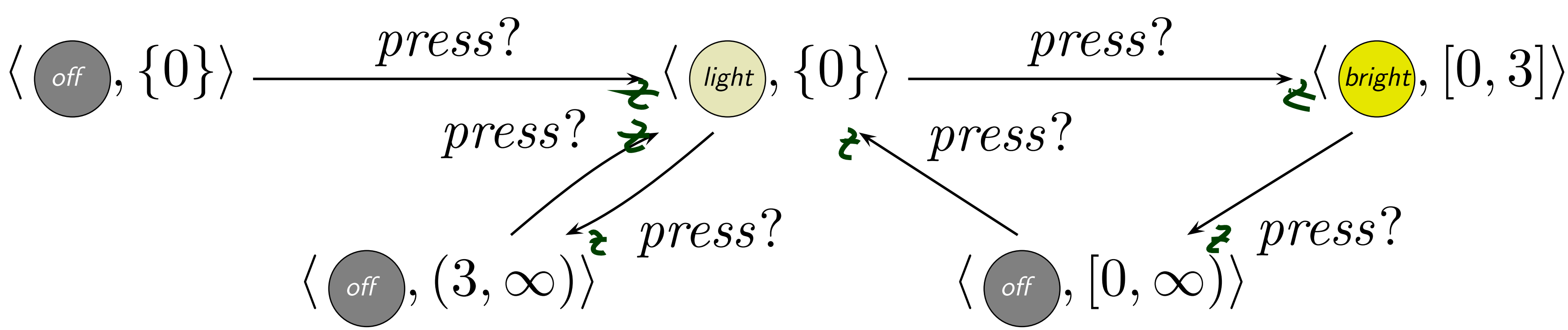
So: seems there are even **equivalence classes** of undistinguishable regions.

Wanted: Zones instead of Regions

- In $\mathcal{R}(\mathcal{L})$ we have transitions:
  - $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, \{0\}\rangle$, $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, (0,1)\rangle$,
  - $\ldots$,
  - $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, (2,3)\rangle$, $\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, \{3\}\rangle$
- Which seems to be a complicated way to write just:

$$\langle \text{light}, \{0\}\rangle \xrightarrow{press?} \langle \text{bright}, [0,3]\rangle$$

- Can't we **constructively** abstract $\mathcal{L}$ to:



What is a Zone?

Definition. A (**clock**) **zone** is a set $z \subseteq (X \to \mathrm{Time})$ of valuations of clocks X such that there exists $\varphi \in \Phi(X)$ with

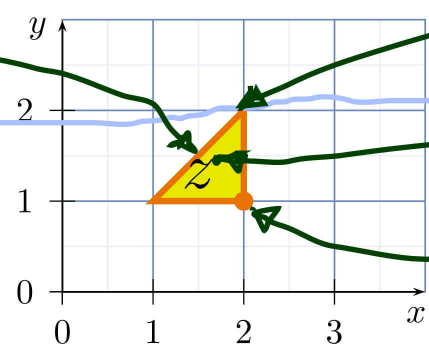
$$\nu \in z \text{ if and only if } \nu \models \varphi.$$

Example:

$x=1, y=1.5$ is in z

$x=2, y=2$ is not in z

$x \leq 2$



$x=1.75, y=1.25$ is in z

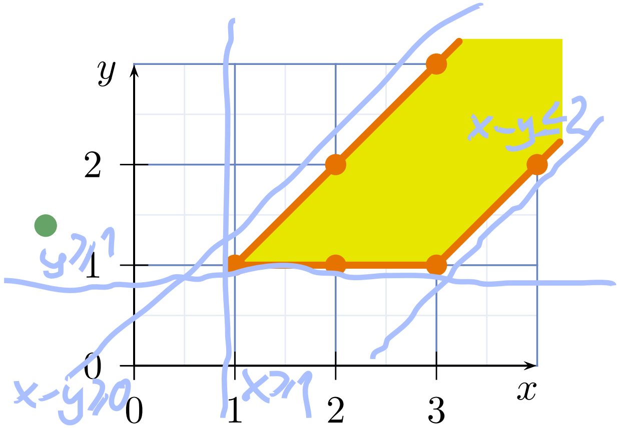
$(x=2, y=1)$ is in z

is a clock zone by

$$\varphi = (x \leq 2) \wedge (x > 1) \wedge (y \geq 1) \wedge (y < 2) \wedge (x - y \geq 0)$$

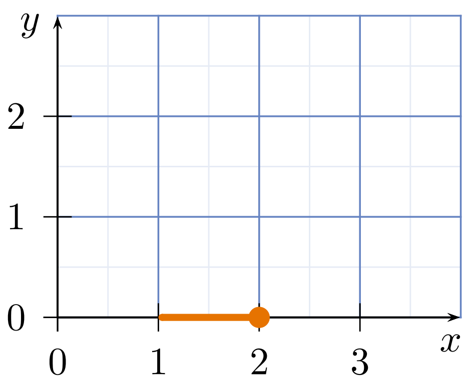
- Note: Each clock constraint φ is a **symbolic representation** of a zone.
- But: There's no one-on-one correspondence between clock constraints and zones. The zone $z = \emptyset$ corresponds to $(x > 1 \wedge x < 1)$, $(x > 2 \wedge x < 2)$, ...

More Examples: Zone or Not?



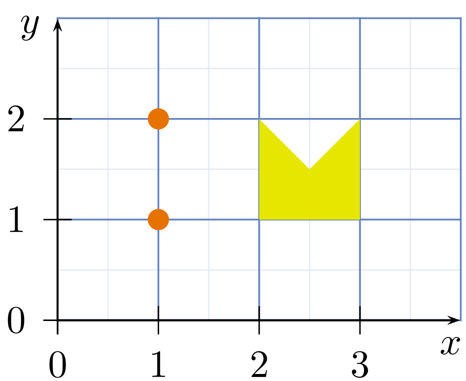
YES by

$x \geq 1 \wedge y \geq 1 \wedge x-y \geq 0 \wedge x-y \leq 2$



YES by

$y = 0 \wedge x \leq 2 \wedge x > 1$

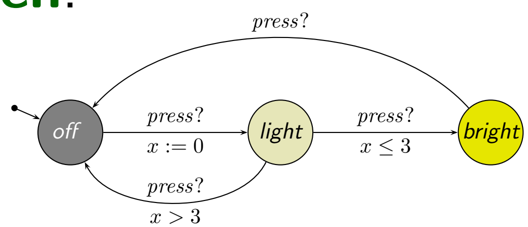


NO

(not convex)

Zone-based Reachability

Given:

- 

and initial configuration $\langle$ off $, \{0\}\rangle$

Assume a function

edge

$$\mathrm{Post}_e : (L \times \text{Zones}) \to (L \times \text{Zones})$$

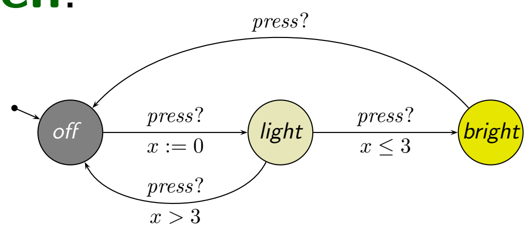
such that $\mathrm{Post}_e(\langle \ell, z\rangle)$ yields the configuration $\langle \ell', z'\rangle$ such that

- zone z' denotes exactly those clock valuations ν'
- which are reachable from a configuration $\langle \ell, \nu\rangle$, $\nu \in z$,
- by taking edge $e = (\ell, \alpha, \varphi, Y, \ell') \in E$.

first delaying

Zone-based Reachability

Given:

- 

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- which are reachable from a configuration $\langle \ell, \nu\rangle$, $\nu \in z$,
- by taking edge $e = (\ell, \alpha, \varphi, Y, \ell') \in E$.

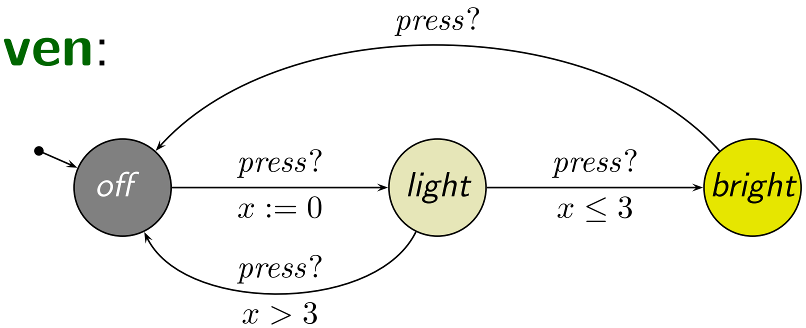
Then $\ell \in L$ is reachable in $\mathcal{A}$ if and only if

$$\mathrm{Post}_{e_n}(\ldots(\mathrm{Post}_{e_1}(\langle \ell_{ini}, z_{ini}\rangle)\ldots)) = \langle \ell, z\rangle$$

for some $e_1, \ldots, e_n \in E$ and some z.

Zone-based Reachability: In Other Words

Given:

- 

 and initial configuration $\langle$ off $, \{0\}\rangle$

Wanted: A procedure to compute the set

- $\langle$ light $, \{0\}\rangle$
- $\langle$ bright $, [0,3]\rangle$
- $\langle$ off $, [0,\infty)\rangle$

- Set $R := \{\langle \ell_{ini}, z_{ini}\rangle\} \subset L \times$ Zones
- Repeat
  - pick
    - a pair $\langle \ell, z\rangle$ from R and
    - an edge $e \in E$ with source ℓ

    such that $\mathrm{Post}_e(\langle \ell, z\rangle)$ is not already subsumed by R
  - add $\mathrm{Post}_e(\langle \ell, z\rangle)$ to R

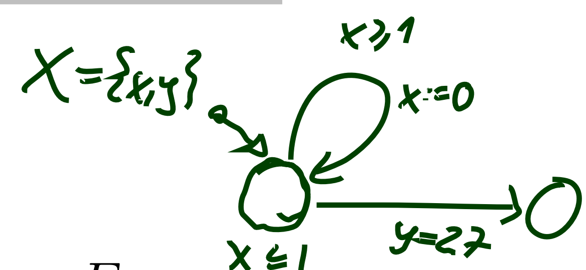
  until no more such $\langle \ell, z\rangle \in R$ and $e \in E$ are found.

Stocktaking: What's Missing?

- Set $R := \{\langle \ell_{ini}, z_{ini} \rangle\} \subset L \times$ Zones
- Repeat
  - pick
    - a pair $\langle \ell, z \rangle$ from R and
    - an edge $e \in E$ with source ℓ

    such that $\mathrm{Post}_e(\langle \ell, z \rangle)$ is not already **subsumed** by R
  - add $\mathrm{Post}_e(\langle \ell, z \rangle)$ to R

  until no more such $\langle \ell, z \rangle \in R$ and $e \in E$ are found.



Missing:

- Algorithm to effectively compute $\mathrm{Post}_e(\langle \ell, z \rangle)$ for given configuration $\langle \ell, z \rangle \in L \times$ Zones and edge $e \in E$.
- Decision procedure for whether configuration $\langle \ell', z' \rangle$ is **subsumed** by a given subset of $L \times$ Zones.

Note: Algorithm in general **terminates only if** we apply **widening** to zones, that is, roughly, to take maximal constants c_x into account (not in lecture).

What is a Good "Post"?

- If z is given by a constraint $\varphi \in \Phi(X)$, then the zone component z' of $\mathrm{Post}_e(\ell, z) = \langle \ell', z' \rangle$ should also be a constraint from $\Phi(X)$.
  (Because sets of clock valuations are soo unhandily. . .)

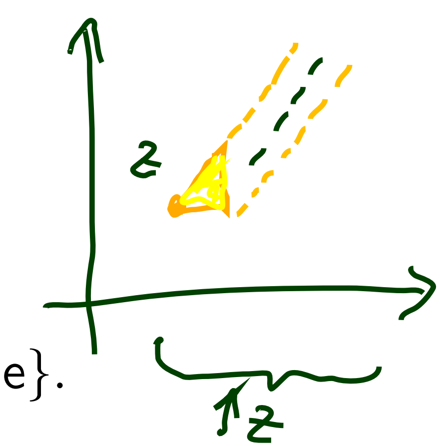
Good news: the following operations can be carried out by manipulating φ.

- The **elapse time** operation:

$$\uparrow : \Phi(X) \to \Phi(X)$$

Given a constraint φ, the constraint $\uparrow(\varphi)$, or $\varphi\uparrow$ in postfix notation, is supposed to denote the set of clock valuations

$$\{\nu + t \mid \nu \models \varphi, t \in \mathsf{Time}\}.$$



In other symbols: we **want**

$$[\![\uparrow(\varphi)]\!] = [\![\varphi\uparrow]\!] = \{\nu + t \mid \nu \in [\![\varphi]\!], t \in \mathsf{Time}\}.$$

To this end: remove all upper bounds $x \leq c$, $x < c$ from φ and add diagonals.

Good News Cont'd

Good news: the following operations can be carried out by manipulating φ.

- **elapse time** $\varphi \uparrow$ with

$$[\![\varphi \uparrow]\!] = \{\nu + t \mid \nu \models \varphi, t \in \mathsf{Time}\}$$

- **zone intersection** $\varphi_1 \wedge \varphi_2$ with

$$[\![\varphi_1 \wedge \varphi_2]\!] = \{\nu \mid \nu \models \varphi_1 \text{ and } \nu \models \varphi_2\}$$

- **clock hiding** $\exists\, x.\varphi$ with

$$[\![\exists\, x.\varphi]\!] = \{\nu \mid \text{there is } t \in \mathsf{Time} \text{ such that } \nu[x := t] \models \varphi\}$$

- **clock reset** $\varphi[x := 0]$ with

$$[\![\varphi[x := 0]]\!] = [\![x = 0 \wedge \exists\, x.\varphi]\!]$$

This is Good News...

...because given $\langle \ell, z\rangle = \langle \ell, \varphi_0\rangle$ and $e = (\ell, \alpha, \varphi, \{y_1, \dots, y_n\}, \ell') \in E$ we have

$$\mathrm{Post}_e(\langle \ell, z\rangle) = \langle \ell', \varphi_5\rangle$$

where

- $\varphi_1 = \varphi_0 \uparrow$

  let **time elapse** starting from φ_0: φ_1 represents all valuations reachable by waiting in ℓ for an arbitrary amount of time.

- $\varphi_2 = \varphi_1 \wedge I(\ell)$

  intersect with invariant of ℓ: φ_2 represents the reachable good valuations.

- $\varphi_3 = \varphi_2 \wedge \varphi$

  intersect with guard: φ_3 are the reachable good valuations where e is enabled.

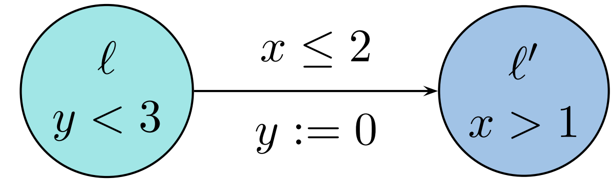
- $\varphi_4 = \varphi_3[y_1 := 0] \dots [y_n := 0]$

  reset clocks: φ_4 are all possible outcomes of taking e from φ_3

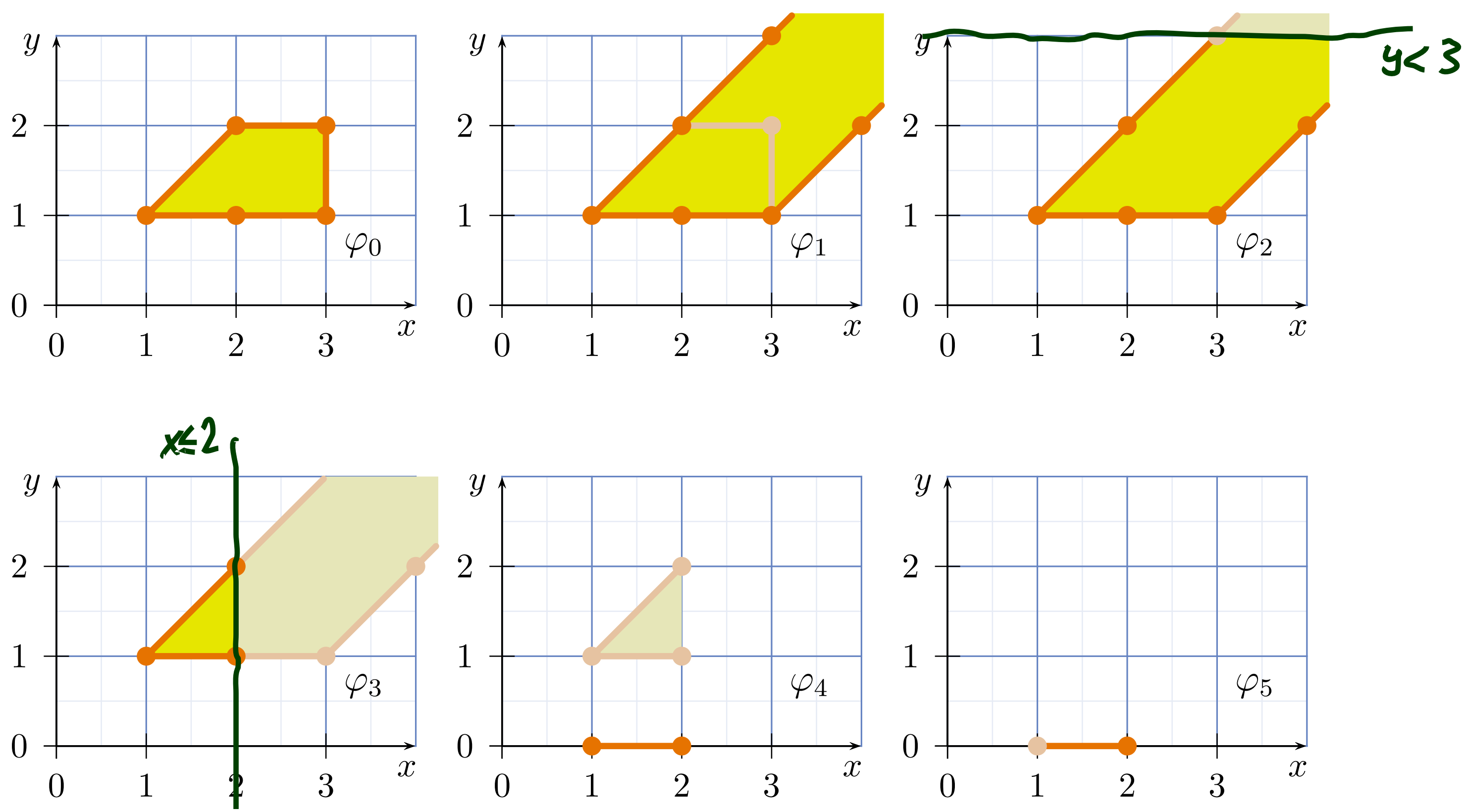
- $\varphi_5 = \varphi_4 \wedge I(\ell')$

  intersect with invariant of ℓ': φ_5 are the good outcomes of taking e from φ_3

Example



- $\varphi_1 = \varphi_0 \uparrow$ let **time elapse**.
- $\varphi_2 = \varphi_1 \wedge I(\ell)$ **intersect with invariant** of ℓ
- $\varphi_3 = \varphi_2 \wedge \varphi$ **intersect with guard**
- $\varphi_4 = \varphi_3[y_1 := 0] \dots [y_n := 0]$ **reset clocks**
- $\varphi_5 = \varphi_4 \wedge I(\ell')$ **intersect with invariant** of ℓ'



Difference Bound Matrices

disjoint union

- Given a finite set of clocks X, a **DBM** over X is a mapping

$$M : (X \mathbin{\dot\cup} \{x_0\} \times X \mathbin{\dot\cup} \{x_0\}) \to (\{<, \leq\} \times \mathbb{Z} \cup \{(<, \infty)\})$$

- $M(x, y) = (\sim, c)$ encodes the conjunct $x - y \sim c$ (x and y can be x_0).

	x_0	x	y
x_0			
x			
y			

$M(x_0, y) \in \{<,\leq\} \times \mathbb{Z} \cup \{(<,\infty)\}$

e.g.

$(<, -5)$

encodes

$x_0 - y < -5$

i.e. $y > 5$

Difference Bound Matrices

- Given a finite set of clocks X, a **DBM** over X is a mapping

$$M : (X \,\dot{\cup}\, \{x_0\} \times X \,\dot{\cup}\, \{x_0\}) \to (\{<, \leq\} \times \mathbb{Z} \cup \{(<, \infty)\})$$

- $M(x, y) = (\sim, c)$ encodes the conjunct $x - y \sim c$ (x and y can be x_0).

- If M and N are DBM encoding φ_1 and φ_2 (representing zones z_1 and z_2), then we can efficiently compute $M\uparrow$, $M \wedge N$, $M[x := 0]$ such that
  - all three are again DBM,
  - $M\uparrow$ encodes $\varphi_1\uparrow$,
  - $M \wedge N$ encodes $\varphi_1 \wedge \varphi_2$, and
  - $M[x := 0]$ encodes $\varphi_1[x := 0]$.

- And there is a **canonical form** of DBM — canonisation of DBM can be done in cubic time (**Floyd-Warshall** algorithm).

- Thus: we can define our 'Post' on DBM, and let our algorithm run on DBM.

Pros and cons

- **Zone-based** reachability analysis usually is explicit wrt. discrete locations:
  - maintains a list of location/zone pairs or
  - maintains a list of location/DBM pairs
  - **confined wrt. size of discrete state space**
  - **avoids blowup by number of clocks and size of clock constraints through symbolic representation of clocks**

- **Region-based** analysis provides a finite-state abstraction, amenable to finite-state symbolic MC
  - **less dependent on size of discrete state space**
  - **exponential in number of clocks**

Contents & Goals

Last Lecture:

- Decidability of the location reachability problem:
  - region automaton & zones

This Lecture:

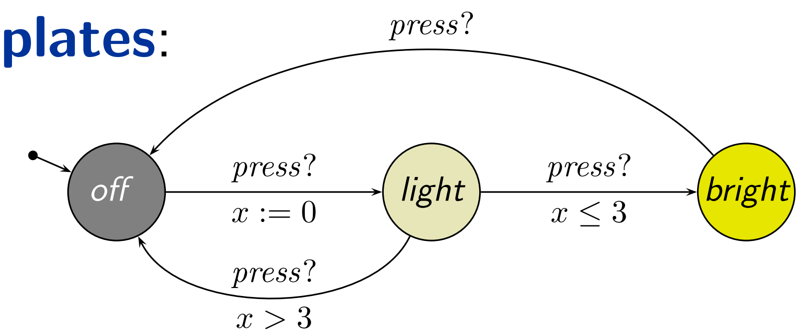
- **Educational Objectives:** Capabilities for following tasks/questions.
  - By what are TA extended? Why is that useful?
  - What's an urgent/committed location? What's the difference?
  - What's an urgent channel?
  - Where has the notion of "input action" and "output action" correspondences in the formal semantics?
- **Content:**
  - Extended TA:
    - Data-Variables, Structuring Facilities, Restriction of Non-Determinism
  - The Logic of Uppaal

Extended Timed Automata

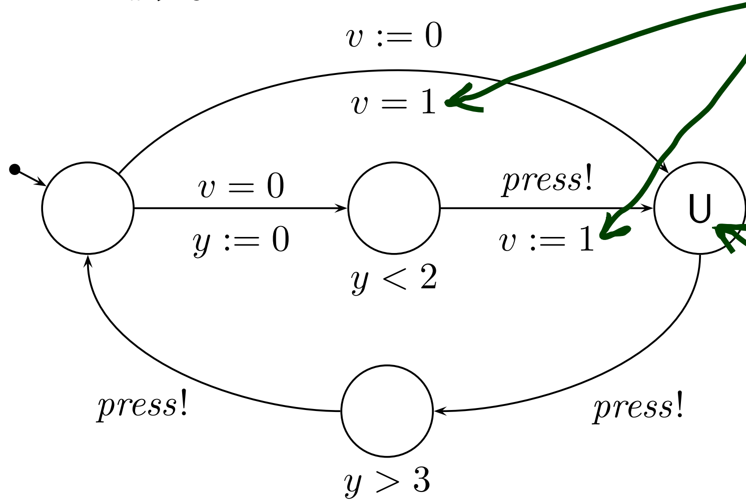
Example (Partly Already Seen in Uppaal Demo)

Templates:

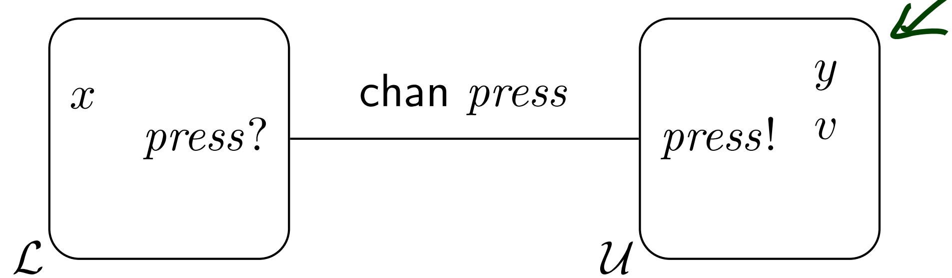
- $\mathcal{L}$:



- $\mathcal{U}$:



System:



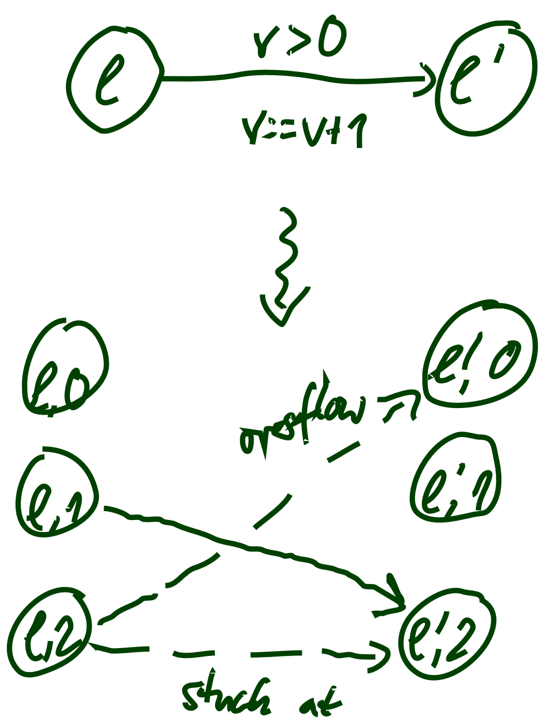
Extensions:

- Data Variables (Expressions, Constraints, Updates)
- Structuring
- Urgent/Committed Location, Urgent Channel

Data-Variables

- When modelling controllers as timed automata, it is sometimes desirable to have (local and shared) variables.
  E.g. count number of open doors, or intermediate positions of gas valve.

$\mathcal{D}(v) = \{0, 1, 2\}$



Data-Variables

- When modelling controllers as timed automata, it is sometimes desirable to have (local and shared) variables.
  E.g. count number of open doors, or intermediate positions of gas valve.
- Adding variables with **finite** range (possibly grouped into **finite** arrays) to any finite-state automata concept is straighforward:
  - If we have control locations $L_0 = \{\ell_1, \dots, \ell_n\}$,
  - and want to model, e.g., the valve range as a variable v with $\mathcal{D}(v) = \{0, \dots, 2\}$,
  - then just use $L = L_0 \times \mathcal{D}(v)$ as control locations, i.e. encode the current value of v in the control location, and consider updates of v in the $\xrightarrow{\lambda}$.

  L is still finite, so we still have a proper TA.
- But: writing $\xrightarrow{\lambda}$ is tedious.
- So: have variables as "first class citizens" and let compilers do the work.
- **Interestingly**, many examples in the literature live without variables: the more abstract the model is, i.e., the fewer information it keeps track of (e.g. in data variables), the easier the verification task.

Data Variables and Expressions

- Let $(v, w \in)\ V$ be a set of (integer) variables.

  $(\psi_{int} \in)\ \Psi(V)$: **integer expressions** over V using func. symb. $+, -, \ldots$

  $(\varphi_{int} \in)\ \Phi(V)$: **integer** (or **data**) **constraints** over V using **integer expressions**, predicate symbols $=, <, \leq, \ldots$, and boolean logical connectives.

- Let $(x, y \in)\ X$ be a set of clocks.

  $(\varphi \in)\ \Phi(X, V)$: (**extended**) **guards**, defined by

$$\varphi ::= \varphi_{clk} \mid \varphi_{int} \mid \varphi_1 \wedge \varphi_2$$

where $\varphi_{clk} \in \Phi(X)$ is a simple clock constraint (as defined before) and $\varphi_{int} \in \Phi(V)$ an **integer (or data) constraint**.

Examples: Extended guard or not extended guard? Why?

(a) $\underbrace{x < y}_{\varphi_{clk}} \wedge \underbrace{v > 2}_{\varphi_{int}}$, ✓ (b) $\underbrace{x < y}_{\varphi_{clk}} \lightning \vee \underbrace{v > 2}_{\varphi_{int}}$, ✗ (c) $\underbrace{v < 1 \vee v > 2}_{\varphi_{int}}$, ✓ (d) $x < v$ ✗

Modification or Reset Operation

- **New**: a **modification** or **reset (operation)** is

$$x := 0, \qquad x \in X,$$

or

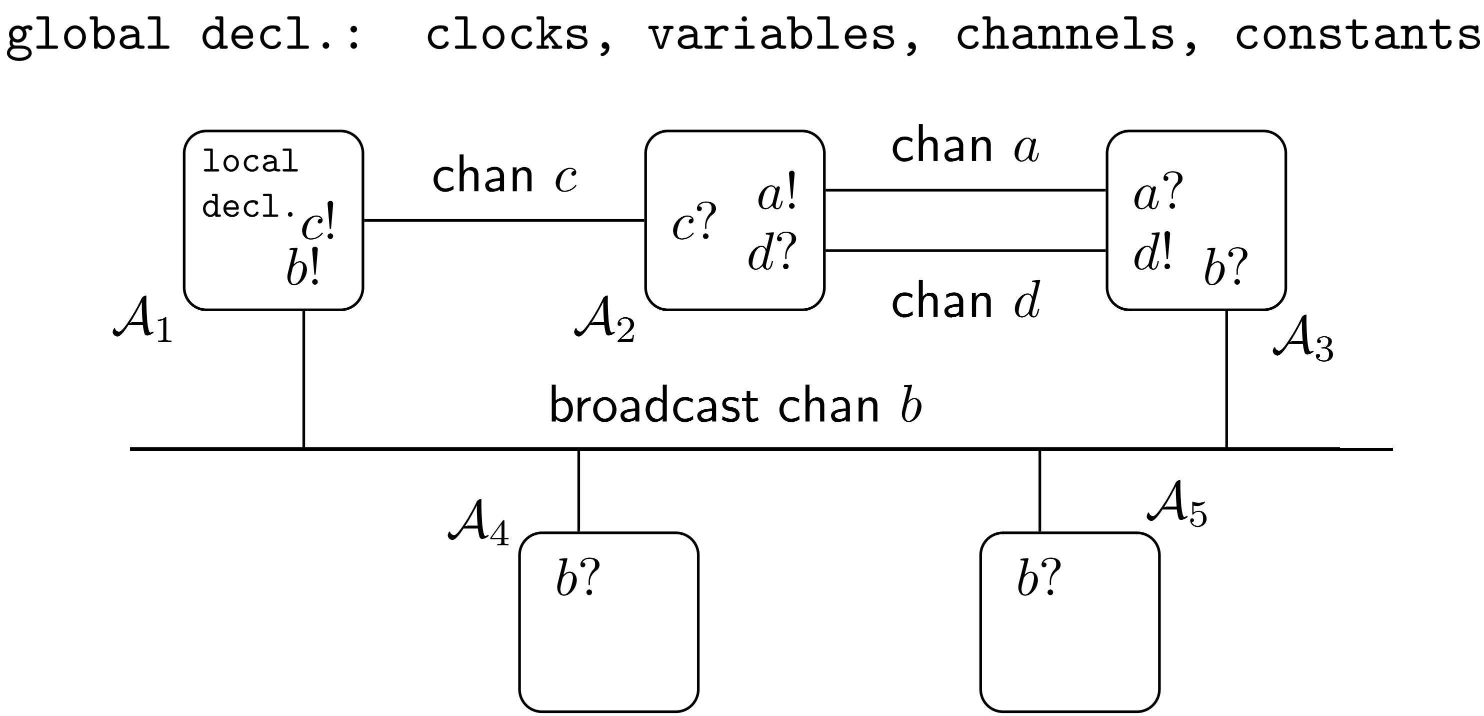
$$v := \psi_{int}, \qquad v \in V, \quad \psi_{int} \in \Psi(V).$$

- By $R(X, V)$ we denote the set of all resets.
- By $\vec{r}$ we denote a finite list $\langle r_1, \dots, r_n \rangle$, $n \in \mathbb{N}_0$, of reset operations $r_i \in R(X, V)$; $\langle\rangle$ is the empty list.
- By $R(X, V)^*$ we denote the set of all such lists of reset operations.

Examples: Modification or not? Why?

(a) $x := y$, ✗ (b) $x := v$, ✗ (c) $v := x$, ✗ (d) $v := w$, ✓ (e) $v := 0$ ✓

Structuring Facilities



- Global declarations of of clocks, data variables, channels, and constants.
- Binary and broadcast channels: chan c and broadcast chan b.
- Templates of timed automata.
- Instantiation of templates (instances are called **process**).
- System definition: list of processes.

Restricting Non-determinism

- **Urgent locations** — enforce local immediate progress.

U

- **Committed locations** — enforce **atomic** immediate progress.

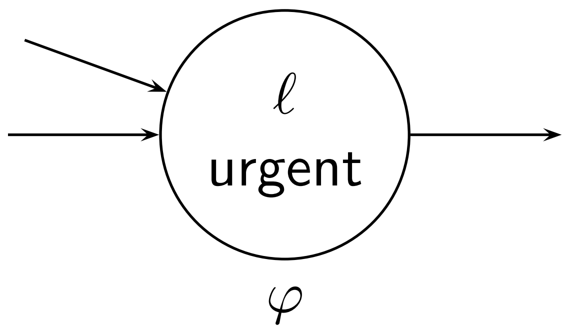
C

- **Urgent channels** — enforce cooperative immediate progress.

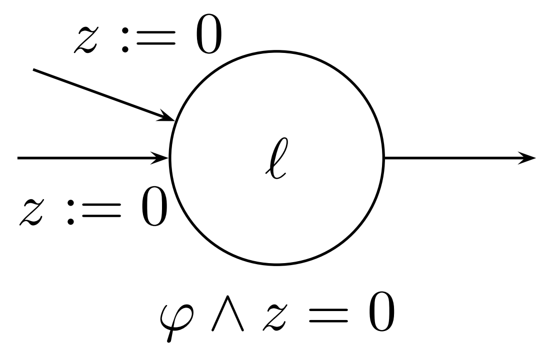
```
urgent chan press;
```

Urgent Locations: Only an Abbreviation...

Replace



with



where z is a fresh clock:

- reset z on all in-going egdes,
- add $z = 0$ to invariant.

Question: How many fresh clocks do we need in the worst case for a network of N extended timed automata?

Extended Timed Automata

Definition 4.39. An **extended timed automaton** is a structure

$$\mathcal{A}_e = (L, C, B, U, X, V, I, E, \ell_{ini})$$

where L, B, X, I, ℓ_{ini} are as in Def. 4.3, except that location invariants in I are **downward closed**, and where

- $C \subseteq L$: **committed locations**,
- $U \subseteq B$: **urgent channels**,
- V: a set of data variables,
- $E \subseteq L \times B_{!?} \times \Phi(X, V) \times R(X, V)^* \times L$: a set of **directed edges** such that

$$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \mathsf{chan}(\alpha) \in U \implies \varphi = \mathit{true}.$$

Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a list $\vec{r}$ of **reset operations**.

References

[Fränzle, 2007] Fränzle, M. (2007). Formale methoden eingebetteter systeme. Lecture, Summer Semester 2007, Carl-von-Ossietzky Universität Oldenburg.

[Olderog and Dierks, 2008] Olderog, E.-R. and Dierks, H. (2008). Real-Time Systems - Formal Specification and Automatic Verification. Cambridge University Press.