Exercise 1: Postcondition
We say that $post$ distributes over the connective $\odot$ wrt. the first argument if the following equation holds.

$$post(\phi_1 \odot \phi_2, \rho) = post(\phi_1, \rho) \odot post(\phi_2, \rho)$$

We say that $post$ distributes over the connective $\odot$ wrt. the second argument if the following equation holds.

$$post(\phi, \rho_1 \odot \rho_2) = post(\phi, \rho_1) \odot post(\phi, \rho_2)$$

Determine for $\odot \in \{\land, \lor, \rightarrow\}$ if $post$ distributes over $\odot$ wrt. the first argument or wrt. the second argument.

Give a proof for each positive answer, give a counterexample for each negative answer.

Exercise 2: Reachability
Consider the following program with input variables $i$ and $j$.

\[
\begin{align*}
\ell_0 & : x := i; \\
\ell_1 & : y := j; \\
\ell_2 & : \textbf{while } x \neq 0 \textbf{ do} \{ \\
\ell_3 & : \quad x := x - 1; \\
\ell_4 & : \quad y := y - 1; \\
\ell_5 & : \} \\
\ell_6 & : \textbf{assert}(i = j \rightarrow y = 0);
\end{align*}
\]

(a) Compute the set of reachable states $\varphi_{\text{reach}}$.

*Hint:* If you only apply the $post$ operator, your algorithm will not terminate. You need to find a relation between all variables which is true before and after each loop iteration (a loop invariant). Then use this to “jump over the loop”.

(b) Is the program safe?