Softwaretechnik / Software-Engineering

Lecture 13: Behavioural Software Modelling

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Topic Area Architecture & Design: Content

Content

Communicating Finite Automata

presentation follows (Olderog and Dierks, 2008)

Channel Names and Actions

To define communicating finite automata, we need the following sets of symbols:

- A set $(a, b \in)$ Chan of **channel names** or **channels**.
- For each channel $a \in$ Chan, two **visible actions**: $a?$ and $a!$ denote **input** and **output** on the **channel** ($a?, a! \notin$ Chan).
- $\tau \notin$ Chan represents an **internal action**, not visible from outside.
- $(\alpha, \beta \in)\ Act := \{a? \mid a \in \text{Chan}\} \cup \{a! \mid a \in \text{Chan}\} \cup \{\tau\}$ is the set of **actions**.

- An **alphabet** B is a set of **channels**, i.e. $B \subseteq$ Chan.
- For each alphabet B, we define the corresponding **action set**

$$B_{?!} := \{a? \mid a \in B\} \cup \{a! \mid a \in B\} \cup \{\tau\}.$$

Note: $\text{Chan}_{?!} = Act$.

Integer Variables and Expressions, Resets

- Let $(v, w \in)\ V$ be a set of ((finite domain) integer) variables.
  By $(\varphi \in)\ \Psi(V)$ we denote the set of **integer expressions** over V using function symbols $+, -, \ldots$, >, ≤, ...
- A **modification** on v is (or update)

$$v := \varphi, \qquad v \in V, \quad \varphi \in \Psi(V).$$

  example: $x := 1$
  $\underbrace{y}_{\in V} := \underbrace{x+1}_{\in \Psi(V)}$

  By $R(V)$ we denote the set of all modifications.
- By $\vec{r}$ we denote a finite list $\langle r_1, \ldots, r_n \rangle$, $n \in \mathbb{N}_0$, of modifications $r_i \in R(V)$.
  $\langle\rangle$ is the empty list ($n = 0$). (reset vector) or (update vector)
- By $R(V)^*$ we denote the set of all such finite lists of modifications.

Communicating Finite Automata

Definition. A **communicating finite automaton** is a structure

$$\mathcal{A} = (L, B, V, E, \ell_{ini})$$

where

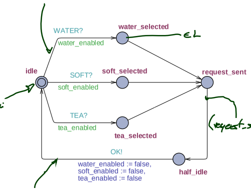
- $(\ell \in)$ L is a finite set of **locations** (or **control states**),
- $B \subseteq$ Chan,
- V: a set of data variables,
- $E \subseteq L \times B_{!?} \times \Phi(V) \times R(V)^* \times L$: a finite set of **directed edges** such that
  $$(\ell, \alpha, \varphi, \vec{r}, \ell') \in E \wedge \mathsf{chan}(\alpha) \in U \implies \varphi = \mathit{true}.$$

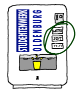
  Edges $(\ell, \alpha, \varphi, \vec{r}, \ell')$ from location ℓ to ℓ' are labelled with an **action** α, a **guard** φ, and a list $\vec{r}$ of **modifications**.
- ℓ_{ini} is the **initial location**.

Example

ChoicePanel:
(simplified)

(idle, WATER?, water_enabled, ⟨⟩, water_selected) $\in E$





(request_sent, τ, true, ⟨⟩, half_idle)

(half_idle, OK!, true, ⟨water_enabled := false, ...⟩, idle)

Operational Semantics of Networks of FCA

Definition.
Let $\mathcal{A}_i = (L_i, B_i, V_i, E_i, \ell_{ini,i})$, $1 \leq i \leq n$, be communicating finite automata.

The **operational semantics** of the **network** of FCA $\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ is the labelled transition system

$$\mathcal{T}(\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)) = (\mathit{Conf}, \underbrace{\mathsf{Chan} \cup \{\tau\}}_{\text{labels}}, \underbrace{\{\xrightarrow{\lambda} \mid \lambda \in \mathsf{Chan} \cup \{\tau\}\}}_{\text{labelled transition relations}}, C_{ini})$$

where

location vector
valuation
$\vec{\ell} = (\ell_1, \dots, \ell_n)$

- $V = \bigcup_{i=1}^n V_i$,
- $\mathit{Conf} = \{\langle \vec{\ell}, \nu \rangle \mid \ell_i \in L_i, \nu : V \to \mathscr{D}(V)\}$,
- $C_{ini} = \langle \vec{\ell}_{ini}, \nu_{ini} \rangle$ with $\nu_{ini}(v) = 0$ for all $v \in V$.

The transition relation consists of transitions of the following two types.

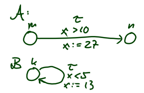
Helpers: Extended Valuations and Effect of Resets

- $\nu : V \to \mathscr{D}(V)$ is a **valuation** of the variables,
- A valuation ν of the variables canonically assigns an integer value $\nu(\varphi)$ to each integer expression $\varphi \in \Phi(V)$.
- $\models \subseteq (V \to \mathscr{D}(V)) \times \Phi(V)$ is the canonical **satisfaction relation** between valuations and integer expressions from $\Phi(V)$. e.g. $\nu \models x > 10$
- **Effect of modification** $r \in R(V)$ **on** ν, denoted by $\nu[r]$:
  $$\underbrace{\big(\nu[v := \varphi]\big)}_{: V \to \mathscr{D}(V)}(a) := \begin{cases} \nu(\varphi), & \text{if } a = v, \\ \nu(a), & \text{otherwise} \end{cases}$$
- We set $\nu[\langle r_1, \dots, r_n \rangle] := \nu[r_1] \dots [r_n] = (((\nu[r_1])[r_2]) \dots)[r_n]$.

  That is, modifications are executed sequentially from left to right.

Operational Semantics of Networks of FCA

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \dots, n\}$ and
  - there is a τ-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell'_i) \in E_i$ such that $\quad \vec{\ell} = (\ell_1, \dots, \ell_i, \dots, \ell_n)$
    - $\nu \models \varphi$,
    - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell'_i]$,
    - $\nu' = \nu[\vec{r}]$.



$\ell_1 \quad \ell_2$

$\langle \vec{\ell}, \nu \rangle = \langle (m, k), x = 8 \rangle \quad \not\xrightarrow{\tau} \quad \langle (n, k), x = 27 \rangle = \langle \vec{\ell}'_1, \nu'_1 \rangle$

$\not\xrightarrow{\tau} \quad \langle (m, k), x = 13 \rangle = \langle \vec{\ell}'_2, \nu'_2 \rangle$

NONE: ||||| |

$\langle \vec{\ell}, \nu \rangle = \langle (m, k), x = 11 \rangle \xrightarrow{\tau} \langle (n, k), x = 27 \rangle$

Operational Semantics of Networks of FCA

- An **internal transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{\tau} \langle \vec{\ell}', \nu' \rangle$ occurs if there is $i \in \{1, \dots, n\}$ and
  - there is a τ-edge $(\ell_i, \tau, \varphi, \vec{r}, \ell'_i) \in E_i$ such that
    - $\nu \models \varphi$, "source valuation satisfies guard"
    - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell'_i]$, "automaton i changes location"
    - $\nu' = \nu[\vec{r}]$. "ν' is ν modified by $\vec{r}$"
- A **synchronisation transition** $\langle \vec{\ell}, \nu \rangle \xrightarrow{b} \langle \vec{\ell}', \nu' \rangle$ occurs if there are $i, j \in \{1, \dots, n\}$ with $i \neq j$ and
  - there are edges $(\ell_i, b!, \varphi_i, \vec{r}_i, \ell'_i) \in E_i$ and $(\ell_j, b?, \varphi_j, \vec{r}_j, \ell'_j) \in E_j$ such that
    - $\nu \models \varphi_i \wedge \varphi_j$,
    - $\vec{\ell}' = \vec{\ell}[\ell_i := \ell'_i][\ell_j := \ell'_j]$,
    - $\nu' = (\nu[\vec{r}_i])[\vec{r}_j]$. sender updates first

This style of communication is known under the names "**rendezvous**", "**synchronous**", "**blocking**" communication (and possibly many others).

Transition Sequences

- A **transition sequence** of $\mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n)$ is any (in)finite sequence of the form

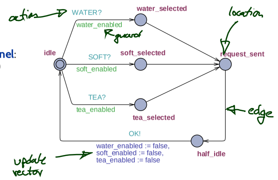
$$\langle \vec{\ell}_0, \nu_0\rangle \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1\rangle \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2\rangle \xrightarrow{\lambda_3} \dots$$

with

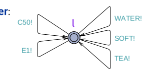
- $\langle \vec{\ell}_0, \nu_0\rangle = C_{ini}$,
- for all $i \in \mathbb{N}$, there is $\xrightarrow{\lambda_{i+1}}$ in $\mathcal{T}(\mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n))$ with $\langle \vec{\ell}_i, \nu_i\rangle \xrightarrow{\lambda_{i+1}} \langle \vec{\ell}_{i+1}, \nu_{i+1}\rangle$.

Example

ChoicePanel:
(simplified)



User:



$\langle (idle, l), \begin{smallmatrix} w=1 \\ s=1 \\ t=0 \end{smallmatrix}\rangle \xrightarrow{WATER} \langle (w\_sel, l), \begin{smallmatrix} w=1 \\ s=1 \\ t=0 \end{smallmatrix}\rangle \xrightarrow{\tau} \langle (req\_s, l), \begin{smallmatrix} w=1 \\ s=1 \\ t=0 \end{smallmatrix}\rangle \xrightarrow{\tau} \langle (half\_i, l), \begin{smallmatrix} w=1 \\ s=1 \\ t=0 \end{smallmatrix}\rangle$

C_{ini}^{*} $\xrightarrow{SOFT} \langle (s\_sel, l), \begin{smallmatrix} w=1 \\ s=1 \\ t=0 \end{smallmatrix}\rangle$ ...

here nothing

else: guard tea_enabled not satisfied, and no comm. partners for C50 or E1

Deadlock, Reachability

- A **configuration** $\langle \ell, \nu\rangle$ of $\mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n)$ is called **deadlock** if and only if there are no transitions from $\langle \ell, \nu\rangle$, i.e. if

$$\neg(\exists\, \lambda \in \Lambda\, \exists\, \langle \ell', \nu'\rangle \in Conf \bullet \langle \ell, \nu\rangle \xrightarrow{\lambda} \langle \ell', \nu'\rangle).$$

The **network** $\mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n)$ is said to **have a deadlock** if and only if there is a configuration $\underbrace{\langle \ell, \nu\rangle}_{\text{reachable}}$ which is a deadlock.

- A **configuration** $\langle \vec{\ell}, \nu\rangle$ is called **reachable** (in $\mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n)$) if and only if there is a transition sequence of the form

$$\langle \vec{\ell}_0, \nu_0\rangle \xrightarrow{\lambda_1} \langle \vec{\ell}_1, \nu_1\rangle \xrightarrow{\lambda_2} \langle \vec{\ell}_2, \nu_2\rangle \xrightarrow{\lambda_3} \dots \xrightarrow{\lambda_n} \langle \vec{\ell}_n, \nu_n\rangle = \langle \vec{\ell}, \nu\rangle.$$

A **location** $\ell \in L_i$ is called **reachable** if and only if **any** configuration $\langle \vec{\ell}, \nu\rangle$ with $\ell_i = \ell$ is reachable, i.e. there exist $\vec{\ell}$ and ν such that $\ell_i = \ell$ and $\langle \vec{\ell}, \nu\rangle$ is reachable.

Uppaal

(*Larsen et al., 1997*; *Behrmann et al., 2004*)

Tool Demo

The Uppaal Query Language

Consider $\mathcal{N} = \mathcal{C}(\mathcal{A}_1,\dots,\mathcal{A}_n)$ over data variables V.

- **basic formula**:

$$atom ::= \mathcal{A}_i.\ell \mid \varphi \mid \texttt{deadlock}$$

where $\ell \in L_i$ is a location and φ an expression over V.

- **configuration formulae**:

$$term ::= atom \mid \texttt{not}\ term \mid term_1\ \texttt{and}\ term_2$$

- **existential path formulae**:

$e\text{-}formula ::= \exists\Diamond\, term$ (**exists finally**)
$\quad\mid \exists\Box\, term$ (**exists globally**)

- **universal path formulae**:

$a\text{-}formula ::= \forall\Diamond\, term$ (**always finally**)
$\quad\mid \forall\Box\, term$ (**always globally**)
$\quad\mid term_1\ \texttt{-->}\ term_2$ (**leads to**)

- **formulae** (or **queries**):

$$F ::= e\text{-}formula \mid a\text{-}formula$$

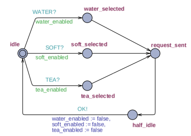
Satisfaction of Uppaal Queries by Configurations

- The **satisfaction relation**
$$\langle \vec{\ell}, \nu \rangle \models F$$
between **configurations**
$$\langle \vec{\ell}, \nu \rangle = \langle (\ell_1, \dots, \ell_n), \nu \rangle$$
of a network $\mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ and **formulae** F of the Uppaal logic is defined **inductively** as follows:

- $\langle \vec{\ell}, \nu \rangle \models \texttt{deadlock}$ iff $\langle \vec{\ell}, \nu \rangle$ is a deadlock
- $\langle \vec{\ell}, \nu \rangle \models \mathcal{A}_i.\ell$ iff $\ell_i = \ell$
- $\langle \vec{\ell}, \nu \rangle \models \varphi$ iff $\nu \models \varphi$
- $\langle \vec{\ell}, \nu \rangle \models \texttt{not}\ term$ iff $\langle \vec{\ell}, \nu \rangle \not\models term$
- $\langle \vec{\ell}, \nu \rangle \models term_1 \ \texttt{and}\ term_2$ iff $\langle \vec{\ell}, \nu \rangle \models term_1$ and $\langle \vec{\ell}, \nu \rangle \models term_2$

Example: Computation Paths vs. Computation Tree

ChoicePanel:



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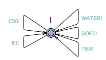

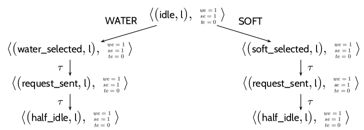

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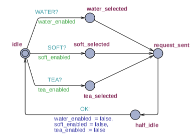

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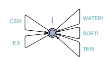
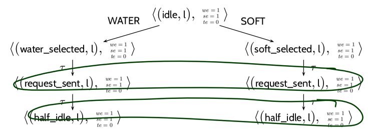
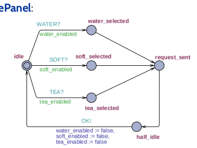




Example: Computation Paths vs. Computation Graph

ChoicePanel:



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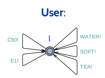
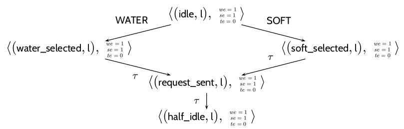




Satisfaction of Uppaal Queries by Configurations

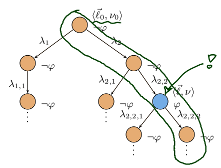
Exists finally:

- $\langle \vec{\ell}_0, \nu_0 \rangle \models \exists\Diamond\ term$ iff $\exists$ path ξ of $\mathcal{C}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle$ $\exists\, i \in \mathbb{N}_0 \bullet \xi^i \models term$

i-th configuration in ξ

"some configuration satisfying $term$ is reachable"

Example: $\langle \vec{\ell}_0, \nu_0 \rangle \models \exists\Diamond\, \varphi$



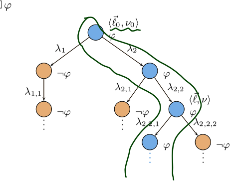
Satisfaction of Uppaal Queries by Configurations

Exists globally:

- $\langle \vec{\ell}_0, \nu_0 \rangle \models \exists\Box\ term$ iff $\exists$ path ξ of $\mathcal{C}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle$ $\forall\, i \in \mathbb{N}_0 \bullet \xi^i \models term$

"on some computation path, all configurations satisfy $term$"

Example: $\langle \vec{\ell}_0, \nu_0 \rangle \models \exists\Box\, \varphi$



Satisfaction of Uppaal Queries by Configurations

- **Always globally:**
  - $\langle \vec{\ell}_0, \nu_0 \rangle \models \forall\Box\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle \not\models \exists\Diamond\, \neg term$

    "not (some configuration satisfying $\neg term$ is reachable)"
    or: "all reachable configurations satisfy $term$"

- **Always finally:**
  - $\langle \vec{\ell}_0, \nu_0 \rangle \models \forall\Diamond\, term$ iff $\langle \vec{\ell}_0, \nu_0 \rangle \not\models \exists\Box\, \neg term$

    "not (on some computation path, all configurations satisfy $\neg term$)"
    or: "on all computation paths, there is a configuration satisfying $term$"

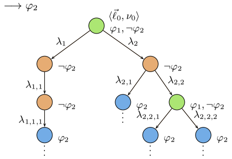
Satisfaction of Uppaal Queries by Configurations

Leads to:

- $\langle \vec{\ell}_0, \nu_0 \rangle \models term_1 \longrightarrow term_2$ iff $\forall$ path ξ of $\mathcal{N}$ starting in $\langle \vec{\ell}_0, \nu_0 \rangle$ $\forall\, i \in \mathbb{N}_0 \bullet$ $\xi^i \models term_1 \implies \xi^i \models \forall\Diamond\, term_2$

  "on all paths, from each configuration satisfying $term_1$,
  a configuration satisfying $term_2$ is reachable" (response pattern)

Example: $\langle \vec{\ell}_0, \nu_0 \rangle \models \varphi_1 \longrightarrow \varphi_2$



CFA Model-Checking

Definition. Let $\mathcal{N} = \mathcal{C}(\mathcal{A}_1, \dots, \mathcal{A}_n)$ be a network and F a query.

(i) We say $\mathcal{N}$ **satisfies** F, denoted by $\mathcal{N} \models F$, if and only if $C_{ini} \models F$.

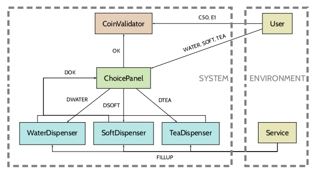
(ii) The **model-checking problem** for $\mathcal{N}$ and F is to decide whether $(\mathcal{N}, F) \in\, \models$.

Proposition.
The model-checking problem for communicating finite automata is **decidable**.

Content

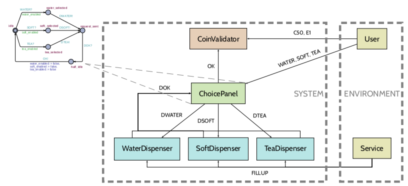
CFA and Queries at Work

Model Architecture — Who Talks What to Whom



- **Shared variables:**
  - `bool water_enabled, soft_enabled, tea_enabled;`
  - `int w = 3, s = 3, t = 3;`
- **Note:** Our model does not use scopes ("information hiding") for channels.
  That is, 'Service' could send 'WATER' if the modeler wanted to.

Model Architecture — Who Talks What to Whom



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  - `int w = 3, s = 3, t = 3;`
- **Note**: Our model does not use scopes ("information hiding") for channels.
  That is, 'Service' could send 'WATER' if the modeler wanted to.

Design Sanity Check: Drive to Configuration

- **Question**: Is is (at all) possible to have no water in the vending machine model?
  (Otherwise, the design is definitely broken.)
- **Approach**: Check whether a configuration satisfying

$$w = 0$$

is reachable, i.e. check

$$\mathcal{N}_{\mathrm{VM}} \models \exists\Diamond\, w = 0.$$

for the vending machine model $\mathcal{N}_{\mathrm{VM}}$.

References

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