Softwaretechnik / Software-Engineering

Lecture 12: Structural Software Modelling

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Topic Area Architecture & Design: Content

VL 11	• **Introduction and Vocabulary**
	• **Principles of Design**
	(i) modularity
	(ii) separation of concerns
	(iii) information hiding and data encapsulation
⋮	(iv) abstract data types, object orientation
	• **Software Modelling**
	(i) views and viewpoints, the 4+1 view
	(ii) model-driven/-based software engineering
	(iii) Unified Modelling Language (UML)
VL 12	(iv) **modelling structure**
	a) (simplified) class diagrams
⋮	b) (simplified) object diagrams
	c) (simplified) object constraint logic (OCL)
VL 13	(v) **modelling behaviour**
	a) communicating finite automata
⋮	b) Uppaal query language
	c) basic state-machines
VL 14	d) an outlook on hierarchical state-machines
⋮	• **Design Patterns**

... so, off to " 'technological paradise' where [...] everything happens according to the blueprints".

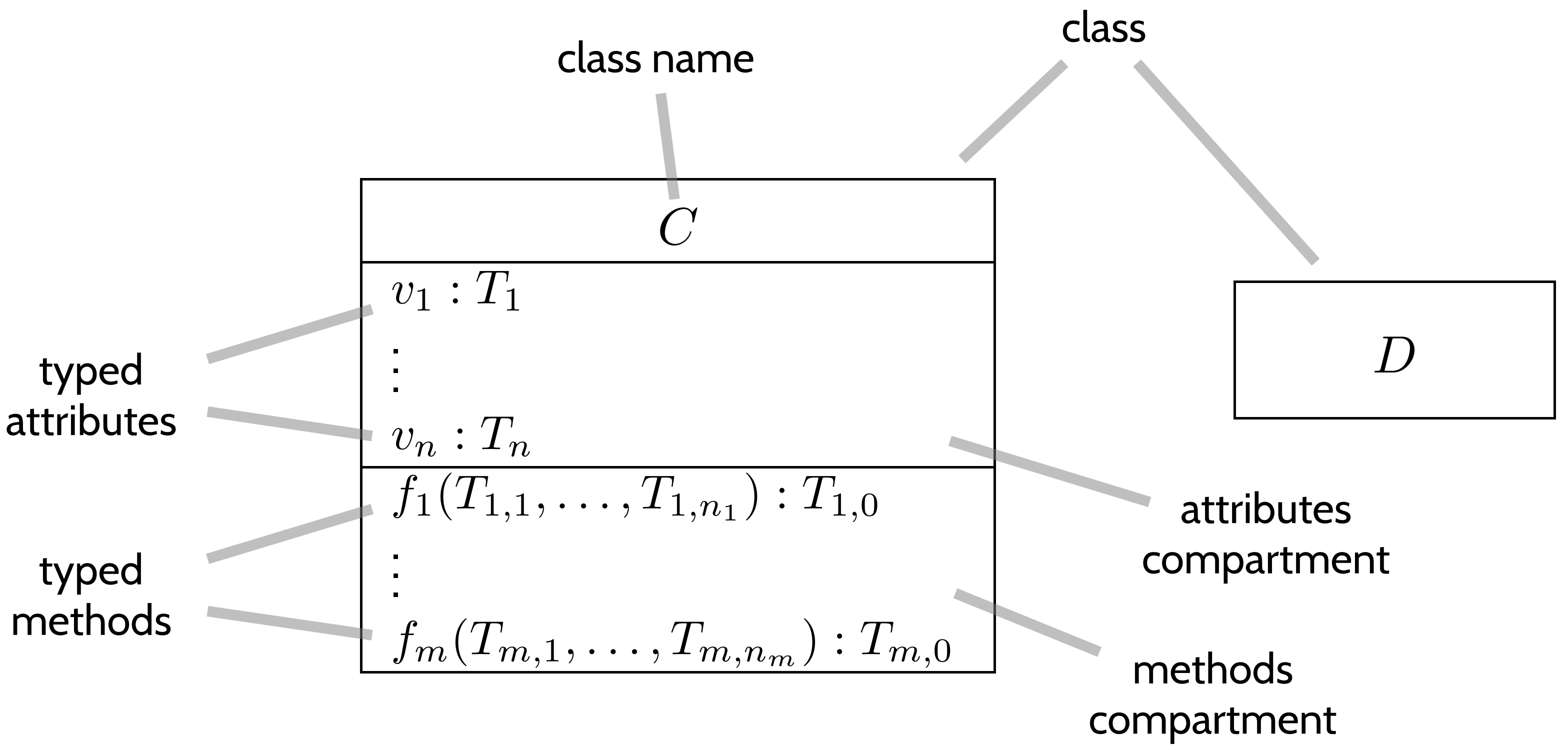
(Kopetz, 2011; Lovins and Lovins, 2001)

Content

- **Class Diagrams**
  - concrete syntax,
  - abstract syntax,
  - class diagrams at work,
  - semantics: system states.
- **Object Diagrams**
  - concrete syntax,
  - dangling references,
  - partial vs. complete,
  - object diagrams at work.
- **Proto-OCL**
  - syntax,
  - semantics,
  - Proto-OCL vs. OCL.
- Putting it All Together:
  Proto-OCL vs. Software

Class Diagrams

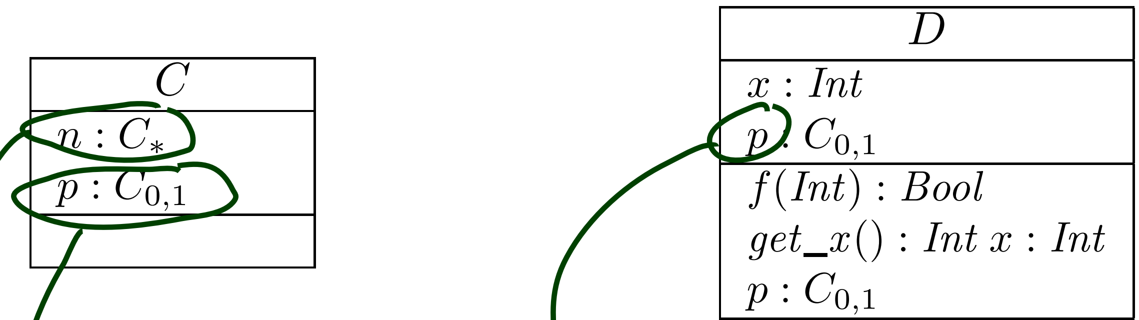
Class Diagrams: Concrete Syntax



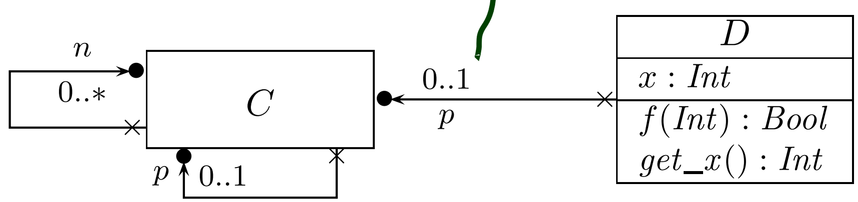
where

- $T_1, \dots, T_{m,0} \in \mathscr{T} \cup \{C_{0,1}, C_* \mid C$ a class name$\}$
- $\mathscr{T}$ is a set of **basic types**, e.g. $Int, Bool, \dots$.

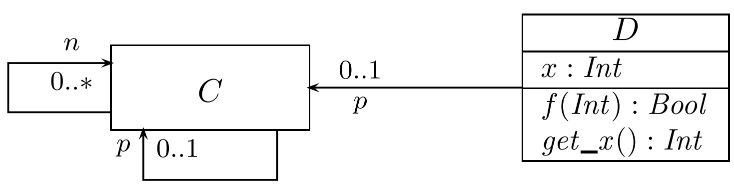
Concrete Syntax: Example



Alternative notation for $C_{0,1}$ and C_* typed attributes:



Alternative lazy notation for **alternative notation**:



And nothing else! This is **the** concrete syntax of **class diagrams** for the **scope of the course**.

Abstract Syntax: Object System Signature

Definition. An **(Object System) Signature** is a 6-tuple

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, F, mth)$$

where

powerset of V

- $\mathscr{T}$ is a set of (basic) **types**,
- $\mathscr{C}$ is a finite set of **classes**,
- V is a finite set of **typed attributes** $v : T$, i.e., each $v \in V$ has type T,
- $atr : \mathscr{C} \to 2^V$ maps each class to its set of attributes.
- F is a finite set of **typed behavioural features** $f : T_1, \ldots, T_n \to T$,
- $mth : \mathscr{C} \to 2^F$ maps each class to its set of behavioural features.

we will discuss these not so much

- A type can be a basic type $\tau \in \mathscr{T}$, or $C_{0,1}$, or C_*, where $C \in \mathscr{C}$.

Note: Inspired by OCL 2.0 standard OMG (2006), Annex A.

Object System Signature Example

Definition. An **(Object System) Signature** is a 6-tuple

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, F, mth)$$

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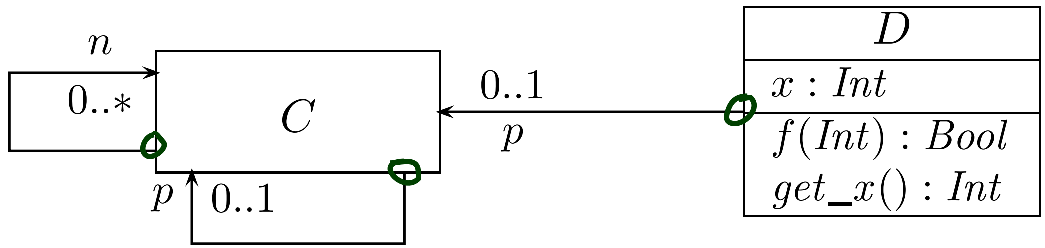
- A type can be a basic type $\tau \in \mathscr{T}$, or $C_{0,1}$, or C_*, where $C \in \mathscr{C}$.

$$\begin{aligned}
\mathscr{S}_0 = (&\{Int, Bool\}, && \mathscr{T}\\
&\{C, D\}, && \mathscr{C}\\
&\{\underbrace{x : Int}, \underbrace{p : C_{0,1}}, \underbrace{n : C_*}\}, && V\\
&\{\underbrace{C \mapsto \{p, n\}}, \underbrace{D \mapsto \{p, x\}}\}, && atr\\
&\{\underbrace{f : Int \to Bool}, \underbrace{get_x : Int}\}, && F\\
&\{\underbrace{C \mapsto \emptyset}, \underbrace{D \mapsto \{f, get_x\}}\}) && mth
\end{aligned}$$

From Abstract to Concrete Syntax



$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, F, mth)$$

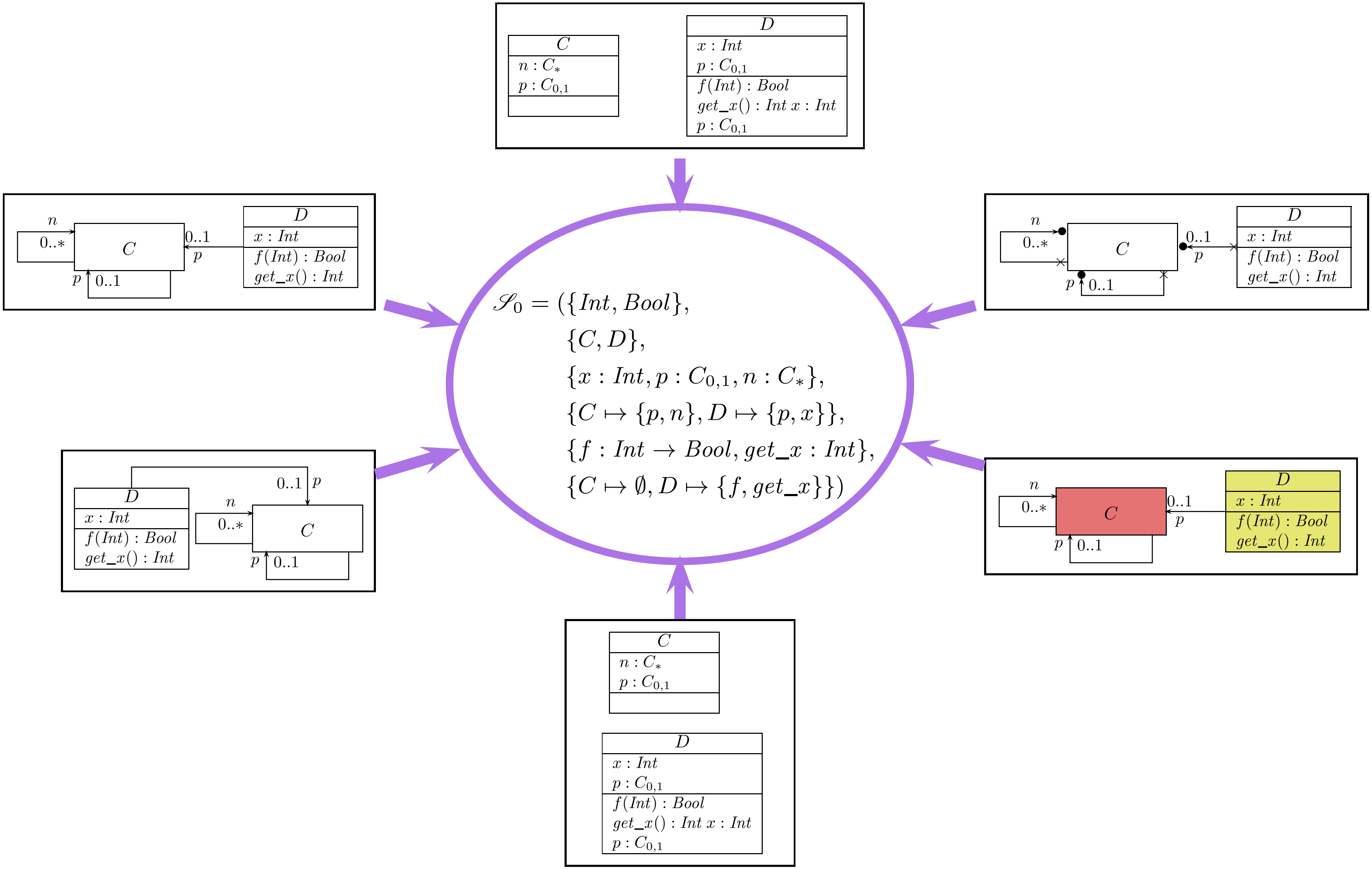
- $\mathscr{T} = \{Int, Bool\}$,
- $\mathscr{C} = \{C, D\}$,
- $V = \{x : Int,\ p : C_{0,1},\ n : C_*\}$,
- $atr = \{C \mapsto \{p, n\},\ D \mapsto \{x, p\}\}$,
- $F = \{f : Int \to Bool,\ get\_x : Int\}$,
- $\underbrace{mth = \{C \mapsto \emptyset,\ D \mapsto \{f, get\_x\}\}}_{\substack{mth(C) = \emptyset \\ mth(D) = \{f,\ get\_x\}}}$

$m : T_1, \dots, T_n \to T,\ n \geq 0$

$m : \to T$ if $n = 0$

$m : T$ if $n = 0$ also ok

Once Again: Concrete vs. Abstract Syntax



Class Diagrams at Work

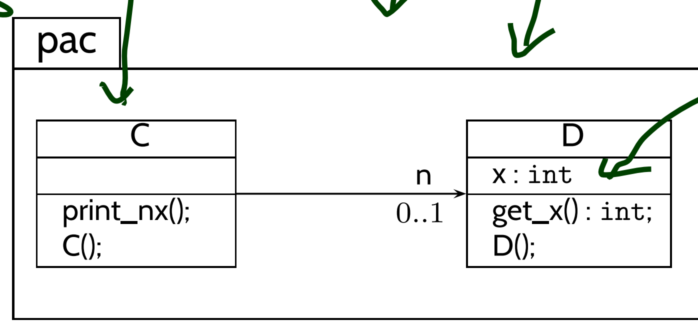
Visualisation of Implementation

- The class diagram syntax can be used to **visualise code**:
  provide rules which map (parts of) the code to class diagram elements.

```
package pac;

import pac.D;

public class C {

  public D n;

  public void print_nx() {
    System.out.printf(
      "%i\n", n.get_x() ); };

  public C() {};
}
```

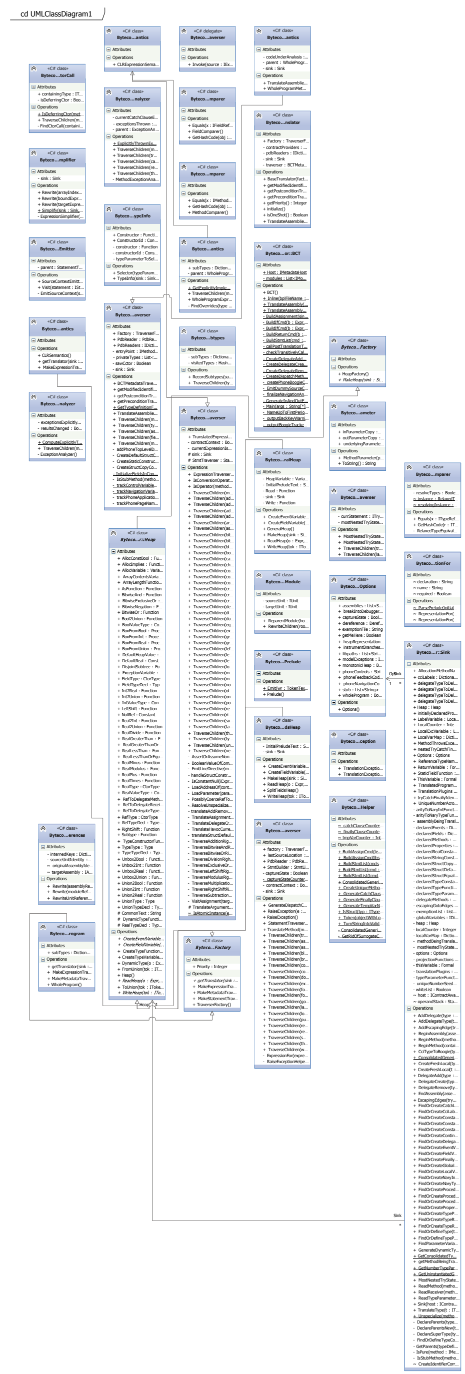
```
package pac;

import pac.C;

public class D {

  private int x;

  public int get_x()
    { return x; };

  public D() {};
}
```

package



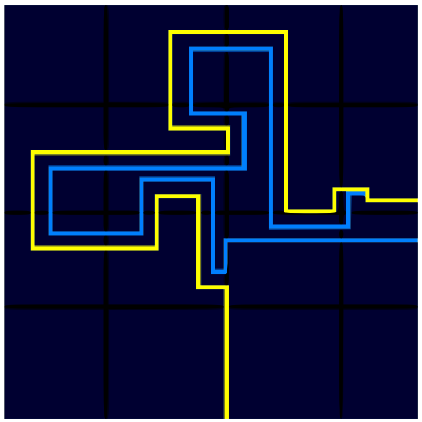
Visualisation of Implementation: (Useless) Example

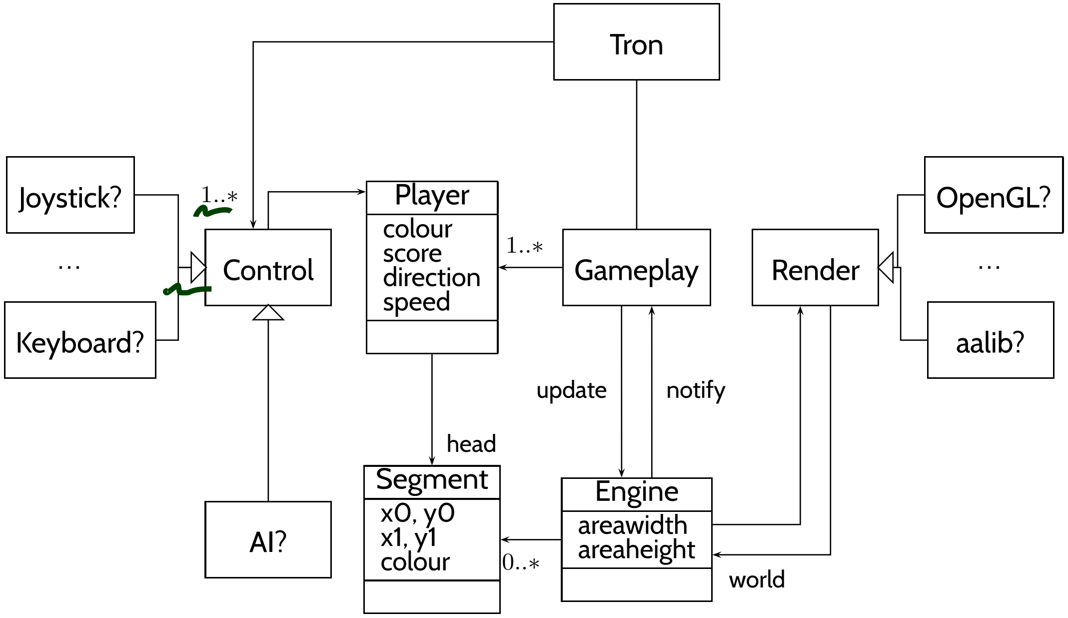
- open favourite IDE,
- open favourite **project**,
- press "**generate class diagram**"
- **wait**…**wait**…**wait**…



- ca. 35 classes,
- ca. 5,000 LOC C#

Visualisation of Implementation: (Useful) Example





- **Note**: a class **diagram** for visualisation may be partial.

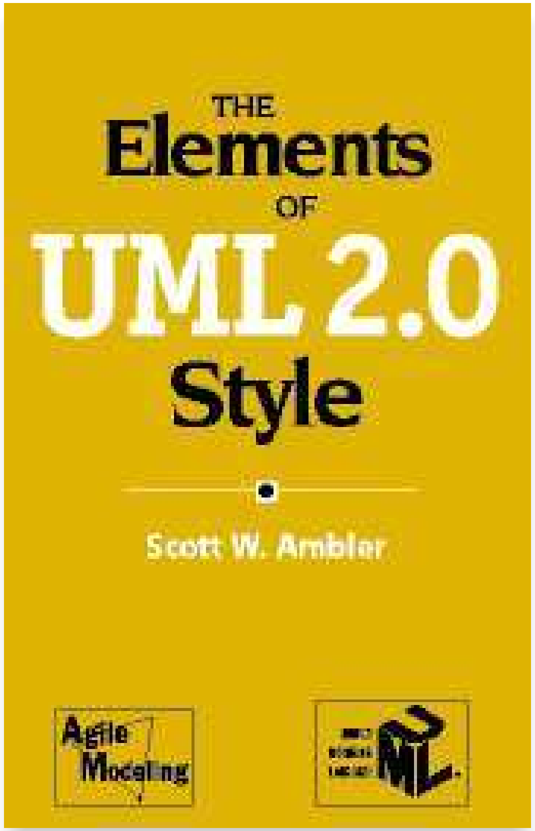
  → show only the **most relevant** classes and attributes (for the given purpose).

- **Note**: a signature can be defined by **a set of** class diagrams.

  → use multiple class diagrams with **a manageable** number of classes for different purposes.

- A diagram is **a good diagram** if (and only if?) it serves its **purpose**!

Literature Recommendation



(Ambler, 2005)

A More Abstract Class Diagram Semantics

Object System Structure

Definition. A Object System **Structure** of signature

$$\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, F, mth)$$

is a domain function $\mathscr{D}$ which assigns to each type a domain, i.e.

- $\tau \in \mathscr{T}$ is mapped to $\mathscr{D}(\tau)$,
- $C \in \mathscr{C}$ is mapped to an infinite set $\mathscr{D}(C)$ of **(object) identities**.
  - object identities of different classes are disjoint, i.e. $\forall C, D \in \mathscr{C} : C \neq D \rightarrow \mathscr{D}(C) \cap \mathscr{D}(D) = \emptyset$,
  - on object identities, (only) comparision for equality "$=$" is defined.
- C_* **and** $C_{0,1}$ for $C \in \mathscr{C}$ are mapped to $2^{\mathscr{D}(C)}$.

We use $\mathscr{D}(\mathscr{C})$ to denote $\bigcup_{C \in \mathscr{C}} \mathscr{D}(C)$; analogously $\mathscr{D}(\mathscr{C}_*)$.

Note: We identify **objects** and **object identities**,
because both uniquely determine each other (cf. OCL 2.0 standard).

Basic Object System Structure Example

Wanted: a structure for signature

Flower y: Flower y

$$\mathscr{S}_0 = (\{Int, Bool\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{p, x\}\},$$
$$\{f : Int \to Bool, get\_x : Int\}, \{C \mapsto \emptyset, D \mapsto \{f, get\_x\}\})$$

> A structure $\mathscr{D}$ maps
>
> - $\tau \in \mathscr{T}$ to **some** $\mathscr{D}(\tau)$, $C \in \mathscr{C}$ to **some** identities $\mathscr{D}(C)$ (infinite, pairwise disjoint),
> - C_* and $C_{0,1}$ for $C \in \mathscr{C}$ to $\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) = 2^{\mathscr{D}(C)}$.

$\mathscr{D}(Flower) = \{rose, daisy, lily\}$

$$\begin{aligned}
\mathscr{D}(Int) &= \mathbb{Z} \\
\mathscr{D}(C) &= \mathbb{N}^+ \times \{C\} = \{1_C, 2_C, 3_C, \dots\} \\
\mathscr{D}(D) &= \mathbb{N}^+ \times \{D\} = \{1_D, 2_D, 3_D, \dots\} \\
\mathscr{D}(C_{0,1}) = \mathscr{D}(C_*) &= 2^{\mathscr{D}(C)} \\
\mathscr{D}(D_{0,1}) = \mathscr{D}(D_*) &= 2^{\mathscr{D}(D)}
\end{aligned}$$

System State

partial function

attribute

Definition. Let $\mathscr{D}$ be a structure of $\mathscr{S} = (\mathscr{T}, \mathscr{C}, V, atr, F, mth)$.
A system state of $\mathscr{S}$ wrt. $\mathscr{D}$ is a type-consistent mapping

object identities

$$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*))).$$

value

That is, for each $u \in \mathscr{D}(C)$, $C \in \mathscr{C}$, if $u \in \mathrm{dom}(\sigma)$

- $\mathrm{dom}(\sigma(u)) = atr(C)$
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$
- $\sigma(u)(v) \in \mathscr{D}(D_*)$ if $v : D_{0,1}$ or $v : D_*$ with $D \in \mathscr{C}$

We call $u \in \mathscr{D}(\mathscr{C})$ alive in σ if and only if $u \in \mathrm{dom}(\sigma)$.

We use $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ to denote the set of all system states of $\mathscr{S}$ wrt. $\mathscr{D}$.

System State Examples

Flower $\qquad\qquad\qquad$ $y : Flower$ $\qquad\qquad\qquad$ y

$$\mathscr{S}_0 = (\{Int, Bool\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{p, x\}\},$$

$$\{f : Int \to Bool, get\_x : Int\}, \{C \mapsto \emptyset, D \mapsto \{f, get\_x\}\})$$

$$\mathscr{D}(Int) = \mathbb{Z}, \quad \mathscr{D}(C) = \{1_C, 2_C, 3_C, ...\}, \quad \mathscr{D}(D) = \{1_D, 2_D, 3_D, ...\}$$

$\mathscr{D}(Flower) = \{rose, daisy, lily\}$

A system state is a partial function $\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow (\mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}_*)))$ such that

- $\mathrm{dom}(\sigma(u)) = atr(C)$,
- $\sigma(u)(v) \in \mathscr{D}(\tau)$ if $v : \tau, \tau \in \mathscr{T}$,
- $\sigma(u)(v) \in \mathscr{D}(C_*)$ if $v : D_*$ or $v : D_{0,1}$ with $D \in \mathscr{C}$.

$\sigma_1 = \emptyset$

$$\sigma_2 = \{\boxed{1_C \mapsto \{y \mapsto rose, p \mapsto \{5_C\}, n \mapsto \emptyset\}},$$
$$\boxed{5_C \mapsto \{y \mapsto lily, p \mapsto \emptyset, n \mapsto \{1_C\}},$$
$$\boxed{4_D \mapsto \{x \mapsto 27, p \mapsto \{5_C\}}\}$$

Content

- **Class Diagrams**
  - concrete syntax,
  - abstract syntax,
  - class diagrams at work,
  - semantics: system states.
- **Object Diagrams**
  - concrete syntax,
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  - partial vs. complete,
  - object diagrams at work.
- **Proto-OCL**
  - syntax,
  - semantics,
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- Putting it All Together:
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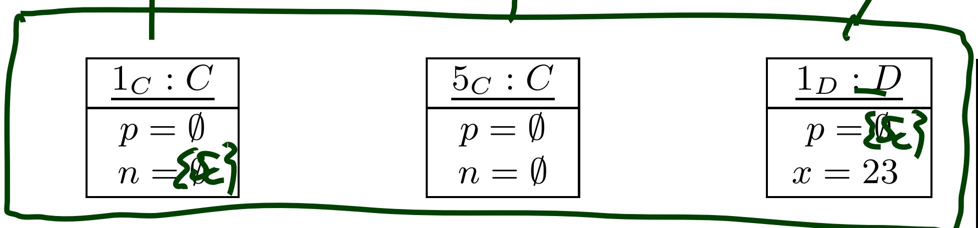
Object Diagrams

Object Diagrams

$$\mathscr{S}_0 = (\{Int, Bool\}, \{C, D\}, \{x : Int, p : C_{0,1}, n : C_*\}, \{C \mapsto \{p, n\}, D \mapsto \{p, x\}\},$$
$$\{f : Int \to Bool, get\_x : Int\}, \{C \mapsto \emptyset, D \mapsto \{f, get\_x\}\}), \qquad \mathscr{D}(Int) = \mathbb{Z}$$

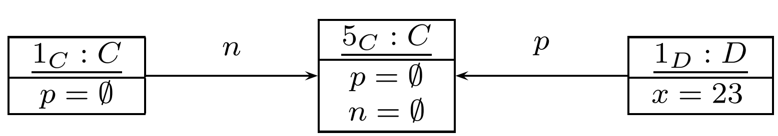
$$\sigma = \{\underbrace{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}}, \underbrace{5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}}, \underbrace{1_D \mapsto \{p \mapsto \{5_C\}, x \mapsto 23\}}\}$$

- We may **represent** σ graphically as follows:

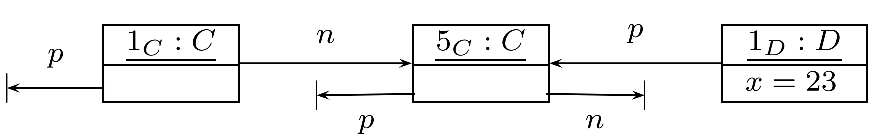


This is an **object diagram**.

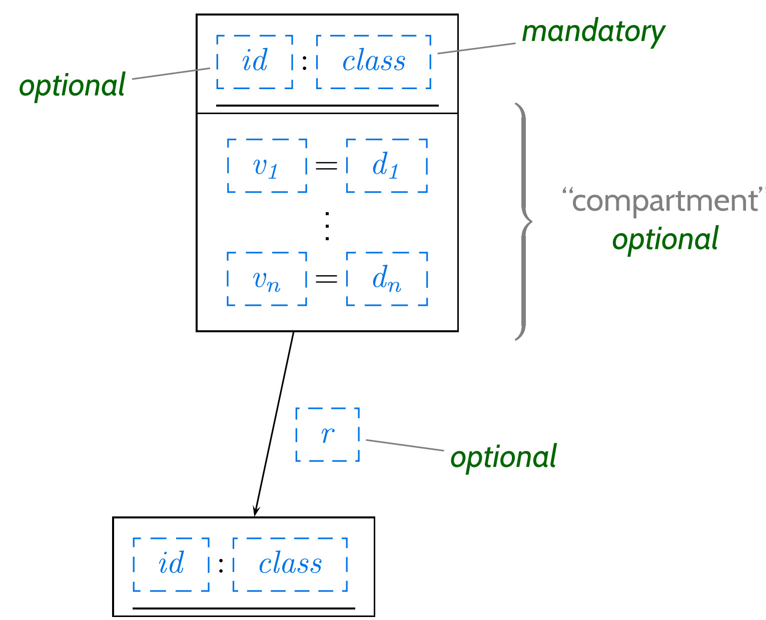
- Alternative notation:



- Alternative **non-standard** notation:



Concrete Syntax:



Special Case: Dangling Reference

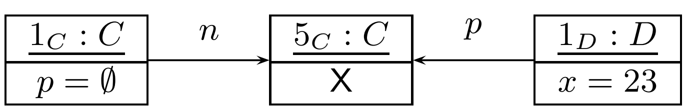
Definition.
Let $\sigma \in \Sigma_\mathscr{S}^\mathscr{D}$ be a system state and $u \in \mathrm{dom}(\sigma)$ an alive object of class C in σ.

We say $r \in atr(C)$ **is a dangling reference in** u if and only if $r : C_{0,1}$ or $r : C_*$ and u refers to a **non-alive** object via v, i.e.

$$\sigma(u)(r) \not\subseteq \mathrm{dom}(\sigma).$$

Example:

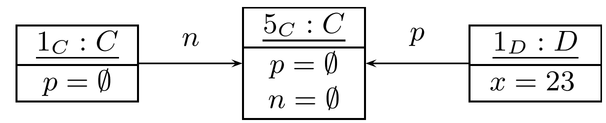
- $\sigma = \{\underbrace{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}}, \underbrace{1_D \mapsto \{p \mapsto \{5_C\}, x \mapsto 23\}}\}$ $5_C \notin \mathrm{dom}(\sigma)$
- Object diagram representation:



Partial vs. Complete Object Diagrams

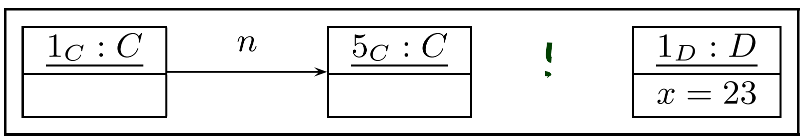
- By now we discussed "**object diagram represents system state**":

$$\begin{aligned}\{1_C &\mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\},\\ 5_C &\mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\},\\ 1_D &\mapsto \{p \mapsto \{5_C\}, x \mapsto 23\}\}\end{aligned} \quad \rightsquigarrow$$

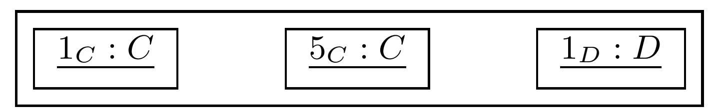


What about the other way round...?

- **Object diagrams** can be **partial**, e.g.

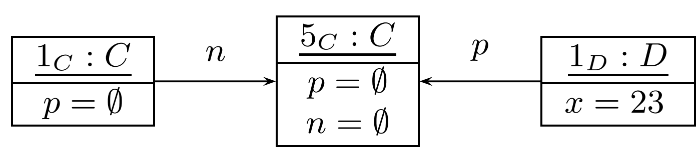


or



$\rightarrow$ we may omit information.

- Is the following object diagram **partial** or **complete**? (wrt. given signature $\mathcal{S}$)

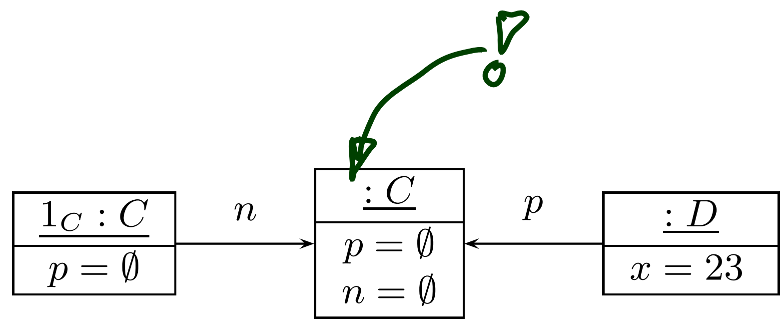


- If an object diagram
  - has values for **all** attributes of **all** objects in the diagram, and
  - if we **say that** it is meant to be complete

  then we can **uniquely** reconstruct a system state σ.

Special Case: Anonymous Objects

If the object diagram



is considered as **complete**, then it denotes the set of all system states

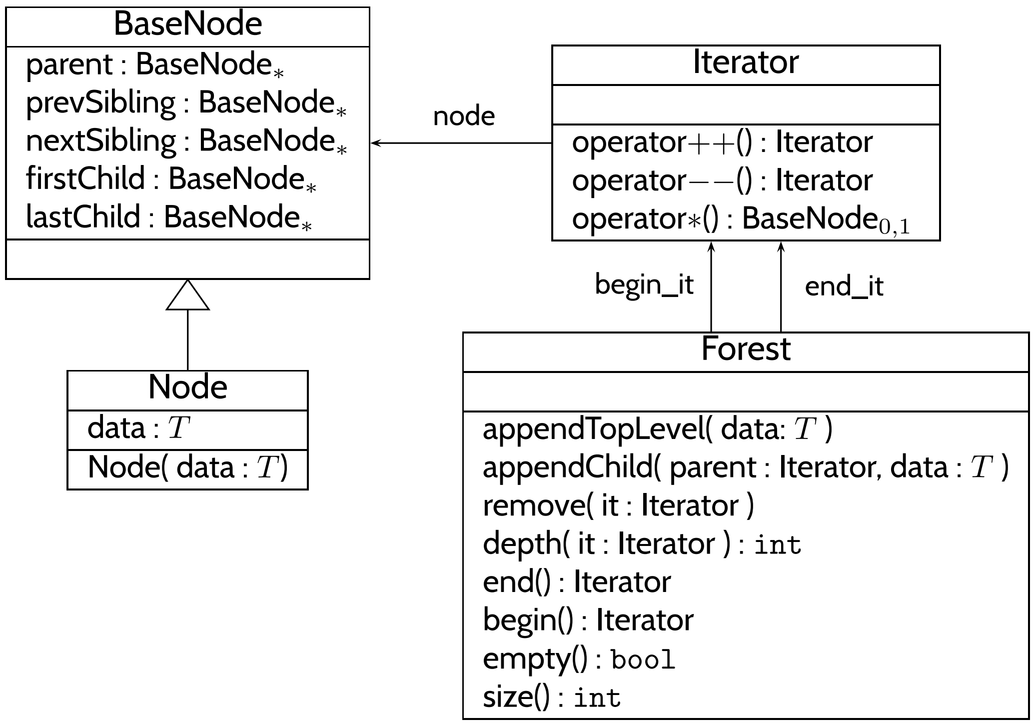
$$\{c_1 \mapsto \{p \mapsto \emptyset, n \mapsto \{c_2\}\}\}, c_2 \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, d \mapsto \{p \mapsto \{c_2\}, x \mapsto 23\}\}$$

where $\quad c \in \mathscr{D}(C), \quad d \in \mathscr{D}(D), \quad c \neq 1_C.$

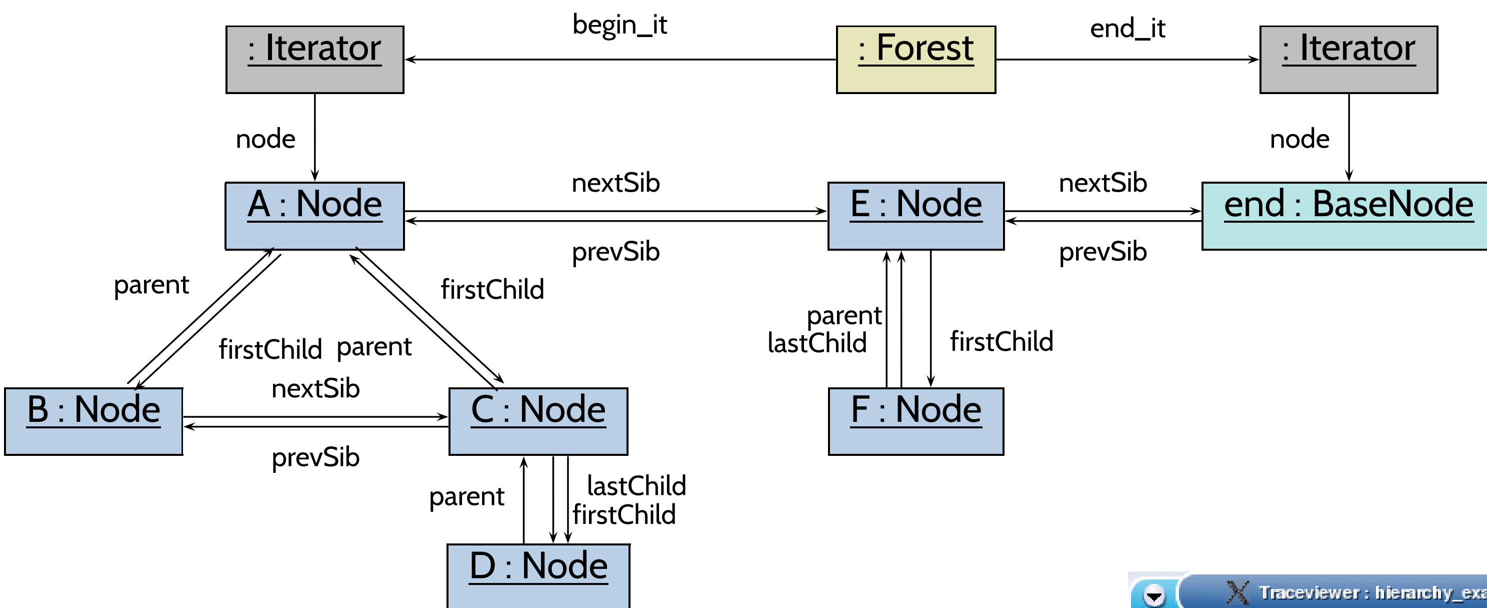
Intuition: different boxes represent different objects.

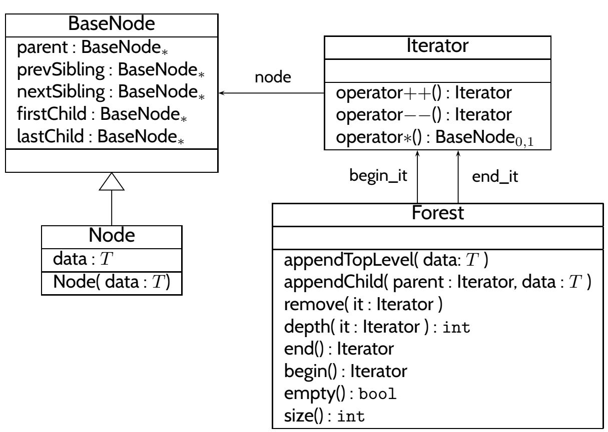
Object Diagrams at Work

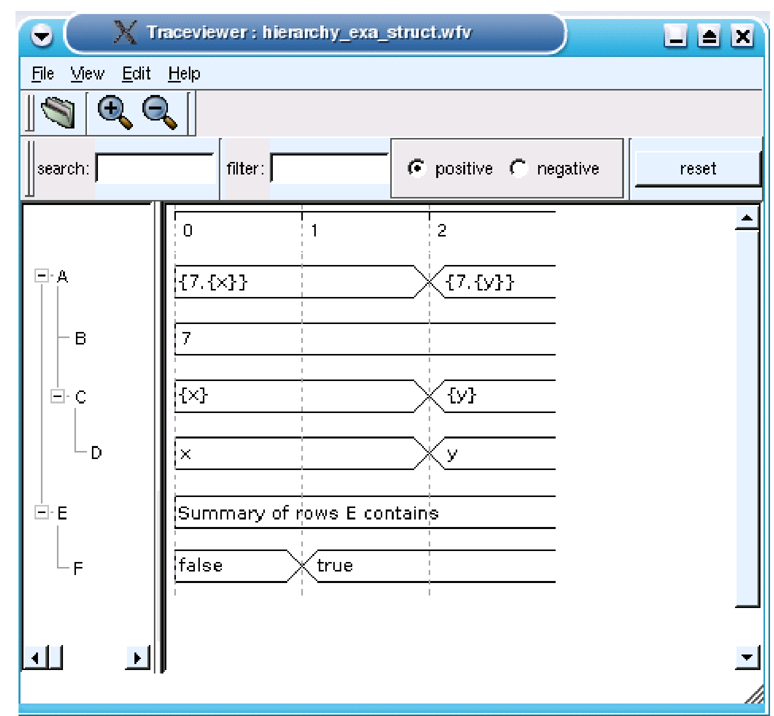
Example: Data Structure (Schumann et al., 2008)



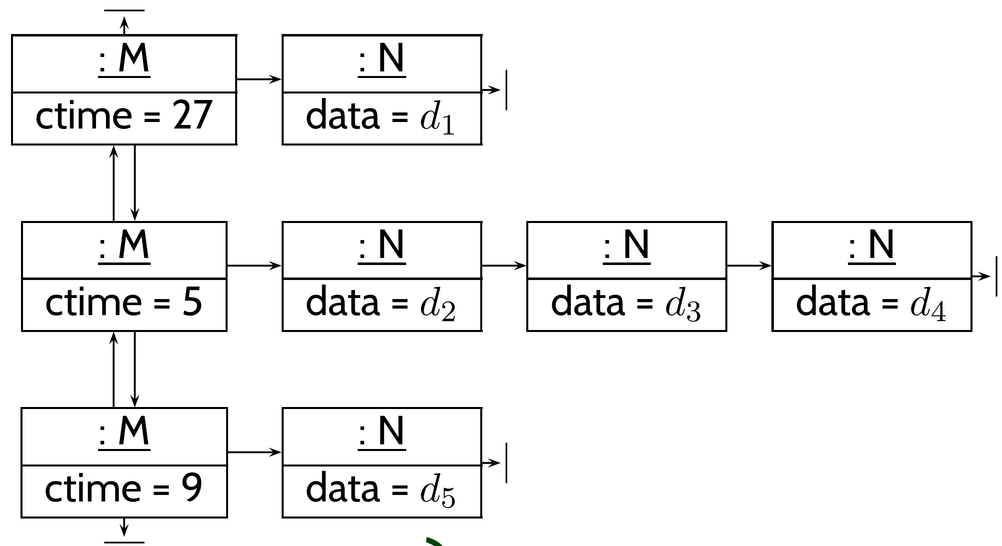
Example: Illustrative Object Diagram (Schumann et al., 2008)

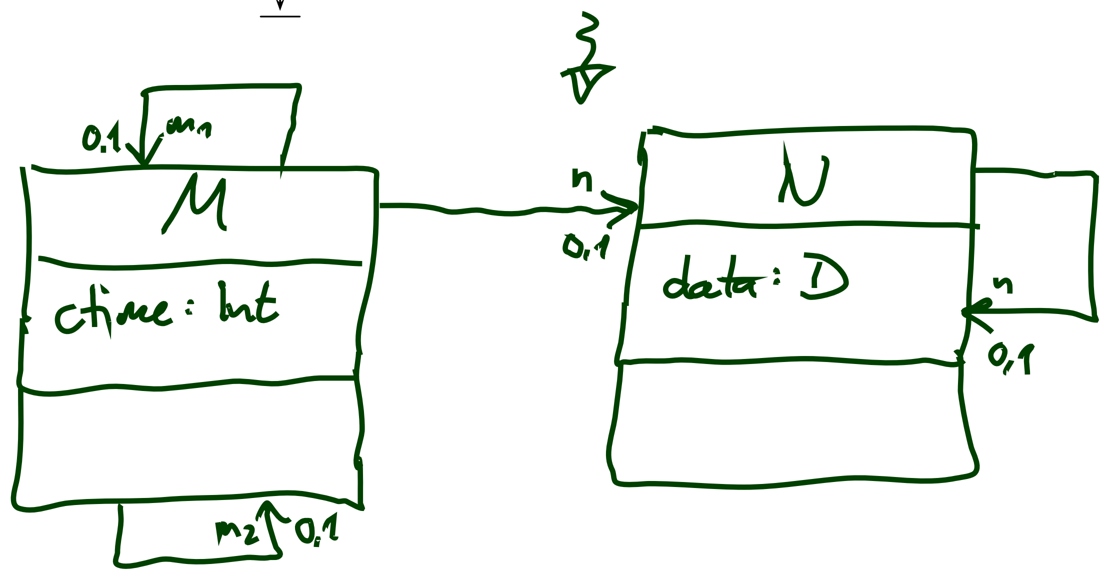






Object Diagrams for Analysis





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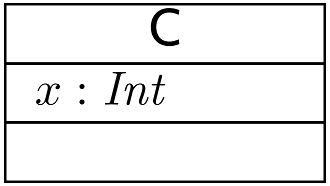
Towards Object Constraint Logic (OCL)
— “Proto-OCL” —

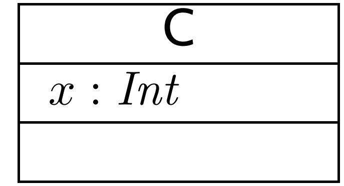
Constraints on System States



- **Example**: for all C-instances, x should never have the value 27.

$$\forall\, c \in allInstances_C \bullet x(c) \neq 27$$

Constraints on System States



- **Example**: for all C-instances, x should never have the value 27.

$$\forall\, c \in allInstances_C \bullet x(c) \neq 27$$

- **Proto-OCL Syntax** wrt. signature $(\mathscr{T}, \mathscr{C}, V, atr, F, mth)$, c is a **logical variable**, $C \in \mathscr{C}$:

$$\begin{array}{rlll}
F ::= & c & : \tau_C & \\
| & allInstances_C & : 2^{\tau_C}, \quad C \in \mathscr{C} & \\
| & v(F) & : \tau_C \to \tau_\perp, & \text{if } v : \tau \in atr(C), \quad \tau \in \mathscr{T} \\
| & v(F) & : \tau_C \to \tau_D, & \text{if } v : D_{0,1} \in atr(C) \\
| & v(F) & : \tau_C \to 2^{\tau_D}, & \text{if } v : D_* \in atr(C) \\
| & f(F_1, \dots, F_n) & : \tau_1 \times \dots \times \tau_n \to \tau, & \text{if } f : \tau_1 \times \dots \times \tau_n \to \tau \\
| & \forall\, c \in F_1 \bullet F_2 & : \tau_C \times 2^{\tau_C} \times \mathbb{B}_\perp \to \mathbb{B}_\perp &
\end{array}$$

C
$x : Int$

- **Example**: for all C-instances, x should never have the value 27.

$$\forall\, c \in allInstances_C \bullet x(c) \neq 27$$

- **Proto-OCL Syntax** wrt. signature $(\mathscr{T}, \mathscr{C}, V, atr, F, mth)$, c is a **logical variable**, $C \in \mathscr{C}$:

$$
\begin{array}{rlll}
F ::= & c & : \tau_C & \\
| & allInstances_C & : 2^{\tau_C}\,, \quad C \in \mathscr{C} & \\
| & v(F) & : \tau_C \to \tau_\perp, & \text{if } v : \tau \in atr(C),\ \tau \in \mathscr{T} \\
| & v(F) & : \tau_C \to \tau_D, & \text{if } v : D_{0,1} \in atr(C) \\
| & v(F) & : \tau_C \to 2^{\tau_D}, & \text{if } v : D_* \in atr(C) \\
| & f(F_1, \dots, F_n) & : \tau_1 \times \dots \times \tau_n \to \tau, & \text{if } f : \tau_1 \times \dots \times \tau_n \to \tau \quad (*) \\
| & \forall\, c \in F_1 \bullet F_2 & : \tau_C \times 2^{\tau_C} \times \mathbb{B}_\perp \to \mathbb{B}_\perp &
\end{array}
$$

- The formula above in **prefix normal form**: $\forall\, c \in allInstances_C \bullet \overset{f}{\neq} (\underbrace{x(c)}_{F_1}, \underbrace{27}_{F_2})$ $\quad (*)$

$\sigma : \mathscr{D}(\mathscr{C}) \nrightarrow (V \nrightarrow \mathscr{D}(\mathscr{T}) \cup \mathscr{D}(\mathscr{C}))$

Semantics

- **Proto-OCL Types:**

  disjoint union

  - $\mathcal{I}[\![\tau_C]\!] = \mathscr{D}(C) \,\dot\cup\, \{\bot\}, \quad \mathcal{I}[\![\tau_\bot]\!] = \mathscr{D}(\tau) \,\dot\cup\, \{\bot\}, \quad \mathcal{I}[\![2^{\tau_C}]\!] = \mathscr{D}(C_*) \,\dot\cup\, \{\bot\}$
  - $\mathcal{I}[\![\mathbb{B}_\bot]\!] = \{\mathit{true}, \mathit{false}\} \,\dot\cup\, \{\bot\}, \quad \mathcal{I}[\![\mathbb{Z}_\bot]\!] = \mathbb{Z} \,\dot\cup\, \{\bot\}$

- **Functions:**
  - We assume $f_\mathcal{I}$ given for each function symbol f ($\rightarrow$ in a minute).

- **Proto-OCL Semantics** (interpretation function):
  - $\mathcal{I}[\![c]\!](\sigma, \beta) = \beta(c)$ (assuming β is a type-consistent valuation of the logical variables),
  - $\mathcal{I}[\![\mathit{allInstances}_C]\!](\sigma, \beta) = \mathrm{dom}(\sigma) \cap \mathscr{D}(C)$,

    i.e. if $\mathcal{I}[\![F]\!](\sigma,\beta)$ is alive in σ

  - $\mathcal{I}[\![v(F)]\!](\sigma, \beta) = \begin{cases} \sigma(\mathcal{I}[\![F]\!](\sigma,\beta))(v) & \text{, if } \mathcal{I}[\![F]\!](\sigma,\beta) \in \mathrm{dom}(\sigma) \\ \bot & \text{, otherwise} \end{cases}$ (if not $v : C_{0,1}$)
  - $\mathcal{I}[\![v(F)]\!](\sigma, \beta) = \begin{cases} \sigma(u')(v) & \text{, if } \mathcal{I}[\![F]\!](\sigma,\beta) = \{u'\} \subseteq \mathrm{dom}(\sigma) \\ \bot & \text{, otherwise} \end{cases}$ (if $v : C_{0,1}$)
  - $\mathcal{I}[\![f(F_1, \dots, F_n)]\!](\sigma, \beta) = f_\mathcal{I}(\mathcal{I}[\![F_1]\!](\sigma,\beta), \dots, \mathcal{I}[\![F_n]\!](\sigma,\beta))$,
  - $\mathcal{I}[\![\forall\, c \in F_1 \bullet F_2]\!](\sigma, \beta) = \begin{cases} \mathit{true} & \text{, if } \mathcal{I}[\![F_2]\!](\sigma, \beta[c := u]) = \mathit{true} \text{ for all } u \in \mathcal{I}[\![F_1]\!](\sigma,\beta) \\ \mathit{false} & \text{, if } \mathcal{I}[\![F_2]\!](\sigma, \beta[c := u]) = \mathit{false} \text{ for some } u \in \mathcal{I}[\![F_1]\!](\sigma,\beta) \\ \bot & \text{, otherwise} \end{cases}$

Semantics Cont'd

- Proto-OCL is a **three-valued** logic: a formula evaluates to *true*, *false*, or $\bot$.
- **Example**: $\wedge_\mathcal{I}(\cdot,\cdot) : \{\mathit{true}, \mathit{false}, \bot\} \times \{\mathit{true}, \mathit{false}, \bot\} \to \{\mathit{true}, \mathit{false}, \bot\}$ is defined as follows:

x_1	*true*	*true*	*true*	*false*	*false*	*false*	$\bot$	$\bot$	$\bot$
x_2	*true*	*false*	$\bot$	*true*	*false*	$\bot$	*true*	*false*	$\bot$
$\wedge_\mathcal{I}(x_1, x_2)$	*true*	*false*	$\bot$	*false*	*false*	*false*	$\bot$	*false*	$\bot$

  We assume common logical connectives $\neg, \wedge, \vee, \dots$ with canonical 3-valued interpretation.

- **Example**: $+_\mathcal{I}(\cdot,\cdot) : (\mathbb{Z} \,\dot\cup\, \{\bot\}) \times (\mathbb{Z} \,\dot\cup\, \{\bot\}) \to \mathbb{Z} \,\dot\cup\, \{\bot\}$

$$+_\mathcal{I}(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{, if } x_1 \neq \bot \text{ and } x_2 \neq \bot \\ \bot & \text{, otherwise} \end{cases}$$

  We assume common arithmetic operations $-, /, *, \dots$
  and relation symbols $>, <, \leq, \dots$ with **monotone** 3-valued interpretation.

- And we assume the special unary function symbol $\mathit{isUndefined}$:

$$\mathit{isUndefined}_\mathcal{I}(x) = \begin{cases} \mathit{true} & \text{, if } x = \bot, \\ \mathit{false} & \text{, otherwise} \end{cases}$$

  $\mathit{isUndefined}_\mathcal{I}$ is **definite**: it never yields $\bot$.

Example: Evaluate Formula for System State

σ : $\underline{1_C : \mathsf{C}}$ | $x = 13$

$\mathcal{S}$: C | $x : Int$

$F = \forall\, c \in allInstances_C \bullet x(c) \neq 27$

- Recall **prefix notation**: $\forall\, c \in allInstances_C \bullet \neq(x(c), 27)$

  Note: $\neq$ is a binary function symbol, 27 is a 0-ary function symbol.

- **Example**:

  $\mathcal{I}[\![\forall\, c \in allInstances_C \bullet \neq(x(c), 27)]\!](\sigma, \emptyset) =$ *true*, because…

  $\mathcal{I}[\![\neq(x(c), 27)]\!](\sigma, \beta)$, $\quad \beta := \emptyset[c := 1_C] = \{c \mapsto 1_C\}$

  $= \neq_{\mathcal{I}}(\ \mathcal{I}[\![x(c)]\!](\sigma, \beta),\ \mathcal{I}[\![27]\!](\sigma, \beta)\ )$

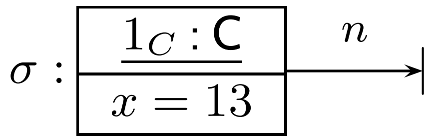
  $= \neq_{\mathcal{I}}(\ \sigma(\ \mathcal{I}[\![c]\!](\sigma, \beta)\ )(x),\ 27_{\mathcal{I}}\ )$

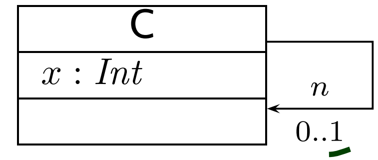
  $= \neq_{\mathcal{I}}(\ \sigma(\ \beta(c)\ )(x),\ 27_{\mathcal{I}}\ )$

  $= \neq_{\mathcal{I}}(\ \sigma(\ 1_C\ )(x),\ 27_{\mathcal{I}}\ )$

  $= \neq_{\mathcal{I}}(\ 13,\ 27\ ) =$ *true* $\qquad$ …and 1_C is the only C-object in σ: $\mathcal{I}[\![allInstances_C]\!](\sigma, \emptyset) = \{1_C\}$.

More Interesting Example





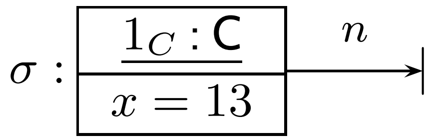
$$\forall c \overset{\in allInstances_C}{\cancel{\in C}} \bullet \underbrace{x(n(c)) \neq 27}_{\neq(x(n(c)),\,27)}$$

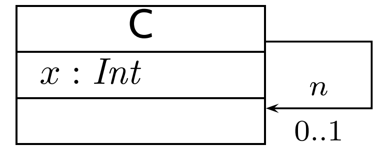
- Similar to the previous slide, we need the value of

$\beta = \{c \mapsto 1_C\}$

$$\underbrace{\sigma\,(\,\underbrace{\sigma(\,\underbrace{\mathcal{I}[\![c]\!](\sigma,\beta)}_{\beta(c)=1_C}\,)(n)}_{\sigma(1_C)(n)=\emptyset}\,)\,(x)}_{=\perp}$$

More Interesting Example





$\in allInstances_C$

$$\forall\, c \bullet x(n(c)) \neq 27$$

- Similar to the previous slide, we need the value of

$$\sigma\ (\ \sigma(\ \mathcal{I}[\![c]\!](\sigma, \beta)\ )(n)\ )\ (x)$$

  - $\mathcal{I}[\![c]\!](\sigma, \beta) = \beta(c) = 1_C$
  - $\sigma(\ \mathcal{I}[\![c]\!](\sigma, \beta)\ )(n) = \sigma(\ 1_C\ )(n) = \emptyset$
  - $\sigma\ (\ \sigma(\ \mathcal{I}[\![c]\!](\sigma, \beta)\ )(n)\ )\ (x) = \bot$

by the following rule:

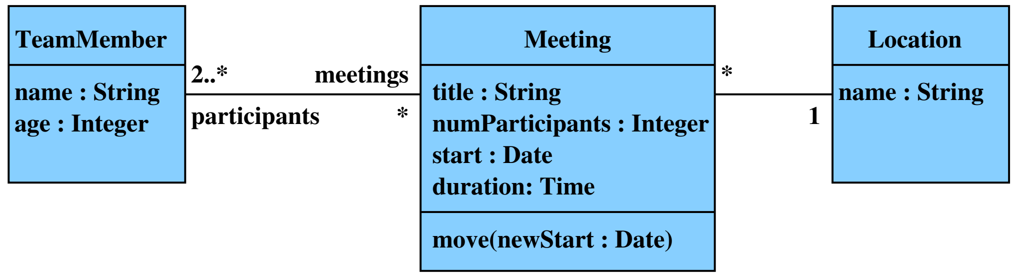
$$\mathcal{I}[\![v(F)]\!](\sigma, \beta) = \begin{cases} \sigma(u')(v) & \text{, if } \mathcal{I}[\![F]\!](\sigma, \beta) = \{u'\} \subseteq \mathrm{dom}(\sigma) \\ \bot & \text{, otherwise} \end{cases} \qquad (\text{if } v : C_{0,1})$$

Object Constraint Language (OCL)

OCL is the same – just with less readable (?) syntax.

Literature: (OMG, 2006; Warmer and Kleppe, 1999).

Examples *(from lecture “Softwaretechnik 2008”)*



- **context** Meeting
  - **inv:** self.participants->size() = self.numParticipants
- **context** Location
  - **inv:** name="Lobby" **implies** meeting->isEmpty()

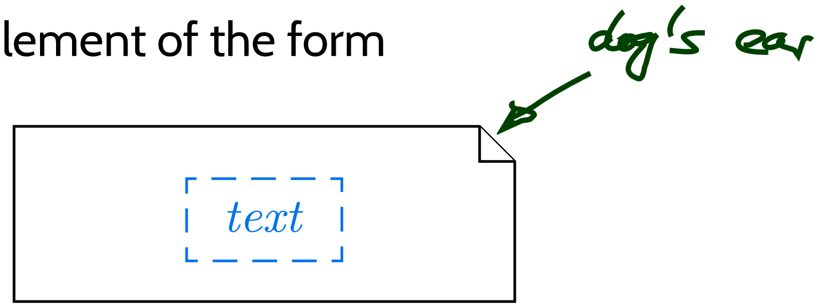
Prof. Dr. P. Thiemann, http://proglang.informatik.uni-freiburg.de/teaching/swt/2008/

$\forall\, self \in allInstances_{Meeting} \bullet size(participants(self)) = numParticipants(self)$

$\forall\, self \in allInstances_{Location} \bullet name(self) = \text{“Lobby”} \Rightarrow isEmpty(meeting(self))$

Where To Put OCL Constraints?

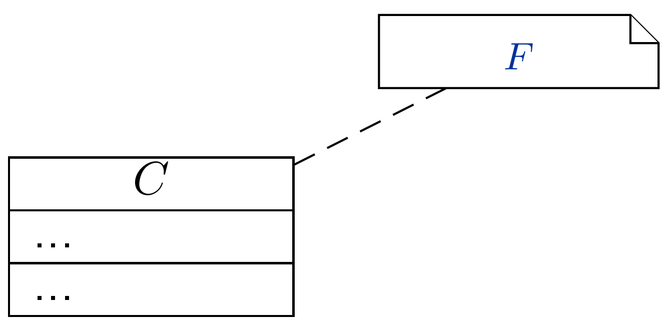
- **Notes**: A UML **note** is a diagram element of the form



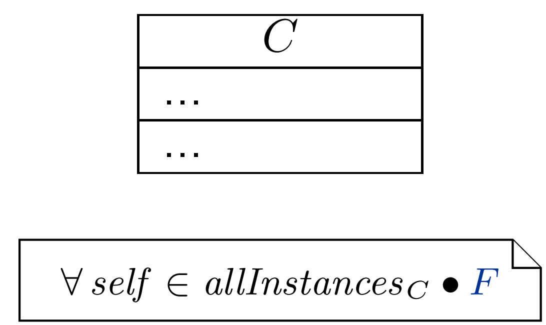
  $text$ can principally be **everything**, in particular **comments** and **constraints**.

  Sometimes, content is **explicitly classified** for clarity:

- Conventions:



stands for



Content

- **Class Diagrams**
  - concrete syntax,
  - abstract syntax,
  - class diagrams at work,
  - semantics: system states.
- **Object Diagrams**
  - concrete syntax,
  - dangling references,
  - partial vs. complete,
  - object diagrams at work.
- **Proto-OCL**
  - syntax,
  - semantics,
  - Proto-OCL vs. OCL.
- Putting it All Together:
  Proto-OCL vs. Software

Putting It All Together

Modelling Structure with Class Diagrams

Definition. **Software** is a finite description S of a (possibly infinite) set $[\![S]\!]$ of (finite or infinite) computation paths of the form $\sigma_0 \xrightarrow{\alpha_1} \sigma_1 \xrightarrow{\alpha_2} \sigma_2 \cdots$ where

- $\sigma_i \in \Sigma$, $i \in \mathbb{N}_0$, is called **state** (or **configuration**), and
- $\alpha_i \in A$, $i \in \mathbb{N}_0$, is called **action** (or **event**).

The (possibly partial) function $[\![\,\cdot\,]\!] : S \mapsto [\![S]\!]$ is called **interpretation** of S.

- The set of **states** Σ could be the set of **system states** as defined by a class diagram, e.g.

$$\Sigma := \Sigma_{\mathscr{S}}^{\mathscr{D}} \qquad\qquad \mathscr{S} : \begin{array}{|l|}\hline \quad\text{C} \\ \hline x : \mathit{Int} \\ \hline \\ \hline\end{array}$$

- A corresponding **computation path** of a software S could be

$$\begin{array}{|c|}\hline \underline{27_C : \text{C}} \\ \hline x = 0 \\ \hline\end{array} \xrightarrow{\tau} \begin{array}{|c|}\hline \underline{27_C : \text{C}} \\ \hline x = 1 \\ \hline\end{array} \xrightarrow{\tau} \begin{array}{|c|}\hline \underline{27_C : \text{C}} \\ \hline x = 3 \\ \hline\end{array} \xrightarrow{\tau} \begin{array}{|c|}\hline \underline{27_C : \text{C}} \\ \hline x = 4 \\ \hline\end{array} \xrightarrow{\tau} \ldots$$

- If a requirement is formalised by the Proto-OCL constraint

$$F = \forall\, c \in \mathit{allInstances}_C \bullet x(c) < 4$$

then S **does not** satisfy the requirement.

More General: Software vs. Proto-OCL

- Let $\mathscr{S}$ be an **object system signature** and $\mathscr{D}$ a **structure**.
- Let S be a **software** with
  - states $\Sigma \subseteq \Sigma_{\mathscr{S}}^{\mathscr{D}}$, and
  - **computation paths** $[\![S]\!]$.
- Let F be a Proto-OCL constraint over $\mathscr{S}$.
- We say $[\![S]\!]$ **satisfies** F, denoted by $[\![S]\!] \models F$, if and only if for all

$$\sigma_0 \xrightarrow{\alpha_1} \sigma_1 \xrightarrow{\alpha_2} \sigma_2 \cdots \in [\![S]\!]$$

  and all $i \in \mathbb{N}_0$,

$$\mathcal{I}[\![F]\!](\sigma_i, \emptyset) = \textit{true}.$$

- We say $[\![S]\!]$ **does not satisfy** F, denoted by $[\![S]\!] \not\models F$, if and only if there exists $\sigma_0 \xrightarrow{\alpha_1} \sigma_1 \xrightarrow{\alpha_2} \sigma_2 \cdots \in [\![S]\!]$ and $i \in \mathbb{N}_0$, such that $\mathcal{I}[\![F]\!](\sigma_i, \emptyset) = \textit{false}$.
- **Note**: $\neg([\![S]\!] \not\models F)$ does not imply $[\![S]\!] \models F$.

Tell Them What You've Told Them...

- **Class Diagrams** can be used to **graphically**
  - visualise code,
  - define an **object system** ~~structure~~ Sig. $\mathscr{S}$.
- An **Object System** ~~Structure~~ Sig. $\mathscr{S}$ (together with a structure $\mathscr{D}$)
  - defines a set of **system states** $\Sigma_{\mathscr{S}}^{\mathscr{D}}$.
- A **System State** $\sigma \in \Sigma_{\mathscr{S}}^{\mathscr{D}}$
  - can be **visualised** by an **object diagram**.
- **Proto-OCL** constraints can be evaluated on **system states**.
- A **software** over $\Sigma_{\mathscr{S}}^{\mathscr{D}}$ satisfies a Proto-OCL constraint F if and only if F evaluates to *true* in all system states of all the software's computation paths.

References

References

Ambler, S. W. (2005). *The Elements of UML 2.0 Style*. Cambridge University Press.

Kopetz, H. (2011). What I learned from Brian. In Jones, C. B. et al., editors, *Dependable and Historic Computing*, volume 6875 of *LNCS*. Springer.

Lovins, A. B. and Lovins, L. H. (2001). *Brittle Power - Energy Strategy for National Security*. Rocky Mountain Institute.

Ludewig, J. and Lichter, H. (2013). *Software Engineering*. dpunkt.verlag, 3. edition.

OMG (2006). Object Constraint Language, version 2.0. Technical Report formal/06-05-01.

Schumann, M., Steinke, J., Deck, A., and Westphal, B. (2008). Traceviewer technical documentation, version 1.0. Technical report, Carl von Ossietzky Universität Oldenburg und OFFIS.

Warmer, J. and Kleppe, A. (1999). *The Object Constraint Language*. Addison-Wesley.