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## Tutorial for Cyber-Physical Systems - Hybrid Models

### Exercise Sheet 2

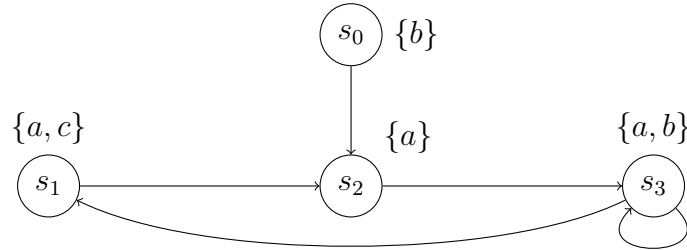


Figure 1: A labeled state transition system with no initial states over the atomic propositions  $\{a, b, c\}$ .

#### Exercise 1: States satisfying LTL formulae

Consider the LSTS from Figure 1 and the following LTL formulae. For each formula, provide the maximal set of initial states such that the formula is satisfied in the corresponding LSTS (if there is no initial state, give the empty set).

- |                                  |  |   |
|----------------------------------|--|---|
| (a) $a \wedge \mathcal{X}b$      | (b) $\mathcal{X}c$                     | (c) $\mathcal{X}\mathcal{X}c$                 |
| (d) $a \mathcal{U} b$            | (e) $b \mathcal{U} a$                  | (f) $b \mathcal{U} \mathcal{G}a$              |
| (g) $a \mathcal{U} \mathcal{G}b$ | (h) $\neg(a \mathcal{U} \mathcal{G}b)$ | (i) $(\mathcal{F}c) \mathcal{U} \mathcal{G}a$ |
| (j) $\mathcal{F}\mathcal{G}a$    | (k) $\mathcal{G}\mathcal{F}b$          | (l) $\mathcal{G}\mathcal{F}c$                 |

#### Exercise 2: States satisfying CTL formulae

Consider the LSTS from Figure 1 and the following CTL formulae. For each formula, provide the maximal set of initial states such that the formula is satisfied in the corresponding LSTS (if there is no initial state, give the empty set).

- |  |  |   |
|--|--|---|
| (a) $a \wedge \mathbf{E}\mathcal{X}b$                  | (b) $\mathbf{E}\mathcal{X}c$                               | (c) $\mathbf{A}\mathcal{X}\mathbf{E}\mathcal{X}c$                             |
| (d) $\mathbf{A}(a \mathcal{U} b)$                      | (e) $\mathbf{A}(b \mathcal{U} a)$                          | (f) $\mathbf{A}(b \mathcal{U} \mathbf{A}\mathcal{G}a)$                        |
| (g) $\mathbf{A}(a \mathcal{U} \mathbf{E}\mathcal{G}b)$ | (h) $\neg\mathbf{E}(a \mathcal{U} \mathbf{E}\mathcal{G}b)$ | (i) $\mathbf{A}((\mathbf{E}\mathcal{F}c) \mathcal{U} \mathbf{A}\mathcal{G}a)$ |
| (j) $\mathbf{A}\mathcal{F}\mathbf{A}\mathcal{G}a$      | (k) $\mathbf{A}\mathcal{G}\mathbf{A}\mathcal{F}b$          | (l) $\mathbf{A}\mathcal{G}\mathbf{E}\mathcal{F}c$                             |

### Exercise 3: Stating properties in LTL

Consider a lift system that services  $N$  floors numbered 0 through  $N - 1$ . Assume atomic proposition  $door(i)$  indicates that the doors on the  $i$ -th floor are open,  $lift(i)$  indicates that the lift is at floor  $i$ , and  $req(i)$  indicates that the request button at floor  $i$  was pressed and is lit. In the lift cabin there are  $N$  buttons for the floors and  $send(i)$  indicates that the  $i$ -th send button was pressed and is lit.

State the following properties in LTL.

- (a) The floor doors are never open if the cabin is not present at that floor.
- (b) A requested floor will be served at some time.
- (c) The lift returns to floor 0 infinitely often.
- (d) The lift does not move unless some request or send button was pressed.

### Exercise 4: Equivalence of LTL formulae

Consider the following potential equivalences between LTL formulae. If the equivalence holds, provide a proof. If it does not hold, provide a counterexample.

(a)  $(\mathcal{G}\varphi_1) \wedge (\mathcal{G}\varphi_2) \stackrel{?}{\equiv} \mathcal{G}(\varphi_1 \wedge \varphi_2)$

(b)  $(\mathcal{G}\varphi_1) \vee (\mathcal{G}\varphi_2) \stackrel{?}{\equiv} \mathcal{G}(\varphi_1 \vee \varphi_2)$

*Hint:* To show  $\psi_1 \equiv \psi_2$  you should show  $\pi \models \psi_1 \Leftrightarrow \pi \models \psi_2$  for any path  $\pi$ .