Exercise 1: Postcondition
We say that $\text{post}$ distributes over the connective $\odot$ wrt. the first argument if the following equation holds.

$$\text{post}(\varphi_1 \odot \varphi_2, \rho) = \text{post}(\varphi_1, \rho) \odot \text{post}(\varphi_2, \rho)$$

We say that $\text{post}$ distributes over the connective $\odot$ wrt. the second argument if the following equation holds.

$$\text{post}(\varphi, \rho_1 \odot \rho_2) = \text{post}(\varphi, \rho_1) \odot \text{post}(\varphi, \rho_2)$$

Determine for $\odot \in \{\land, \lor, \to\}$ if $\text{post}$ distributes over $\odot$ wrt. the first argument or wrt. the second argument.
Give a proof for each positive answer, give a counterexample for each negative answer.

Exercise 2: Reachability
Consider the following integer program with input variables $i$ and $j$.

$$\ell_0 : \ x := i;$$
$$\ell_1 : \ y := j;$$
$$\ell_2 : \ \text{while } x \neq 0 \text{ do } \{$$
$$\ell_3 : \quad x := x - 1;$$
$$\ell_4 : \quad y := y - 1;$$
$$\}$$
$$\ell_6 : \ \assert(i = j \to y = 0);$$

(a) Compute the set of reachable states $\varphi_{\text{reach}}$.

*Hint:* If you only apply the $\text{post}$ operator, your algorithm will not terminate. You need to find a relation between all variables which is true before and after each loop iteration (a loop invariant). Then use this to “jump over the loop”.

(b) Is the program safe?