



## Tutorial for Cyber-Physical Systems - Hybrid Models

### Exercise Sheet 11

#### Exercise 1: Counterexample-guided discovery of predicates

Consider the following program.

$$\varphi_{\text{init}} \equiv pc = \ell_0$$

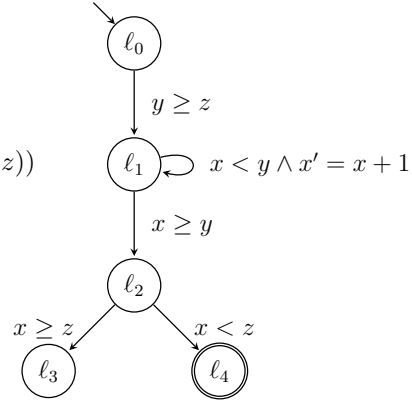
$$\rho_1 \equiv (\text{move}(\ell_0, \ell_1) \wedge y \geq z \wedge \text{skip}(x, y, z))$$

$$\rho_2 \equiv (\text{move}(\ell_1, \ell_1) \wedge x + 1 \leq y \wedge x' = x + 1 \wedge \text{skip}(y, z))$$

$$\rho_3 \equiv (\text{move}(\ell_1, \ell_2) \wedge x \geq y \wedge \text{skip}(x, y, z))$$

$$\rho_4 \equiv (\text{move}(\ell_2, \ell_3) \wedge x \geq z \wedge \text{skip}(x, y, z))$$

$$\rho_5 \equiv (\text{move}(\ell_2, \ell_4) \wedge x + 1 \leq z \wedge \text{skip}(x, y, z))$$



Let  $\text{Preds}_{pc}$  be the set of all predicates on the program counter.

$$\text{Preds}_{pc} = \{pc = \ell_1, pc = \ell_2, pc = \ell_3, pc = \ell_4, pc = \ell_5\}$$

- (a) Given the path  $\rho_1\rho_2\rho_3\rho_5$ , provide a set of predicates  $\text{Preds}$  such that  $\text{Preds} \cup \text{Preds}_{pc}$  is sufficient to prove safety of the program, i.e., every abstract state returned by  $\text{ABSTREACH}(\text{Preds} \cup \text{Preds}_{pc})$  is disjoint from  $\varphi_{\text{err}}$  (the set of error states  $\varphi_{\text{err}}$  is  $pc = \ell_4$ ).

Show that the predicates returned by your algorithm are sufficient to prove safety of the program by providing the abstract reachability graph.

- (b) On page 31 of the handbook article you can find the (general) function  $\text{REFINEPATH}$  which is used in the function  $\text{ABSTREFINELoop}$  and returns a set of predicates  $\text{Preds}$  given a path  $\rho_1, \dots, \rho_n$ .

Let us implement  $\text{REFINEPATH}$  using the following idea:

Let  $\varphi_0 := \varphi_{\text{init}}$  and for the other predicates use the result from the application of  $\text{post}$  (e.g.,  $\varphi_1 := \text{post}(\varphi_0, \rho_1)$ ).

Observe that the predicates satisfy the required constraints of  $\text{REFINEPATH}$ .

Compute  $\text{Preds}$  for the above program and path  $\rho_1\rho_2\rho_3\rho_5$  using this algorithm.

Are the predicates sufficient to prove safety?

- (c) Imagine we had a smart implementation for  $\text{REFINEPATH}$  to find predicates sufficient to prove safety of any safe program. What are the implications? What can you conclude about the existence of such an algorithm?