



Tutorial for Program Verification Exercise Sheet 3

Exercise 1: Formalization in first-order logic 2 Points

Express the following declarative sentences in first-order logic; in each case state what your respective constant, function, and predicate symbols stand for:

- (a) Whatever goes upon four legs, or has wings, is a friend.
- (b) No animal shall kill any other animal.
- (c) All animals are equal, but some animals are more equal than others.
- (d) The array a , whose indices and values are integers, is sorted between position 0 and position l .

Exercise 2: Quantifiers 2 Points

- (a) Show that the following first-order logic formula is not valid.

$$((\forall x. P(x)) \rightarrow Q) \rightarrow (\forall x. P(x) \rightarrow Q)$$

- (b) Is the other direction of the implication (s. below) valid?

$$(\forall x. P(x) \rightarrow Q) \rightarrow ((\forall x. P(x)) \rightarrow Q)$$

A short argument is sufficient.

Exercise 3: Minimal unsatisfiable core 2 Points

Definition (Minimal unsatisfiable core) Let Γ be a finite set of formulas such that the conjunction $\bigwedge_{\phi \in \Gamma} \phi$ is unsatisfiable. A subset $\Gamma' \subseteq \Gamma$ is called *unsatisfiable core* of Γ if $\bigwedge_{\phi \in \Gamma'} \phi$ is also unsatisfiable. An unsatisfiable core Γ' is called *minimal unsatisfiable core* if for each proper subset $\Gamma'' \subsetneq \Gamma'$ the conjunction $\bigwedge_{\phi \in \Gamma''} \phi$ is satisfiable.

- (a) Give a minimal unsatisfiable core for the following set of formulas.

$$\{ \neg(X \rightarrow \neg Z), \quad Y \rightarrow \neg U, \quad X \rightarrow Y, \quad X, \quad Z \rightarrow U \}$$

- (b) Is the minimal unsatisfiable core of a set of formulas unique? (Are there sets of formulas $\Gamma, \Gamma_1, \Gamma_2$ such that $\Gamma_1 \neq \Gamma_2$ but both Γ_1 and Γ_2 are minimal unsatisfiable cores of Γ ?)