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Tutorial for Program Verification
Exercise Sheet 8

Exercise 1: Precondition function 1 Point

We use $pre(\varphi, \rho)$ to denote the predecessor states from a set of states φ under a transition relation ρ. In other words, $pre(\varphi, \rho)$ is the biggest set of states such that after executing ρ we *can* arrive at a state in φ.

(a) Write down a formula that describes $pre(\varphi, \rho)$.

(b) In Exercise 4 on Exercise Sheet 7 we used the following formula to describe $pre(\varphi, \rho)$:

$$\neg wp(\neg\varphi, \rho)$$

Substitute the definition of wp and simplify the formula by eliminating negations. You should obtain the same formula as in part (a).

(c) What is, intuitively speaking, the difference between pre and wp?

(d) Give formulas $\varphi_1, \varphi_2, \varphi_3, \rho_1, \rho_2, \rho_3$ such that the claims below hold.
(We write $\subseteq$ for $\implies$ here.)

$$wp(\varphi_1, \rho_1) \not\subseteq pre(\varphi_1, \rho_1)$$

$$wp(\varphi_2, \rho_2) \not\supseteq pre(\varphi_2, \rho_2)$$

$$wp(\varphi_3, \rho_3) = pre(\varphi_3, \rho_3)$$

Exercise 2: Predicate transformers 1 Point

We consider two variables x, y over the binary domain $\{0, 1\}$.

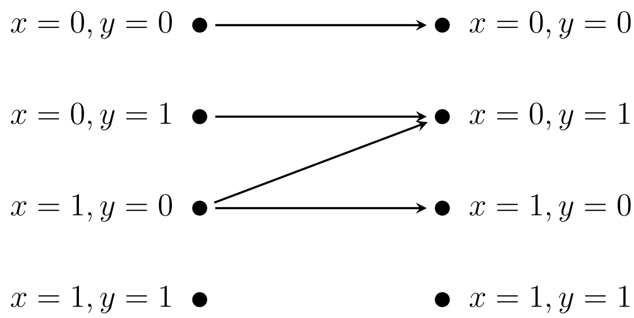
(a) The following diagram shows the four possible states on the left and on the right.

$x = 0, y = 0$ •	• $x = 0, y = 0$
$x = 0, y = 1$ •	• $x = 0, y = 1$
$x = 1, y = 0$ •	• $x = 1, y = 0$
$x = 1, y = 1$ •	• $x = 1, y = 1$

Draw the transitions that correspond to the following statements.

(i) $x := 1$ (ii) $havoc(x)$ (iii) $assume(x = 0)$

(b) Consider the transition relation ρ that is given by the following diagram.



Find a formula for ρ.
Furthermore, compute the following sets.

(i) $wp(true, \rho)$	(iii) $wp(y = 1, \rho)$	(v) $wp(y = 0, \rho)$
(ii) $pre(true, \rho)$	(iv) $pre(y = 1, \rho)$	(vi) $pre(y = 0, \rho)$

Exercise 3: Relational composition 1 Point

Find a formula that denotes the relational composition $\rho_1 \circ \rho_2$ of the two relations denoted by the formulas ρ_1 and ρ_2. Here ρ_1 and ρ_2 are formulas in the variables $V \cup V'$, where V' consists of the primed versions of the variables in V.

Careful: The algebraic definition may be counterintuitive because one first applies ρ_2:

$$\rho_1 \circ \rho_2 = \{(s_1, s_3) \mid \exists s_2.\ (s_1, s_2) \in \rho_2 \wedge (s_2, s_3) \in \rho_1\}$$

Exercise 4: Properties of $post^{\#}$ 1 Point

Give a counterexample for those of the following propositions that are wrong.

(a) $post^{\#}(\varphi, \rho_1 \circ \rho_2) \subseteq post^{\#}(post^{\#}(\varphi, \rho_2), \rho_1)$

(b) $post^{\#}(\varphi, \rho_1 \circ \rho_2) \supseteq post^{\#}(post^{\#}(\varphi, \rho_2), \rho_1)$

(c) $post^{\#}(\varphi, \rho_1 \vee \rho_2) \subseteq post^{\#}(\varphi, \rho_1) \vee post^{\#}(\varphi, \rho_2)$

(d) $post^{\#}(\varphi, \rho_1 \vee \rho_2) \supseteq post^{\#}(\varphi, \rho_1) \vee post^{\#}(\varphi, \rho_2)$

(e) $post^{\#}(\varphi_1 \vee \varphi_2, \rho) \subseteq post^{\#}(\varphi_1, \rho_1) \vee post^{\#}(\varphi_2, \rho)$

(f) $post^{\#}(\varphi_1 \vee \varphi_2, \rho) \supseteq post^{\#}(\varphi_1, \rho_1) \vee post^{\#}(\varphi_2, \rho)$