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Tutorial for Program Verification Bonus Sheet 13 (not discussed in the tutorial)

Exercise 1: Transition predicate abstraction

Consider the following modified version of the TPA algorithm. (Modification underlined: T composed with the *abstraction* of ρ_τ)

Algorithm (TPA^{cl})

Input: program $P = (\Sigma, \mathcal{T}, \rho)$
set of transition predicates \mathcal{P}
abstraction α defined by \mathcal{P}

Output: set of abstract transitions $P^\# = \{T_1, \dots, T_n\}$
such that $T_1 \cup \dots \cup T_n$ is a transition invariant

$P^\# := \{\alpha(\rho_\tau) \mid \tau \in \mathcal{T}\}$

repeat

$P^\# := P^\# \cup \{\alpha(T \circ \underline{\alpha(\rho_\tau)}) \mid T \in P^\#, \tau \in \mathcal{T}, \underline{\alpha(T \circ \alpha(\rho_\tau))} \neq \emptyset\}$

until no change

- (a) Prove or refute the following claim:

The set of abstract transitions computed by TPA^{cl} is a disjunctively well-founded transition invariant iff the set of abstract transitions computed by TPA is a disjunctively well-founded transition invariant.

- (b) Think about a setting where we reapply the algorithm multiple times for the same set of transition predicates. What can be a possible advantage of TPA^{cl} over TPA?

Exercise 2: TPA with initial states

So far we considered only programs $P = (\Sigma, \mathcal{T}, \rho)$ where every state is an initial state. Let us now consider programs $P = (\Sigma, \Sigma_{\text{init}}, \mathcal{T}, \rho)$ where only the states in $\Sigma_{\text{init}} \subseteq \Sigma$ are initial states.

- (a) Give a program $P = (\Sigma, \Sigma_{\text{init}}, \mathcal{T}, \rho)$ whose transition relation R_P is not well-founded, but R_P restricted to the reachable states of P is well-founded. Give an informal explanation why for each set of transition predicates \mathcal{P} the set of abstract transitions $P^\#$ is not disjunctively well-founded.
- (b) Assume you have a tool that does a reachability analysis and you have a tool that computes the TPA algorithm. Describe a termination analysis that uses both tools and can be used to show termination of your program stated in part (a).