Softwaretechnik / Software-Engineering

Lecture 15: Testing

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Prof. Dr. Andreas Podelski, **Dr. Bernd Westphal**

Albert-Ludwigs-Universität Freiburg, Germany

Topic Area Code Quality Assurance: Content

VL 14	• **Introduction and Vocabulary**
⋮	• Test case, test suite, test execution.
	• Positive and negative outcomes.
VL 15	• **Limits of Software Testing**
	• **Glass-Box Testing**
⋮	• Statement-, branch-, term-**coverage**.
	• **Other Approaches**
	• **Model-based testing**,
	• **Runtime verification**.
	• **Software quality assurance** in a **larger scope**.
VL 16	• **Program Verification**
⋮	• partial and total **correctness**,
	• **Proof System PD**.
VL 17 ⋮	• **Review**

Content

Recall: Test Case, Test Execution

Test Case

Definition. A **test case** T over Σ and A is a pair $(In, Soll)$ consisting of

- a description In of sets of finite **input sequences**,
- a description $Soll$ of **expected outcomes**,

and an interpretation $[\![\cdot]\!]$ of these descriptions:

- $[\![In]\!] \subseteq (\Sigma_{in} \times A)^*, \quad [\![Soll]\!] \subseteq (\Sigma \times A)^* \cup (\Sigma \times A)^\omega$

Examples:

- Test case for procedure `strlen` : $String \to \mathbb{N}$, s denotes parameter, r return value:

$$T = (\underbrace{s = \texttt{"abc"}}_{In}, \underbrace{r = 3}_{Soll})$$

$[\![s = \texttt{"abc"}]\!] = \{\sigma_0^i \xrightarrow{\tau} \sigma_1^i \mid \sigma_0(s) = \texttt{"abc"}\}, \quad [\![r = 3]\!] = \{\sigma_0 \xrightarrow{\tau} \sigma_1 \mid \sigma_1(r) = 3\},$

Shorthand notation: $T = (\texttt{"abc"}, 3)$.

- "Call `strlen()` with string `"abc"`, expect return value 3."

Executing Test Cases

- A computation path

$$\pi = \begin{pmatrix} \sigma_0^i \\ \sigma_0^o \end{pmatrix} \xrightarrow{\alpha_1} \begin{pmatrix} \sigma_1^i \\ \sigma_1^o \end{pmatrix} \xrightarrow{\alpha_2} \cdots$$

from $[\![S]\!]$ is called **execution** of test case $(In, Soll)$ if and only if

- there is $n \in \mathbb{N}$ such that $\sigma_0 \xrightarrow{\alpha_1} \ldots \xrightarrow{\alpha_n} \sigma_n \downarrow \Sigma_{in} \in [\![In]\!]$.
  ("A prefix of π corresponds to an input sequence").

Execution π of test case T is called

- **successful** (or **positive**) if and only if $\pi \notin [\![Soll]\!]$.
  - Intuition: an an error has been discovered.
  - Alternative: test item S **failed to pass the test**.
  - Confusing: "test failed".
- **unsuccessful** (or **negative**) if and only if $\pi \in [\![Soll]\!]$.
  - Intuition: no error has been discovered.
  - Alternative: test item S **passed the test**.
  - Okay: "test passed".

Tell Them What You've Told Them...

- **Testing** is about
  - finding errors, or
  - demonstrating scenarios.
- A **test case** consists of
  - **input sequences** and
  - **expected outcome(s)**.
- A test case **execution** is
  - **positive** if an error is found,
  - **negative** if no error is found.
- A **test suite** is a set of test cases.
- Distinguish (among others),
  - **glass-box test**: structure (or source code) of test item available,
  - **black-box test**: structure not available.

? ? ?

Not Executing Test Cases

- Consider the test case

$$T = (\texttt{""}, 0)$$

  for procedure `strlen`.

  ("Empty string has length 0.")

$\neq$

- A tester observes the following software behaviour:

$$\pi = \underbrace{\{s \mapsto \texttt{NULL}, r \mapsto 0\}}_{=\sigma_0} \xrightarrow{\tau} \underbrace{\textit{program-abortion}}_{\sigma_1}$$

- Test execution **positive** or **negative**?

Note:

- If a tester does not adhere to an allowed input sequence of T, π **is not** a test execution.

  Thus π is neither positive nor negative (only defined for test executions).

- Same case: power outage (if continuous power supply is considered in input sequence).

By The Way. . . (Good Design)

- **High quality software** should be aware of its specification.
  and "**complain**" if operated outside of specification, e.g.
  - throw an exception,
  - abort program execution,
  - (at least) print an error message,
  - etc.

  Not: "garbage in, garbage out"

- **Example**: `strlen(3)` (C standard)
  - Allowed inputs are C-strings, return value is an integer,
  - `NULL` is not a C-string!
  - Thus, on input `NULL`, "complain" instead of just return an arbitrary number or "crash".

Test Suite

- A **test suite** is a finite set of test cases $\{T_1, \dots, T_n\}$.

- An **execution** of a **test suite** is a set of computation paths, such that there is at least one execution for each test case.

- An **execution** of a **test suite** is called **positive** if and only if at least one test case execution is **positive**.

  Otherwise, it is called **negative**.

Tests vs. Systematic Tests

Systematic Test – a test such that

- (environment) conditions are defined or precisely documented,
- inputs have been chosen systematically,
- results are documented and assessed according to criteria that have been fixed before. (Ludewig and Lichter, 2013)

Test – (one or multiple) execution(s) of a program on a computer with the goal to find errors. (Ludewig and Lichter, 2013)

Not (even) a test (in the sense of this weak definition):

- any **inspection** of the program (no execution),
- **demo** of the program (other goal),
- analysis by software-tools for, e.g., values of **metrics** (other goal),
- **investigation** of the program with a debugger (other goal).

(Our) Synonyms for **non-systematic tests**: Experiment, 'Rumprobieren'.

In the following: **test** means systematic test; if not systematic, call it **experiment**.

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  - Generic **requirements** on good test cases
    - Point vs. range errors
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    - Expected outcomes: Test **Oracle** : -/
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Testing Vocabulary

Specific Testing Notions

- How are the test cases **chosen**?
  - Considering only the specification (**black-box** or **function** test).
  - Considering the structure of the test item (**glass-box** or **structure** test).

- How much **effort** is put into testing?

  execution trial – does the program run at all?

  throw-away-test – invent input and judge output on-the-fly ($\rightarrow$ "**rumprobieren**"),

  systematic test – somebody (not author!) derives test cases, defines input/soll, documents test execution.

  Experience: In the long run, **systematic tests** are more **economic**.

- **Complexity** of the test item:

  unit test – a single program unit is tested (function, sub-routine, method, class, etc.)

  module test – a component is tested,

  integration test – the interplay between components is tested.

  system test – tests a whole system.

Specific Testing Notions Cont'd

- Which **property** is tested?

function test –
 functionality as specified by the requirements documents,

installation test –
 is it possible to **install** the software with the provided documentation and tools?

recommissioning test –
 is it possible to **bring the system back to operation** after operation was stopped?

availability test –
 does the system run for the required amount of time without issues,

load and stress test –
 does the system behave as required under **high or highest load**? … under overload?

"Hey, let's try how many game objects can be handled!" – that's an experiment, not a test.

resource tests –
 response time, minimal **hardware (software) requirements**, etc.

regression test –
 does the new version of the software **behave like the old one**
 on inputs where no behaviour change is expected?

Specific Testing Notions Cont'd

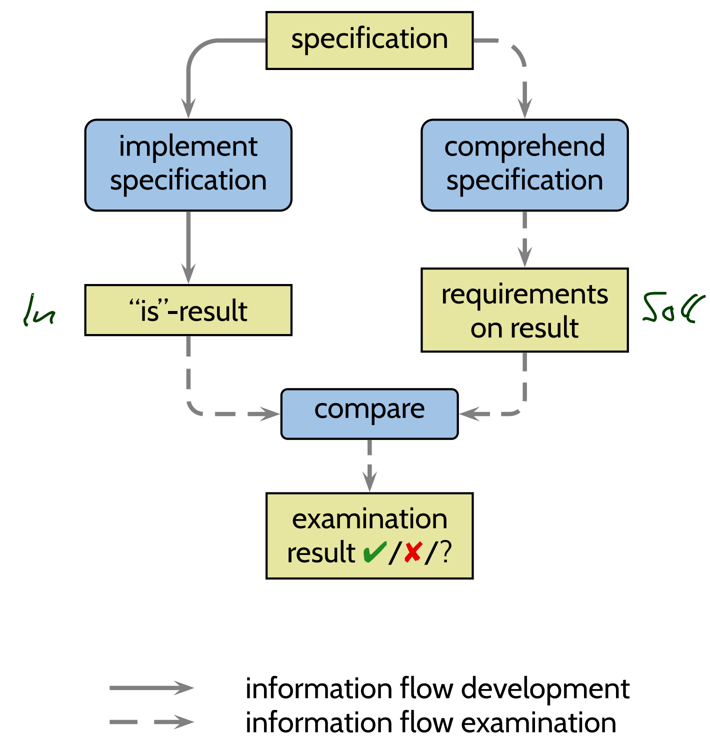
- Which roles are **involved** in testing?
  - **inhouse test** –
    only developers (meaning: quality assurance roles),
  - **alpha** and **beta** test –
    selected (potential) customers,
  - **acceptance test** –
    the customer tests whether the system (or parts of it, at milestones) test whether the system is acceptable.

Content

The Limits of Software Testing

Software Examination *(in Particular Testing)*

- In each examination, there are **two paths** from the specification to results:
  - the **production path** (using model, source code, executable, etc.), and
  - the **examination path** (using requirements specifications).
- A check can only discover errors on **exactly one** of the paths.
- If a **difference is detected**: examination result is **positive**.
- What is not on the paths, is not checked; crucial: **specification** and **comparison**.



(Ludewig and Lichter, 2013)

Recall:

SW broken
test ex. neg.

SW broken
test ex. pos.

		checking procedure: shows no error	checking procedure: reports error
artefact has error	yes	false negative	true positive
	no	true negative	false positive

SW okay
test ex. neg.

SW okay
test ex. pos.

Recall: Quotes On Testing

"Software testing can be used to show the presence of bugs,
but never to show their absence!" (**E. W. Dijkstra, 1970**)

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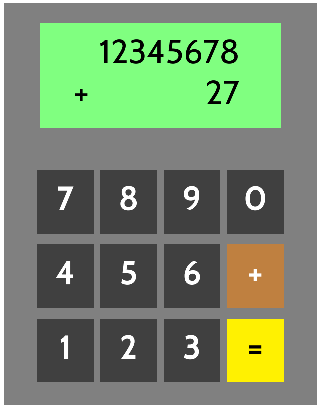
Why Can't We Show The Absence of Errors (in General)?

Consider a **simple pocket calculator** for adding 8-digit decimals:



- **Requirement**: If the display shows x, $+$, and y, then after pressing **=**,
  - the sum of x and y is displayed if $x + y$ has at most 8 digits,
  - otherwise "-E-" is displayed.
- With 8 digits, both x and y range over $[0, 10^8 - 1]$.
- Thus there are $10^{16} = 10,000,000,000,000,000$ possible input pairs (x, y) to be considered
  for **exhaustive testing**, i.e. testing every possible case!
- And if we restart the pocket calculator for each test,
  we **do not know anything** about problems with **sequences** of inputs...

  (Local variables may not be re-initialised properly, for example.)

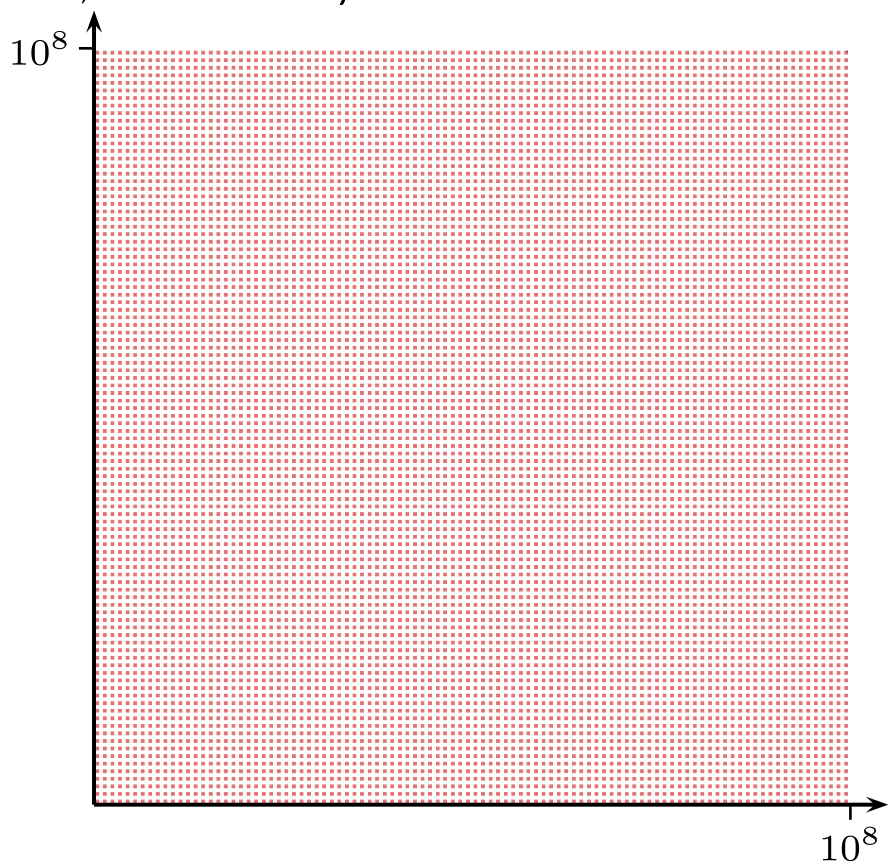
Observation: Software Usually Has Many Inputs

- **Example**: Simple Pocket Calculator.

  With **ten thousand** (10,000) **different** test cases (that's a lot!),
  9,999,999,999,990,000 of the 10^{16} possible inputs remain **uncovered**.

  In other words:
  Only 0.0000000001% of the possible inputs are covered, 99.9999999999% not touched.

- **In diagrams**: (red: uncovered, blue: covered)



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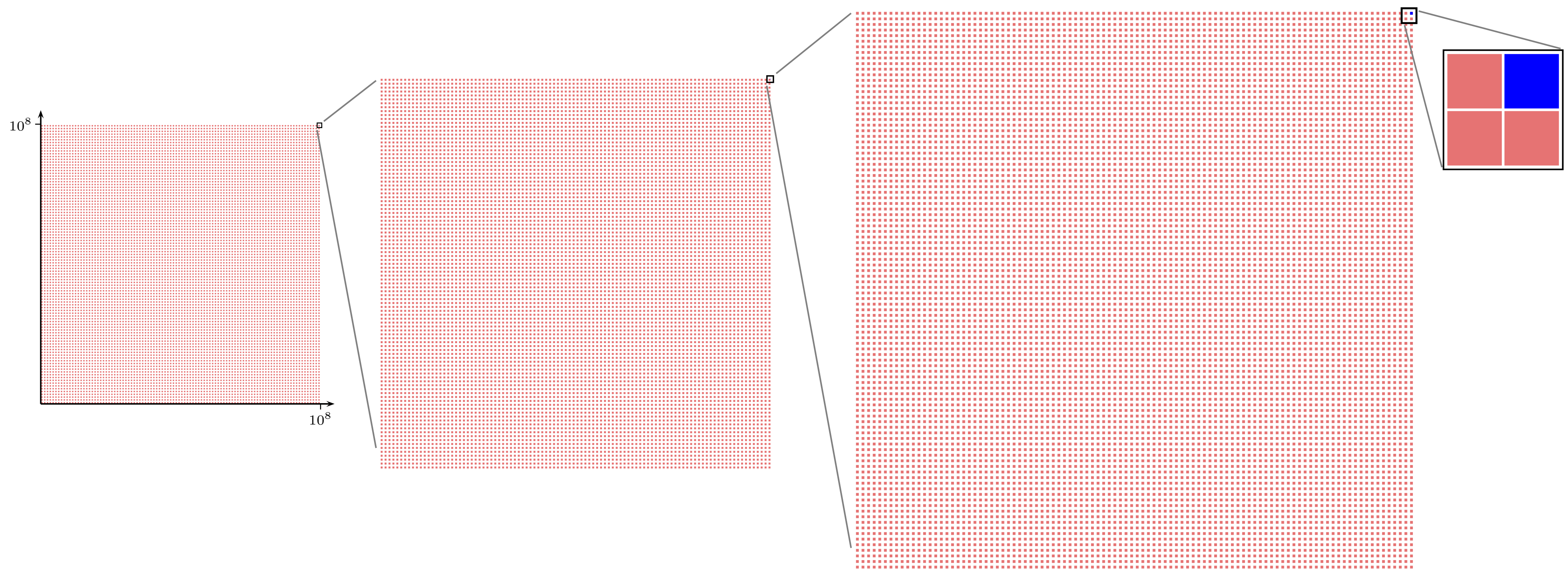

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Choosing Test Cases

Generic Requirements on Good Test Cases: Reproducibility

- Test executions should be (as) **reproducible** and **objective** (as possible).
- So, **strictly speaking**, a test case is a triple $(In, Soll, Env)$
  comprising a description Env of **(environmental) conditions**.

  Env describes any aspects which **could have an effect**
  on the outcome of a test execution and cannot be specified as part of In, such as:
  - Which **program** (version) is tested?
  - **Built** with which compiler, linker, etc.?
  - **Test host** (OS, architecture, memory size, connected devices (configuration?), etc.)?
  - Which **other software** (in which version, configuration) is involved?
  - **Who** is supposed to test **when**?
  - etc. etc.

Full reproducibility is hardly possible **in practice** – obviously (err, why…?).

- **Steps** towards **reproducibility** and **objectivity**:
  - have a fixed build environment,
  - use a fixed test host which does not do any other jobs,
  - execute test cases **automatically** (test scripts).

How to Choose Test Cases?

- **A first rule-of-thumb**:

> "Everything, which is required, **must** be examined/checked. Otherwise it is **uncertain** whether the requirements have been **understood** and **realised**."
>
> (Ludewig and Lichter, 2013)

In other words:

- Not having
  - at least one (systematic) test case
    - for each (required) feature
      - is (**grossly**?) **negligent**. (Dt.: (grob?) fahrlässig).

- **In even other words:**
  Without at least one test case for each feature, we can **hardly speak of** software **engineering**.

- **Good project management**: document for each test case which feature(s) it tests.

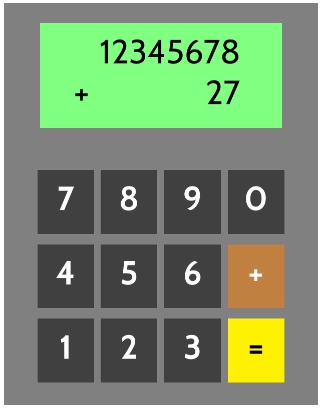
What Else Makes a Test Case a Good Test Case?

A test case is a **good test case** if it discovers – with high probability – an **unknown error**.

An **ideal test case** $(In, Soll)$ would be

- **of low redundancy**, i.e. it does not test what other test cases also test.
- **error sensitive**, i.e. has high probability to detect an error,
  (Probability should at least be greater than 0.)
- **representative**, i.e. represent a whole class of inputs,
  (i.e., software S passes $(In, Soll)$ if and only S behaves well for all In' from the class)

The idea of **representative**:

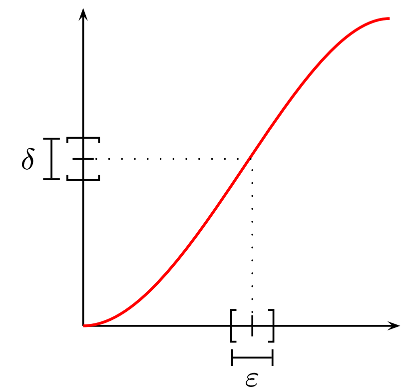


- If $(12345678, 27; 12345705)$ **was** representative for $(0, 27; 27)$, $(1, 27; 28)$, etc.
- then from a **negative** execution of test case $(12345678, 27; 12345705)$
- we **could** conclude that $(0, 27; 27)$, etc. will be negative as well.

Point vs. Range Errors

- Software is (in general) **not continous**.
  - Consider a continuous function, e.g. the one to the right:

    For sufficiently small ε-environments of an input,
    the outputs **differ only by a small amount** δ.

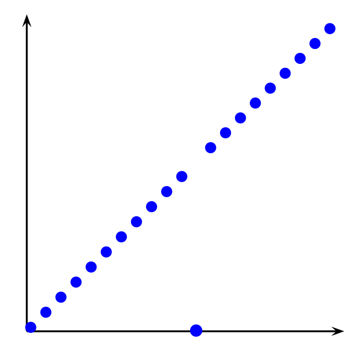


  - Physical systems are (to a certain extent) continous:
    - For example, if a bridge endures a single car of 1000 kg,
      we strongly expect the bridge to endure cars of 990 kg or 1010 kg.
    - And anything of weight smaller than 1000 kg can be expected to be endured.

  - For software, adjacent inputs **may yield arbitrarily distant** output values.

    Vocabulary:

    - **Point error**: an isolated input value triggers the error.
    - **Range error**: multiple "neighbouring" inputs trigger the error.



- For software, we can (in general, without extra information) not **conclude from some values to others**.

What Else Makes a Test Case a Good Test Case?

Thus: The wish for representative test cases is **problematic**:

- In general, we **do not know** which inputs lie in an equivalence class **wrt. a certain error**.
- Yet there is a large body on literature on how to construct representative test cases, **assuming** we know the equivalence classes.

Of course: ***If*** we ***know*** equivalence classes, we should exploit that knowledge to optimise the number of test cases.

But it is **perfectly reasonable** to test representatives of **equivalence classes induced by the specification**, e.g.

- valid and invalid inputs (to check whether input validation works at all),
- different classes of inputs considered in the requirements,
  like "C50", "E1" coins in the vending machine → have at least one test case with each.

Recall: one should have at least one test case per feature.

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Statistical Testing

One Approach: Statistical Tests

Classical **statistical testing** is one approach to deal with

- in practice not exhaustively testable **huge input space**,
- **tester bias**.

  (People tend to choose "good-will" inputs and disregard (tacit?) corner-cases; recall: the developer is not a good tester.)

Procedure:

- Randomly (!) choose test cases $T_1, \ldots, T_n$ for test suite $\mathcal{T}$.
- Execute test suite $\mathcal{T}$.
- **If an error is found**:
  - **good**, we certainly know there is an error,
- **if no error is found**:
  - **refuse hypothesis** "program is **not** correct" with a certain significance niveau.

    (Significance niveau may be unsatisfactory with small test suites.)

  (And: Needs stochastical assumptions on error distribution and truly random test cases.)

Statistical Testing: Discussion

(Ludewig and Lichter, 2013) name the following objections against statistical testing:

- In particular for **interactive software**, the **primary requirement** is often **no failures are experienced by the "typical user"**.

  Statistical testing (in general) may also cover a lot of "**untypical user behaviours**" unless (sophisticated) **user-models** are used.

- Statistical testing needs a method to **compute "soll"-values** for the randomly chosen inputs.

  That is easy for requirement "does not crash", but can be difficult in general.

- There is a high risk for **not finding** **point** or **small-range** errors.

  If they live in their **"natural habitat"**, carefully crafted test cases would probably uncover them.

Findings in the literature can at best be called **inconclusive**.

Getting Soll-Values

Where Do We Get The "Soll"-Values From?

Recall: A test case is a pair $(In, Soll)$ with proper expected (or "soll") values.

- In an **ideal world**, all "soll"-values
  are **defined** by the (formal) requirements specification and effectively **pre-computable**.

- In **this world**,
  - the formal requirements specification may only **reflectively** describe acceptable results without giving a **procedure** to compute the results.
  - there may not be a formal requirements specification, e.g.
    - "**the game objects should be rendered properly**",
    - "the compiler must translate the program correctly",
    - "**the notification message should appear on a proper screen position**",
    - "the data must be available for at least 10 days".
    - **etc.**

    Then: need another instance to decide whether the observation is acceptable.

- The testing community prefers to call **any instance** which decides whether results are acceptable a **(test) oracle**.

  I'd prefer **not to call** automatic derivation of "soll"-values from a **formal specification** an "oracle"... ; -) ("person or agency considered to provide wise and insightful [...] prophetic predictions or precognition of the future, inspired by the gods." says Wikipedia)

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Habitat-based Testing

Choosing Test Cases Habitat-based

Some traditional popular belief on software error habitat:

- Software errors **(seem to) enjoy**
  - **range boundaries**, e.g.
    - 0, 1, 27 if software works on inputs from $[0, 27]$,
    - -1, 28 for error handling,
    - $-2^{31} - 1, 2^{31}$ on 32-bit architectures,
    - boundaries of arrays (first, last element),
    - boundaries of loops (first, last iteration),
    - etc.
  - **special cases** of the problem (empty list, use-case without actor, $\dots$),
  - special cases of the programming language semantics,
  - **complex implementations**.

$\rightarrow$ **Good idea**: for each test case, note down why it has been chosen.
For example, "demonstrate that corner-case handling is not completely broken".

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Glass-Box Testing: Coverage

Statements and Branches by Example

Definition. Software is a finite description S of a (possibly infinite) set $[\![S]\!]$ of (finite or infinite) computation paths of the form $\sigma_0 \xrightarrow{\alpha_1} \sigma_1 \xrightarrow{\alpha_2} \sigma_2 \cdots$ where

- $\sigma_i \in \Sigma$, $i \in \mathbb{N}_0$, is called state (or configuration), and
- $\alpha_i \in A$, $i \in \mathbb{N}_0$, is called action (or event).

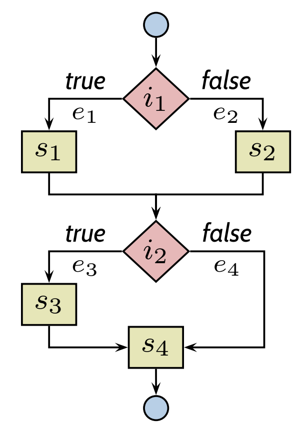
- In the following, we assume that
  - S has a **control flow graph** $(V, E)_S$, and **statements** $Stm_S \subseteq V$ and **branches** $Cnd_S \subseteq E$,
  - each computation path prefix $\sigma_0 \xrightarrow{\alpha_1} \sigma_1 \xrightarrow{\alpha_2} \sigma_2 \cdots \xrightarrow{\alpha_n} \sigma_n$ gives information on statements and control flow graph branch edges **which were executed** right before obtaining σ_n:

$$stm : (\Sigma \times A)^* \to 2^{Stm_S}, \qquad cnd : (\Sigma \times A)^* \to 2^{Cnd_S},$$

```
int f( int x, int y, int z )
{
  i1: if (x > 100 ∧ y > 10)
  s1:     z = z * 2;
      else
  s2:     z = z/2;
  i2: if (x > 500 ∨ y > 50)
  s3:     z = z * 5;
  s4: return z;
}
```

$$Stm_f = \{s_1, s_2, s_3, s_4\}$$

$$Cnd_f = \{e_1, e_2, e_3, e_4\}$$



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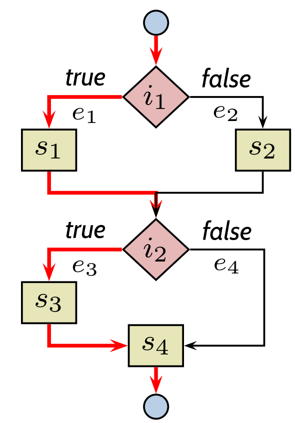
```

$Stm_f = \{s_1, s_2, s_3, s_4\}$

$Cnd_f = \{e_1, e_2, e_3, e_4\}$

i_1/false $\quad$ i_2/false

$x = 501, \; y = 11, \; z = 0$



$z = 0$

	σ_0	$\xrightarrow{\alpha_1}$	σ_1	$\xrightarrow{\alpha_2}$	σ_2	$\xrightarrow{\alpha_2}$	σ_3	$\xrightarrow{\alpha_3}$	σ_4	$\xrightarrow{\alpha_4}$	σ_5	$\xrightarrow{\alpha_5}$	σ_6
	$pc: 1$		$pc: 3$		$pc: 4$		$pc: 7$		$pc: 8$		$pc: 9$		$pc: 10$
	$x: 501$		$x: 501$		$x: 501$		$x: 501$		$x: 501$		$x: 501$		$x: 501$
	$y: 11$		$y: 11$		$y: 11$		$y: 11$		$y: 11$		$y: 11$		$y: 11$
	$z: 0$		$z: 0$		$z: 0$		$z: 0$		$z: 0$		$z: 0$		$z: 0$
stm:	$\{\}$		$\{\}$		$\{\}$		$\{s_1\}$		$\{\}$		$\{s_3\}$		$\{s_4\}$
cnd:	$\{\}$		$\{\}$		$\{e_1\}$		$\{\}$		$\{e_3\}$		$\{\}$		$\{\}$
					i_1/true				i_2/true				

Glass-Box Testing: Coverage

- **Coverage** is a property of **test cases** and **test suites**.

- Execution $\pi = \sigma_0 \xrightarrow{\alpha_1} \cdots$ of test case T achieves $p\,\%$ **statement coverage** if and only if

$$p = cov_{stm}(\pi) := \frac{|\bigcup_{i \in \mathbb{N}_0} stm(\sigma_0 \cdots \sigma_i)|}{|Stm_S|}, \; |Stm_S| \neq 0.$$

  Test case T achieves $p\,\%$ **statement coverage** if and only if $p = \min_{\pi \text{ execution of } T} cov_{stm}(\pi)$.

- Execution π of T achieves $p\,\%$ **branch coverage** if and only if

$$p = cov_{cnd}(\pi) := \frac{|\bigcup_{i \in \mathbb{N}_0} cnd(\sigma_0 \cdots \sigma_i)|}{|Cnd_S|}, \; |Cnd_S| \neq 0.$$

  Test case T achieves $p\,\%$ **branch coverage** if and only if $p = \min_{\pi \text{ execution of } T} cov_{cnd}(\pi)$.

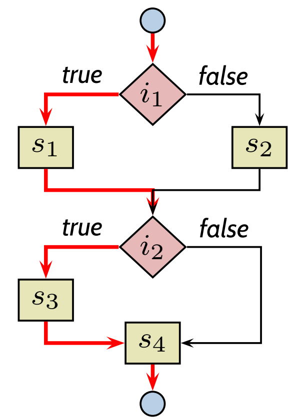
- **Define**: $p = 100$ for empty program. (More precisely: $Stm_S = \emptyset$ and $Cnd_S = \emptyset$, respectively.)

- Statement/branch coverage canonically extends to test suite $\mathcal{T} = \{T_1, \dots, T_n\}$.
  For example, given $\pi_1 = \sigma_0^1 \cdots, \dots, \pi_n = \sigma_0^n \cdots$, then $\mathcal{T}$ achieves

$$p = \frac{|\bigcup_{1 \leq j \leq n} \bigcup_{i \in \mathbb{N}_0} stm(\sigma_0^j \cdots \sigma_i^j)|}{|Stm_S|}, \; |Stm_S| \neq 0, \; \textbf{statement coverage}.$$

Coverage Example

`int` f(`int` x, `int` y, `int` z)
{
i_1: `if` $(x > 100 \land y > 10)$
s_1: $z = z * 2$;
`else`
s_2: $z = z/2$;
i_2: `if` $(x > 500 \lor y > 50)$
s_3: $z = z * 5$;
s_4: `return` z;
}



- **Requirement**: $\{\mathit{true}\}\ f\ \{\mathit{true}\}$ (no abnormal termination), i.e. $Soll = \Sigma^* \cup \Sigma^\omega$.

test suite coverage

In x, y, z	i_1/t	i_1/f	s_1	s_2	i_2/t	i_2/f	c_1	c_2	s_3	s_4	% stm	% cnd	i_2/% term
$501, 11, 0$	✔		✔		✔		✔		✔	✔	75	50	25
12, 11, 0		✓		✓		✓				✓	100	100	

Coverage Example

int $f($ int x, int y, int z)
{
i_1: if $(x > 100 \land y > 10)$
s_1: $z = z * 2$;
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s_2: $z = z/2$;
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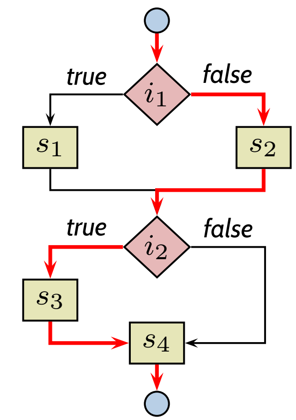

- **Requirement**: $\{$*true*$\}$ f $\{$*true*$\}$ (no abnormal termination), i.e. $Soll = \Sigma^* \cup \Sigma^\omega$.

test suite coverage

In x, y, z	i_1/t	i_1/f	s_1	s_2	i_2/t	i_2/f	c_1	c_2	s_3	s_4	% stm	% cnd	i_2/% term
$501, 11, 0$	✔		✔		✔		✔		✔	✔	75	50	25
$501, 0, 0$		✔		✔	✔		✔		✔	✔	100	75	25
$0, 0, 0$		✔		✔		✔				✔	100	100	75
$0, 51, 0$		✔		✔	✔			✔		✔	100	100	100

Term Coverage

- Consider the statement

$$\texttt{if } (\overbrace{A \wedge (B \vee (C \wedge D)) \vee E}^{expr}) \texttt{ then } \ldots;$$

  where A, …, E are **minimal** boolean terms, e.g. $x > 0$, but not $a \vee b$.

  Branch coverage is easy in this case:

  Use In_1 such that $(A = 0, \ldots, E = 0)$, and In_2 such that $(A = 0, \ldots, E = 1)$.

- **Additional goal**:
  check whether there are useless terms,
  or terms causing abnormal program termination.

- **Term Coverage** (for an expression $expr$):
  - Let $\beta : \{A_1, \ldots, A_n\} \to \mathbb{B}$ be a valuation of the terms.
  - Term A_i is b-**effective** in β for $expr$ if and only if

$$\beta(A_i) = b \text{ and } [\![expr]\!](\beta[A_i/\textit{true}]) \neq [\![expr]\!](\beta[A_i/\textit{false}]).$$

  - $\Xi \subseteq (\{A_1, \ldots, A_n\} \to \mathbb{B})$ achieves p % **term coverage** if and only if

$$p = \frac{|\{A_i^b \mid \exists\, \beta \in \Xi \bullet A_i \text{ is } b\text{-effective in } \beta\}|}{2n}.$$

	A	B	C	D	E	b	%
β_1	1	1	0	0	0	1	20
β_2	1	0	0	1	0	0	50
β_3	1	0	1	1	0	1	70
β_4	0	0	1	0	1	1	80

red: b-effective, black: otherwise

Unreachable Code

$$
\begin{array}{ll}
\multicolumn{2}{l}{\texttt{int}\ f(\ \texttt{int}\ x, \texttt{int}\ y, \texttt{int}\ z\ )} \\
\{ & \\
i_1: & \texttt{if}\ (x \neq x) \\
s_1: & \quad z = y/0; \\
i_2: & \texttt{if}\ (x = x \vee z/0 = 27) \\
s_2: & \quad z = z * 2; \\
s_3: & \texttt{return}\ z; \\
\} &
\end{array}
$$

- Statement s_1 is **never executed** (because $x \neq x \iff$ *false*),
  thus 100 % statement-/branch-/term-coverage is **not achievable**.
- Assume, evaluating $n/0$ causes (undesired) **abnormal program termination**.

  Is statement s_1 an **error** in the program...?
- Term $z/0$ in i_2 also looks critical...

  (In programming languages with short-circuit evaluation, it is never evaluated.)

Conclusions from Coverage Measures

- Assume, test suite $\mathcal{T}$ tests software S for the following property φ:
  - **pre-condition**: p, **post-condition**: q,

  and S passes (!) $\mathcal{T}$, and the execution achieves $100\,\%$ statement / branch / term coverage.

What does this tell us about S? Or: what can we conclude from coverage measures?

- 100 % **statement** coverage:
  - "there is no statement, which **necessarily** violates φ"

    (Still, there may be many, many computation paths which violate φ, and which just have not been touched by $\mathcal{T}$.)
  - "there is no unreachable statement"
- 100 % **branch** (**term**) coverage:
  - "there is no single branch (term) which **necessarily causes** violations of φ"

    In other words: "for each condition (term), there is one computation path satisfying φ where the condition (term) evaluates to *true*, and one for *false*."
  - "there is no unused condition (term)"

Not more ($\rightarrow$ exercises)!

That's definitely **something**, but not as much as "100 %" may sound like...

Coverage Measures in Certification

- (Seems that) DO-178B,

  "Software Considerations in Airborne Systems and Equipment Certification", (which deals with the safety of software used in certain airborne systems)

  requires that certain **coverage measures** are reached,
  in particular something similar to **term coverage** (MC/DC coverage).

  (Next to development process requirements, reviews, unit testing, etc.)

- If not required, ask: what is the effort / gain ratio?
  (Average effort to detect an error; term coverage needs high effort.)

- Currently, the standard moves towards accepting certain verification or static analysis tools to support (or even replace?) some testing obligations.

Tell Them What You've Told Them...

- A check can **only discover** errors on **exactly one** path.
- Software testing is **challenging** because
  - typically **huge input space**,
  - software is **non-continous**,
- There is a **vast amount of literature** on how to choose test cases.

  A good starting point:
  - at least **one test case per feature**,
  - **corner-cases**, extremal values,
  - **error handling**, etc.
- **Glass-box testing**
  - considers the **control flow graph**,
  - defines **coverage measures**.
- **Other approaches**:
  - **statistical** testing, **model-based** testing,
- Define criteria for **"testing done"** (like coverage, or cost per error).
- **Process**: tester and developer should be different persons.

References

References

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