Softwaretechnik / Software-Engineering

Lecture 17: Software Verification

2018-07-12

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Topic Area Code Quality Assurance: Content

VL 15

- **Introduction and Vocabulary**
  - Test case, test suite, test execution.
  - Positive and negative outcomes.

⋮

- **Limits of Software Testing**

VL 16

- **Glass-Box Testing**
  - Statement-, branch-, term-**coverage**.

⋮

- **Other Approaches**
  - **Model-based testing**,
  - **Runtime verification**.
- **Program Verification**

VL 17

  - partial and total **correctness**,

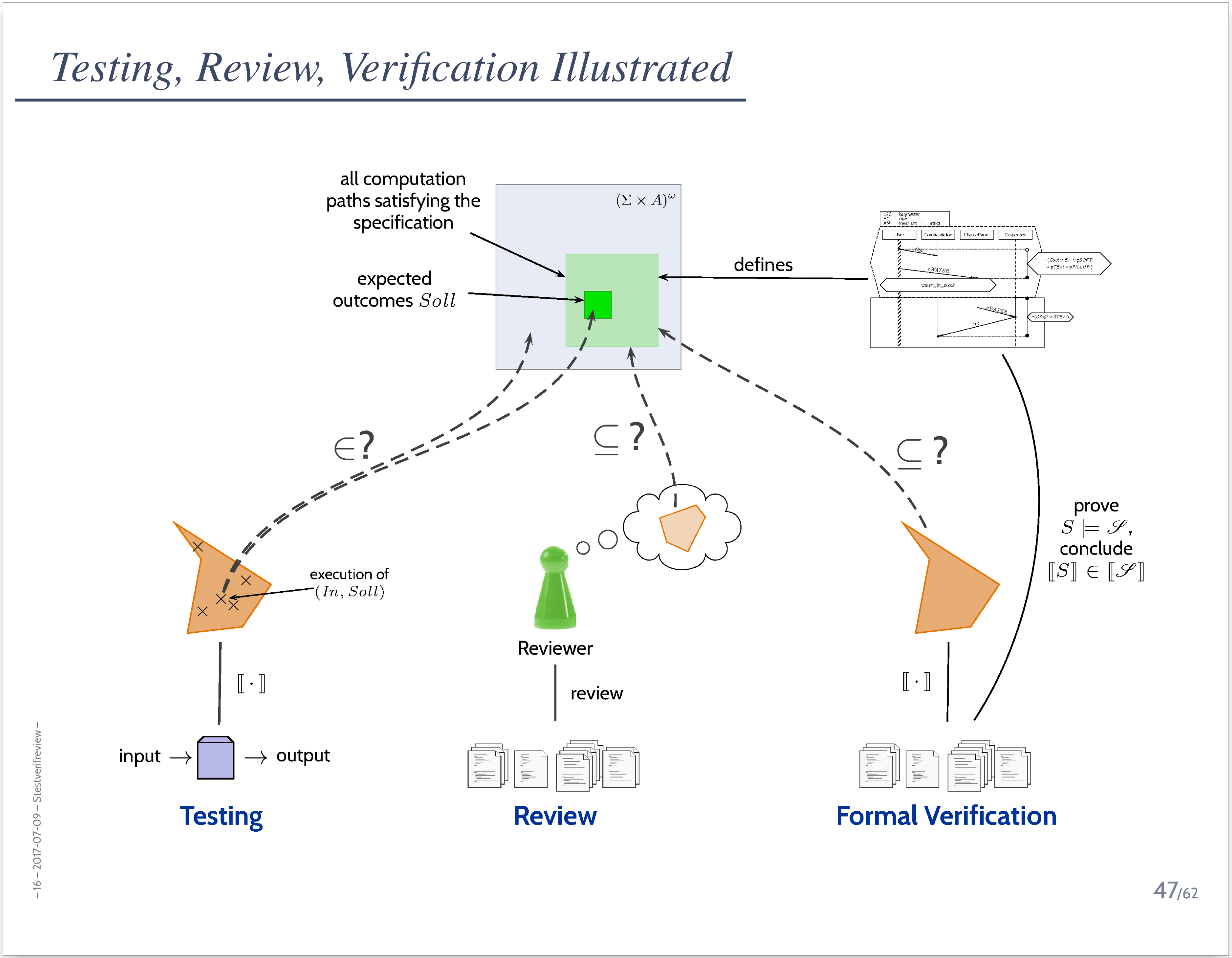
⋮

  - **Proof System PD**.

VL 18

- **Review**

⋮



Testing, Review, Verification Illustrated

all computation paths satisfying the specification

$(\Sigma \times A)^\omega$

expected outcomes $Soll$

defines

$\in$?

$\subseteq$?

$\subseteq$?

execution of $(In, Soll)$

prove $S \models \mathscr{S}$, conclude $[\![S]\!] \in [\![\mathscr{S}]\!]$

Reviewer

review

$[\![\cdot]\!]$

$[\![\cdot]\!]$

input → → output

Testing

Review

Formal Verification

– 16 – 2017-07-09 – Stestverifreview –

Content

- **Formal Program Verification**
  - **Deterministic Programs**
    - **Syntax**
    - **Semantics**
    - Termination, Divergence
  - **Correctness** of deterministic programs
    - **partial** correctness,
    - **total** correctness.
  - **Proof System PD**
- **The Verifier for Concurrent C**

Sequential, Deterministic While-Programs

Deterministic Programs

Syntax:

$$S ::= \mathit{skip} \mid u := t \mid S_1; S_2 \mid \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi} \mid \mathbf{while}\ B\ \mathbf{do}\ S_1\ \mathbf{od}$$

where $u \in V$ is a **variable**, t is a type-compatible **expression**, B is a Boolean **expression**.

Semantics: (is induced by the following transition relation) — $\sigma : V \to \mathcal{D}(V)$

(i) $\langle \mathit{skip}, \sigma \rangle \to \langle E, \sigma \rangle$ empty program

(ii) $\langle u := t, \sigma \rangle \to \langle E, \sigma[u := \sigma(t)] \rangle$

(iii) $\dfrac{\langle S_1, \sigma \rangle \to \langle S_2, \tau \rangle}{\langle S_1; S, \sigma \rangle \to \langle S_2; S, \tau \rangle}$

(iv) $\langle \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}, \sigma \rangle \to \langle S_1, \sigma \rangle$, if $\sigma \models B$,

(v) $\langle \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}, \sigma \rangle \to \langle S_2, \sigma \rangle$, if $\sigma \not\models B$,

(vi) $\langle \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}, \sigma \rangle \to \langle S; \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}, \sigma \rangle$, if $\sigma \models B$,

(vii) $\langle \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}, \sigma \rangle \to \langle E, \sigma \rangle$, if $\sigma \not\models B$,

E denotes the **empty program**; define $E; S \equiv S; E \equiv S$.

Note: the first component of $\langle S, \sigma \rangle$ is a program (**structural operational semantics** (SOS)).

Example

(i) $\langle skip,\ \sigma\rangle \to \langle E,\ \sigma\rangle$ $\qquad E; S \equiv S; E \equiv S$

(ii) $\langle u := t,\ \sigma\rangle \to \langle E,\ \sigma[u := \sigma(t)]\rangle$

(iii) $\dfrac{\langle S_1,\ \sigma\rangle \to \langle S_2,\ \tau\rangle}{\langle S_1; S,\ \sigma\rangle \to \langle S_2; S,\ \tau\rangle}$

(iv) $\langle \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi},\ \sigma\rangle \to \langle S_1,\ \sigma\rangle$, if $\sigma \models B$,

(v) $\langle \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi},\ \sigma\rangle \to \langle S_2,\ \sigma\rangle$, if $\sigma \not\models B$,

(vi) $\langle \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od},\ \sigma\rangle \to \langle S; \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od},\ \sigma\rangle$, if $\sigma \models B$,

(vii) $\langle \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od},\ \sigma\rangle \to \langle E,\ \sigma\rangle$, if $\sigma \not\models B$,

Consider **program**

$$S \equiv a[0] := 1; a[1] := 0; \textbf{while}\ a[x] \neq 0\ \textbf{do}\ x := x + 1\ \textbf{od}$$

and a **state** σ with $\sigma \models x = 0$.

$$\begin{array}{rcl}
\langle S,\ \sigma\rangle & \xrightarrow{(ii),(iii)} & \langle a[1] := 0; \textbf{while}\ a[x] \neq 0\ \textbf{do}\ x := x + 1\ \textbf{od},\ \sigma[a[0] := 1]\rangle \\
& \xrightarrow{(ii),(iii)} & \langle \textbf{while}\ a[x] \neq 0\ \textbf{do}\ \underbrace{x := x + 1}\ \textbf{od},\ \sigma'\rangle \\
& \xrightarrow{(vi)} & \langle \underbrace{x := x + 1}; \textbf{while}\ a[x] \neq 0\ \textbf{do}\ x := x + 1\ \textbf{od},\ \sigma'\rangle \\
& \xrightarrow{(ii),(iii)} & \langle \textbf{while}\ a[x] \neq 0\ \textbf{do}\ x := x + 1\ \textbf{od},\ \sigma'[x := 1]\rangle \\
& \xrightarrow{(vii)} & \langle E,\ \sigma'[x := 1]\rangle
\end{array}$$

where $\sigma' = \sigma[a[0] := 1][a[1] := 0]$.

Another Example

(i) $\langle skip,\, \sigma\rangle \to \langle E,\, \sigma\rangle$ $\qquad E; S \equiv S; E \equiv S$
(ii) $\langle u := t,\, \sigma\rangle \to \langle E,\, \sigma[u := \sigma(t)]\rangle$
(iii) $\dfrac{\langle S_1,\, \sigma\rangle \to \langle S_2,\, \tau\rangle}{\langle S_1; S,\, \sigma\rangle \to \langle S_2; S,\, \tau\rangle}$
(iv) $\langle \textbf{if } B \textbf{ then } S_1 \textbf{ else } S_2 \textbf{ fi},\, \sigma\rangle \to \langle S_1,\, \sigma\rangle$, if $\sigma \models B$,
(v) $\langle \textbf{if } B \textbf{ then } S_1 \textbf{ else } S_2 \textbf{ fi},\, \sigma\rangle \to \langle S_2,\, \sigma\rangle$, if $\sigma \not\models B$,
(vi) $\langle \textbf{while } B \textbf{ do } S \textbf{ od},\, \sigma\rangle \to \langle S; \textbf{while } B \textbf{ do } S \textbf{ od},\, \sigma\rangle$, if $\sigma \models B$,
(vii) $\langle \textbf{while } B \textbf{ do } S \textbf{ od},\, \sigma\rangle \to \langle E,\, \sigma\rangle$, if $\sigma \not\models B$,

Consider **program**

$$S_1 \equiv y := x; y := (x-1)\cdot x + y$$

and a **state** σ with $\sigma \models x = 3$.

$$\begin{aligned}\langle S_1,\, \sigma\rangle &\xrightarrow{(ii),(iii)} \langle y := (x-1)\cdot x + y,\, \{x \mapsto 3, y \mapsto 3\}\rangle\\ &\xrightarrow{(ii)} \langle E,\, \{x \mapsto 3, y \mapsto 9\}\rangle\end{aligned}$$

Consider **program** $S_3 \equiv y := x;\ y := (x-1)\cdot x + y;\ \textbf{while } 1 \textbf{ do } skip \textbf{ od}.$

$$\begin{aligned}\langle S_3,\, \sigma\rangle &\xrightarrow{(ii),(iii)} \langle y := (x-1)\cdot x + y; \textbf{while } 1 \textbf{ do } skip \textbf{ od},\, \{x \mapsto 3, y \mapsto 3\}\rangle\\ &\xrightarrow{(ii),(iii)} \langle \textbf{while } 1 \textbf{ do } skip \textbf{ od},\, \{x \mapsto 3, y \mapsto 9\}\rangle\\ &\xrightarrow{(vi)} \langle skip; \textbf{while } 1 \textbf{ do } skip \textbf{ od},\, \{x \mapsto 3, y \mapsto 9\}\rangle\\ &\xrightarrow{(i),(iii)} \langle \textbf{while } 1 \textbf{ do } skip \textbf{ od},\, \{x \mapsto 3, y \mapsto 9\}\rangle\\ &\xrightarrow{(vi)} \dots\end{aligned}$$

Computations of Deterministic Programs

Definition. Let S be a deterministic program.

(i) A **transition sequence** of S (starting in σ) is a finite or infinite sequence

$$\langle S,\, \sigma\rangle = \langle S_0,\, \sigma_0\rangle \to \langle S_1,\, \sigma_1\rangle \to \ldots$$

(that is, $\langle S_i,\, \sigma_i\rangle$ and $\langle S_{i+1},\, \sigma_{i+1}\rangle$ are in transition relation for all i).

(ii) A **computation (path)** of S (starting in σ) is a maximal transition sequence of S (starting in σ), i.e. infinite or not extendible.

(iii) A computation of S is said to

- a) **terminate** in τ if and only if it is finite and ends with $\langle E,\, \tau\rangle$,
- b) **diverge** if and only if it is infinite.

  S **can diverge from** σ if and only if a diverging computation starts in σ.

(iv) We use $\to^*$ to denote the transitive, reflexive closure of $\to$.

Lemma. For each deterministic program S and each state σ, there is exactly one computation of S which starts in σ.

Input/Output Semantics of Deterministic Programs

Definition.
Let S be a deterministic program.

(i) The semantics of partial correctness is the function

$$\mathcal{M}[\![S]\!] : \Sigma \to 2^{\Sigma}$$

with $\mathcal{M}[\![S]\!](\sigma) = \{\tau \mid \langle S,\, \sigma\rangle \to^{*} \langle E,\, \tau\rangle\}$.

finitely many steps

(ii) The semantics of total correctness is the function

$$\mathcal{M}_{tot}[\![S]\!] : \Sigma \to 2^{\Sigma} \mathbin{\dot{\cup}} \{\infty\}$$

with $\mathcal{M}_{tot}[\![S]\!](\sigma) = \mathcal{M}[\![S]\!](\sigma) \cup \{\infty \mid S$ can diverge from $\sigma\}$.

∞ is an error state representing divergence.

Note: $\mathcal{M}_{tot}[\![S]\!](\sigma)$ has exactly one element, $\mathcal{M}[\![S]\!](\sigma)$ at most one.

Example: $\mathcal{M}[\![S_1]\!](\sigma) = \mathcal{M}_{tot}[\![S_1]\!](\sigma) = \{\tau \mid \tau(x) = \sigma(x) \wedge \tau(y) = \sigma(x)^2\}$, $\sigma \in \Sigma$.

(Recall: $S_1 \equiv y := x; y := (x-1) \cdot x + y$)

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Correctness of While-Programs

Correctness of Deterministic Programs

Definition.
Let S be a program over variables V, and p and q Boolean expressions over V.

(i) The **correctness formula**

$$\{p\}\ S\ \{q\} \qquad \text{(“Hoare triple”)}$$

holds in the sense of partial correctness, denoted by $\models \{p\}\ S\ \{q\}$, if and only if

$$\underbrace{\mathcal{M}\llbracket S\rrbracket(\overbrace{\llbracket p\rrbracket}^{\{\sigma \mid \sigma \models p\} \subseteq (V \to \mathcal{D}(V))})}_{\text{set of states}} \subseteq \underbrace{\llbracket q\rrbracket}_{\text{set of states}}.$$

We say S is **partially correct** wrt. p and q.

(ii) A **correctness formula**

$$\{p\}\ S\ \{q\}$$

holds in the sense of total correctness, denoted by $\models_{tot} \{p\}\ S\ \{q\}$, if and only if

$$\mathcal{M}_{tot}\llbracket S\rrbracket(\llbracket p\rrbracket) \subseteq \llbracket q\rrbracket.$$

We say S is **totally correct** wrt. p and q.

Example: Computing squares (of numbers $0, \dots, 27$)

- **Pre-condition**: $p \equiv 0 \leq x \leq 27$,
- **Post-condition**: $q \equiv y = x^2$.

Program S_1:

```
int y = x;
y = (x - 1) * x + y;
```

$\models^? \{p\}\ S_1\ \{q\}$ ✓

$\models^?_{tot} \{p\}\ S_1\ \{q\}$ ✓

Program S_2:

```
int y = x;
y = (x - 1) * x + y;
while (1);
```

$\models^? \{p\}\ S_2\ \{q\}$ ✓

$\models^?_{tot} \{p\}\ S_2\ \{q\}$ ✗

Program S_3:

```
int y = x;
int z; // uninitialised
y = ((x - 1) * x + y) + z;
```

$\models^? \{p\}\ S_3\ \{q\}$ ✗ e.g. $z = 1$

$\models^?_{tot} \{p\}\ S_3\ \{q\}$ ✗

Program S_4:

p

```
int x = read_input();
int y = x + (x-1) * x;
```

q

	$\mathbb{Z}$ math.	64-bit computer
$\models^? \{p\}\ S_4\ \{q\}$	✓	✗
$\models^?_{tot} \{p\}\ S_4\ \{q\}$	✓	✗

Example: Correctness

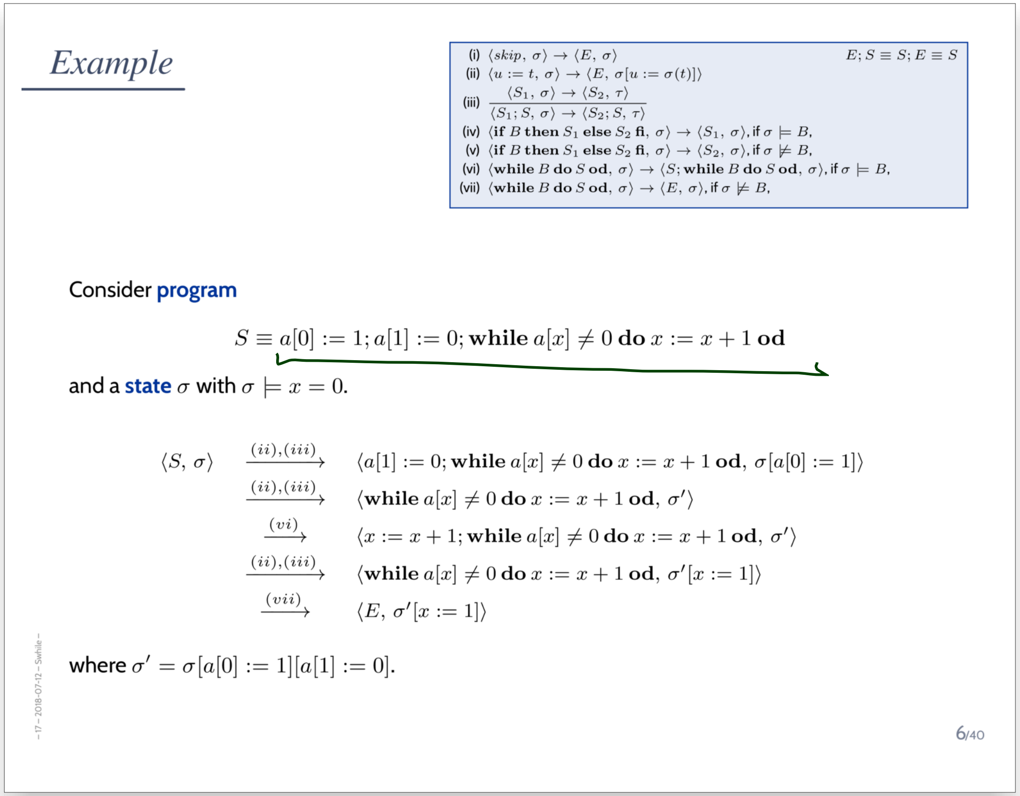
- By the example, **we have shown**

$$\models \{x = 0\}\ S\ \{x = 1\}$$

and

$$\models_{tot} \{x = 0\}\ S\ \{x = 1\}.$$

(because we only assumed $\sigma \models x = 0$ for the example, which is exactly the precondition.)



Example

(i) $\langle skip,\ \sigma\rangle \to \langle E,\ \sigma\rangle$ $\quad E; S \equiv S; E \equiv S$
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Consider **program**

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$$\begin{aligned}
\langle S,\ \sigma\rangle &\xrightarrow{(ii),(iii)} \langle a[1] := 0; \mathbf{while}\ a[x] \neq 0\ \mathbf{do}\ x := x + 1\ \mathbf{od},\ \sigma[a[0] := 1]\rangle\\
&\xrightarrow{(ii),(iii)} \langle \mathbf{while}\ a[x] \neq 0\ \mathbf{do}\ x := x + 1\ \mathbf{od},\ \sigma'\rangle\\
&\xrightarrow{(vi)} \langle x := x + 1; \mathbf{while}\ a[x] \neq 0\ \mathbf{do}\ x := x + 1\ \mathbf{od},\ \sigma'\rangle\\
&\xrightarrow{(ii),(iii)} \langle \mathbf{while}\ a[x] \neq 0\ \mathbf{do}\ x := x + 1\ \mathbf{od},\ \sigma'[x := 1]\rangle\\
&\xrightarrow{(vii)} \langle E,\ \sigma'[x := 1]\rangle
\end{aligned}$$

where $\sigma' = \sigma[a[0] := 1][a[1] := 0]$.

– 17 – 2018-07-12 – Swhile –

6/40

- **We have also shown** (= **proved** (!)):

$$\models \{x = 0\}\ S\ \{x = 1 \wedge a[x] = 0\}.$$

- The correctness formula $\{x = 2\}\ S\ \{\mathit{true}\}$ **does not hold** for S. (in the sense of total correctness)
  (For example, if $\sigma \models a[i] \neq 0$ for all $i > 2$.)

- In the sense of **partial correctness**, $\{x = 2 \wedge \forall\, i \geq 2 \bullet a[i] = 1\}\ S\ \{\mathit{false}\}$ also holds.

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Proof-System PD

Proof-System PD *(for sequential, deterministic programs)*

Axiom 1: **Skip-Statement**

$$\{p\}\ skip\ \{p\}$$

Axiom 2: **Assignment**

$$\{p[u := t]\}\ u := t\ \{p\}$$

Rule 3: **Sequential Composition**

$$\frac{\{p\}\ S_1\ \{r\}, \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$$

Rule 4: **Conditional Statement**

$$\frac{\{p \wedge B\}\ S_1\ \{q\}, \{p \wedge \neg B\}\ S_2\ \{q\},}{\{p\}\ \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}\ \{q\}}$$

Rule 5: **While-Loop**

$$\frac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{p \wedge \neg B\}}$$

Rule 6: **Consequence**

$$\frac{p \rightarrow p_1, \{p_1\}\ S\ \{q_1\}, q_1 \rightarrow q}{\{p\}\ S\ \{q\}}$$

Theorem. PD is correct ("sound") and (relative) complete for partial correctness of deterministic programs, i.e. $\vdash_{PD}\ \{p\}\ S\ \{q\}$ if and only if $\models\ \{p\}\ S\ \{q\}$.

Example Proof

$$DIV \equiv \overbrace{a := 0;\ b := x;}^{=:S_0^D}\ \mathbf{while}\ \overbrace{b \geq y}^{=:B^D}\ \mathbf{do}\ \overbrace{b := b - y;\ a := a + 1}^{=:S_1^D}\ \mathbf{od}$$

(The first (textually represented) program that has been formally verified (Hoare, 1969).

We can prove $\models \{x \geq 0 \land y \geq 0\}\ DIV\ \{a \cdot y + b = x \land b < y\}$

by showing $\vdash_{PD} \underbrace{\{x \geq 0 \land y \geq 0\}}_{=:p^D}\ DIV\ \underbrace{\{a \cdot y + b = x \land b < y\}}_{=:q^D}$, i.e., derivability in PD:

$$\dfrac{\dfrac{\text{(1)}}{\{x \geq 0 \land y \geq 0\}\ a := 0;\ b := x\ \{P\},}\qquad \dfrac{\dfrac{}{P \to P,}\quad \dfrac{\dfrac{\text{(2)}}{\{P \land (b \geq y)\}\ b := b - y;\ a := a + 1\ \{P\}}}{\{P\}\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{P \land \neg(b \geq y)\},}\,\text{(R5)}\quad \dfrac{\text{(3)}}{P \land \neg(b \geq y) \to a \cdot y + b = x \land b < y}}{\{P\}\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{a \cdot y + b = x \land b < y\}}\,\text{(R6)}}{\{x \geq 0 \land y \geq 0\}\ a := 0;\ b := x;\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{a \cdot y + b = x \land b < y\}}\,\text{(R3)}$$

(A1) $\{p\}\ skip\ \{p\}$

(R3) $\dfrac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$

(R5) $\dfrac{\{p \land B\}\ S\ \{p\}}{\{p\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{p \land \neg B\}}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R4) $\dfrac{\{p \land B\}\ S_1\ \{q\},\ \{p \land \neg B\}\ S_2\ \{q\}}{\{p\}\ \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}\ \{q\}}$

(R6) $\dfrac{p \to p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \to q}{\{p\}\ S\ \{q\}}$

Example Proof Cont'd

$$\dfrac{\dfrac{\text{(1)}}{\{x \geq 0 \wedge y \geq 0\}\; a := 0;\ b := x\; \{P\},} \qquad \dfrac{\dfrac{}{P \to P,} \quad \dfrac{\dfrac{\text{(2)}}{\{P \wedge (b \geq y)\}\; b := b - y;\ a := a + 1\; \{P\}}}{\{P\}\; \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\; \{P \wedge \neg(b \geq y)\},}\,\text{(R5)} \quad \dfrac{\text{(3)}}{P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y}}{\{P\}\; \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\; \{a \cdot y + b = x \wedge b < y\}}\,\text{(R6)}}{\{x \geq 0 \wedge y \geq 0\}\; a := 0;\ b := x;\ \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\; \{a \cdot y + b = x \wedge b < y\}}\,\text{(R3)}$$

In the following, we show

(1) $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\; a := 0;\ b := x\; \{P\}$,

(2) $\vdash_{PD} \{P \wedge b \geq y\}\; b := b - y;\ a := a + 1\; \{P\}$,

(3) $\models P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y$.

As **loop invariant**, we choose (**creative act!**):

$$P \equiv a \cdot y + b = x \wedge b \geq 0$$

Proof of (1)

(A1) $\{p\}\ skip\ \{p\}$

(R4) $\dfrac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{q\}}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R5) $\dfrac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{p \wedge \neg B\}}$

(R3) $\dfrac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$

(R6) $\dfrac{p \rightarrow p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \rightarrow q}{\{p\}\ S\ \{q\}}$

- **(1)** claims:

  $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\}$

  where $P \equiv a \cdot y + b = x \wedge b \geq 0$.

- $\vdash_{PD} \{\underbrace{0 \cdot y + x = x \wedge x \geq 0}_{p[u:=t]}\}\ \overset{u}{a} := \overset{t}{0}\ \{\underbrace{a \cdot y + x = x \wedge x \geq 0}_{p}\}$ by (A2),

$$(a \cdot y + x = x \wedge x \geq 0)[a := 0]$$

$$0 \cdot y + x = x \wedge x \geq 0$$

Proof of (1)

(A1) $\{p\}\ \mathit{skip}\ \{p\}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R3) $\dfrac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$

(R4) $\dfrac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{q\}}$

(R5) $\dfrac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{p \wedge \neg B\}}$

(R6) $\dfrac{p \rightarrow p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \rightarrow q}{\{p\}\ S\ \{q\}}$

- **(1)** claims:

  $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\}$

  where $P \equiv a \cdot y + b = x \wedge b \geq 0$.

- $\vdash_{PD} \{0 \cdot y + x = x \wedge x \geq 0\}\ a := 0\ \{a \cdot y + x = x \wedge x \geq 0\}$ by (A2),

- $\vdash_{PD} \{a \cdot y + x = x \wedge x \geq 0\}\ b := x\ \{\underbrace{a \cdot y + b = x \wedge b \geq 0}_{\equiv P}\}$ by (A2),

- thus, $\vdash_{PD} \{0 \cdot y + x = x \wedge x \geq 0\}\ a := 0;\ b := x\ \{P\}$ by (R3),

- using $x \geq 0 \wedge y \geq 0 \rightarrow 0 \cdot y + x = x \wedge x \geq 0$ and $P \rightarrow P$, we obtain

  $$\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\}$$

  by (R6). ☐

Substitution

The rule ‘**Assignment**’ uses (syntactical) **substitution**: $\{p[u := t]\}\ u := t\ \{p\}$

(In formula p, replace all (free) occurences of (program or logical) variable u by term t.)

Defined as usual, only **indexed** and **bound** variables need to be treated specially:

$$a \geq x\,[x := u+3] \rightsquigarrow a \geq u+3$$

$$a \geq x \wedge \forall \underline{x} \bullet b \geq \underline{x}\ [x := u+3] \rightsquigarrow \ ?$$

$$\rightsquigarrow a \geq x \wedge \forall z \bullet b \geq z \rightsquigarrow a \geq u+3 \wedge \forall z \bullet b \geq z$$

Substitution

The rule '**Assignment**' uses (syntactical) **substitution**: $\{p[u := t]\}\ u := t\ \{p\}$

(In formula p, replace all (free) occurences of (program or logical) variable u by term t.)

Defined as usual, only **indexed** and **bound** variables need to be treated specially:

Expressions:

- plain variable x: $x[u := t] \equiv \begin{cases} t & \text{, if } x = u \\ x & \text{, otherwise} \end{cases}$
- constant c:
  $c[u := t] \equiv c$.
- constant op, terms s_i:
  $op(s_1, \dots, s_n)[u := t]$
  $\equiv op(s_1[u := t], \dots, s_n[u := t])$.
- conditional expression:
  $(B\ ?\ s_1 : s_2)[u := t]$
  $\equiv (B[u := t]\ ?\ s_1[u := t] : s_2[u := t])$

Formulae:

- boolean expression $p \equiv s$:
  $p[u := t] \equiv s[u := t]$
- negation:
  $(\neg q)[u := t] \equiv \neg(q[u := t])$
- conjunction etc.:
  $(q \wedge r)[u := t]$
  $\equiv q[u := t] \wedge r[u := t]$
- **quantifier**:
  $(\forall\, x : q)[u := t] \equiv \forall\, y : q[x := y][u := t]$
  y fresh (not in q, t, u), same type as x.

- **indexed variable**, u plain or $u \equiv b[t_1, \dots, t_m]$ and $a \neq b$:

$$(a[s_1, \dots, s_n])[u := t] \equiv a[s_1[u := t], \dots, s_n[u := t]])$$

- **indexed variable**, $u \equiv a[t_1, \dots, t_m]$:

$$(a[s_1, \dots, s_n])[u := t] \equiv (\textstyle\bigwedge_{i=1}^{n} s_i[u := t] = t_i\ ?\ t\ : a[s_1[u := t], \dots, s_n[u := t]])$$

Example Proof Cont'd

$$\dfrac{\dfrac{(1)}{\{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\},} \quad \dfrac{\dfrac{}{P \to P,} \quad \dfrac{\dfrac{(2)}{\{P \wedge (b \geq y)\}\ b := b - y;\ a := a + 1\ \{P\}}}{\{P\}\ \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\ \{P \wedge \neg(b \geq y)\},}\ \text{(R5)} \quad \dfrac{(3)}{P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y}}{\{P\}\ \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\ \{a \cdot y + b = x \wedge b < y\}}\ \text{(R6)}}{\{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x;\ \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}\ \{a \cdot y + b = x \wedge b < y\}}\ \text{(R3)}$$

In the following, we show

(1) $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\}$,

(2) $\vdash_{PD} \{P \wedge b \geq y\}\ b := b - y;\ a := a + 1\ \{P\}$,

(3) $\models P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y$.

As **loop invariant**, we choose (**creative act!**):

$$P \equiv a \cdot y + b = x \wedge b \geq 0$$

(A1) $\{p\}\ skip\ \{p\}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R3) $\dfrac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$

(R4) $\dfrac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{q\}}$

(R5) $\dfrac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{p \wedge \neg B\}}$

(R6) $\dfrac{p \to p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \to q}{\{p\}\ S\ \{q\}}$

Proof of (2)

(A1) $\{p\}\ skip\ \{p\}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R3) $$\frac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$$

(R4) $$\frac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{q\}}$$

(R5) $$\frac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{p \wedge \neg B\}}$$

(R6) $$\frac{p \rightarrow p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \rightarrow q}{\{p\}\ S\ \{q\}}$$

- **(2)** claims:

  $\vdash_{PD} \{P \wedge \underline{b \geq y}\}\ b := b - y;\ a := a + 1\ \{P\}$

  where $P \equiv a \cdot y + b = x \wedge b \geq 0$.

- $\vdash_{PD} \{(a+1) \cdot y + \underline{(b - y)} = x \wedge \underline{(b - y)} \geq 0\}\ b := b - y\ \{(a+1) \cdot y + \underline{b} = x \wedge \underline{b} \geq 0\}$

  by (A2),

- $\vdash_{PD} \{\underline{(a+1)} \cdot y + b = x \wedge b \geq 0\}\ a := a + 1\ \{\underbrace{a \cdot y + b = x \wedge b \geq 0}_{\equiv P}\}$ by (A2),

- $\vdash_{PD} \{(a+1) \cdot y + (b - y) = x \wedge (b - y) \geq 0\}\ b := b - y;\ a := a + 1\ \{P\}$ by (R3),

- using $P \wedge b \geq y \rightarrow (a+1) \cdot y + (b - y) = x \wedge (b - y) \geq 0$ and $P \rightarrow P$ we obtain,

  $$\vdash_{PD} \{P \wedge b \geq y\}\ b := b - y;\ a := a + 1\ \{P\}$$

  by (R6). ☐

Example Proof Cont'd

$$\dfrac{\dfrac{\textbf{(1)}}{\{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\},} \quad \dfrac{\dfrac{}{P \to P,} \quad \dfrac{\dfrac{\textbf{(2)}}{\{P \wedge (b \geq y)\}\ b := b - y;\ a := a + 1\ \{P\}}}{\{P\}\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{P \wedge \neg(b \geq y)\},}\text{(R5)} \quad \dfrac{\textbf{(3)}}{P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y}}{\{P\}\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{a \cdot y + b = x \wedge b < y\}}\text{(R6)}}{\{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x;\ \mathbf{while}\ b \geq y\ \mathbf{do}\ b := b - y;\ a := a + 1\ \mathbf{od}\ \{a \cdot y + b = x \wedge b < y\}}\text{(R3)}$$

In the following, we show

(1) $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\ a := 0;\ b := x\ \{P\}$,

(2) $\vdash_{PD} \{P \wedge b \geq y\}\ b := b - y;\ a := a + 1\ \{P\}$,

(3) $\models P \wedge \neg(b \geq y) \to a \cdot y + b = x \wedge b < y$.

As **loop invariant**, we choose (**creative act!**):

$$P \equiv a \cdot y + b = x \wedge b \geq 0$$

(A1) $\{p\}\ skip\ \{p\}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R3) $\dfrac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$

(R4) $\dfrac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \mathbf{if}\ B\ \mathbf{then}\ S_1\ \mathbf{else}\ S_2\ \mathbf{fi}\ \{q\}}$

(R5) $\dfrac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \mathbf{while}\ B\ \mathbf{do}\ S\ \mathbf{od}\ \{p \wedge \neg B\}}$

(R6) $\dfrac{p \to p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \to q}{\{p\}\ S\ \{q\}}$

Proof of (3)

(3) claims

$$\models P \land \neg(b \geq y) \to a \cdot y + b = x \land b < y.$$

where $P \equiv a \cdot y + b = x \land b \geq 0$.

Proof: easy.

Back to the Example Proof

We have shown:

(1) $\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\; a := 0;\; b := x\; \{P\}$,

(2) $\vdash_{PD} \{P \wedge b \geq y\}\; b := b - y;\; a := a + 1\; \{P\}$,

(3) $\models P \wedge \neg(b \geq y) \rightarrow a \cdot y + b = x \wedge b < y$.

and

$$\dfrac{\dfrac{\text{(1)}\ ✓}{\{x \geq 0 \wedge y \geq 0\}\; a := 0;\; b := x\; \{P\},} \qquad \dfrac{\dfrac{}{P \rightarrow P,} \quad \dfrac{\dfrac{\text{(2)}\ ✓}{\{P \wedge (b \geq y)\}\; b := b - y;\; a := a + 1\; \{P\}}}{\{P\}\; \textbf{while}\; b \geq y\; \textbf{do}\; b := b - y;\; a := a + 1\; \textbf{od}\; \{P \wedge \neg(b \geq y)\},}\,\text{(R5)} \quad \dfrac{\text{(3)}\ ✓}{P \wedge \neg(b \geq y) \rightarrow a \cdot y + b = x \wedge b < y}}{\{P\}\; \textbf{while}\; b \geq y\; \textbf{do}\; b := b - y;\; a := a + 1\; \textbf{od}\; \{a \cdot y + b = x \wedge b < y\}}\,\text{(R6)}}{\{x \geq 0 \wedge y \geq 0\}\; a := 0;\; b := x;\; \textbf{while}\; b \geq y\; \textbf{do}\; b := b - y;\; a := a + 1\; \textbf{od}\; \{a \cdot y + b = x \wedge b < y\}}\,\text{(R3)}$$

thus

$$\vdash_{PD} \{x \geq 0 \wedge y \geq 0\}\; \underbrace{a := 0;\; b := x;\; \textbf{while}\; b \geq y\; \textbf{do}\; b := b - y;\; a := a + 1\; \textbf{od}}_{\equiv DIV}\; \{a \cdot y + b = x \wedge b < y\}$$

and thus (since PD is sound) DIV is **partially correct** wrt.

- **pre-condition**: $x \geq 0 \wedge y \geq 0$,
- **post-condition**: $a \cdot y + b = x \wedge b < y$.

IOW: whenever DIV is called with x and y such that $x \geq 0 \wedge y \geq 0$, then (if DIV terminates) $a \cdot y + b = x \wedge b < y$ will hold.

Once Again

(A1) $\{p\}\ skip\ \{p\}$

(A2) $\{p[u := t]\}\ u := t\ \{p\}$

(R3) $$\frac{\{p\}\ S_1\ \{r\},\ \{r\}\ S_2\ \{q\}}{\{p\}\ S_1;\ S_2\ \{q\}}$$

(R4) $$\frac{\{p \wedge B\}\ S_1\ \{q\},\ \{p \wedge \neg B\}\ S_2\ \{q\}}{\{p\}\ \textbf{if}\ B\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \textbf{fi}\ \{q\}}$$

(R5) $$\frac{\{p \wedge B\}\ S\ \{p\}}{\{p\}\ \textbf{while}\ B\ \textbf{do}\ S\ \textbf{od}\ \{p \wedge \neg B\}}$$

(R6) $$\frac{p \rightarrow p_1,\ \{p_1\}\ S\ \{q_1\},\ q_1 \rightarrow q}{\{p\}\ S\ \{q\}}$$

- $P \equiv a \cdot y + b = x \wedge b \geq 0$

  $\{x \geq 0 \wedge y \geq 0\}$
  $\{0 \cdot y + x = x \wedge x \geq 0\}$
- $a := 0;$

  $\{a \cdot y + x = x \wedge x \geq 0\}$
- $b := x;$

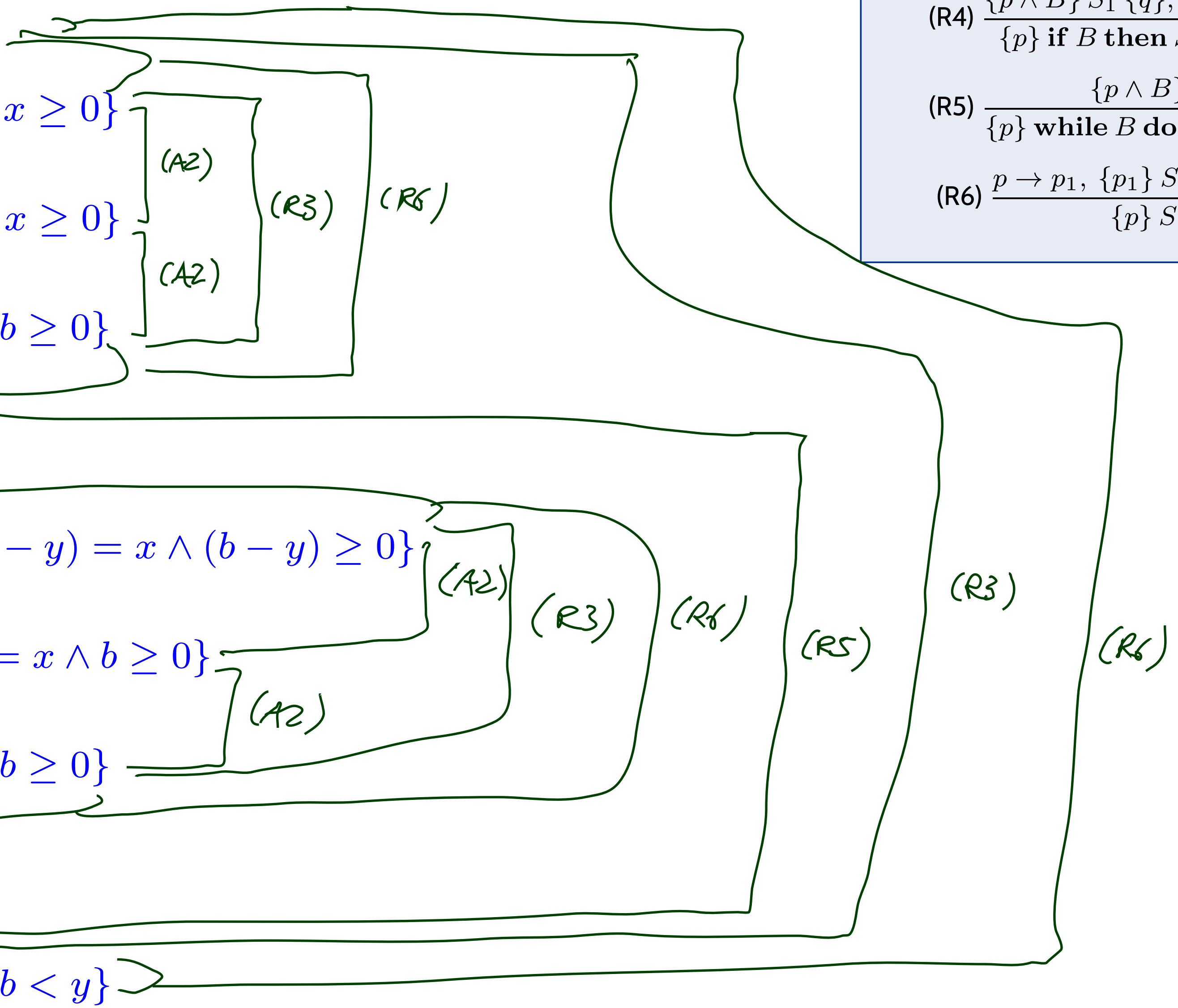
  $\{a \cdot y + b = x \wedge b \geq 0\}$
  $\{P\}$
- **while** $b \geq y$ **do**

  $\{P \wedge b \geq y\}$
  $\{(a+1) \cdot y + (b-y) = x \wedge (b-y) \geq 0\}$
- $b := b - y;$

  $\{(a+1) \cdot y + b = x \wedge b \geq 0\}$
- $a := a + 1$

  $\{a \cdot y + b = x \wedge b \geq 0\}$
  $\{P\}$
- **od**

  $\{P \wedge \neg(b \geq y)\}$
  $\{a \cdot y + b = x \wedge b < y\}$



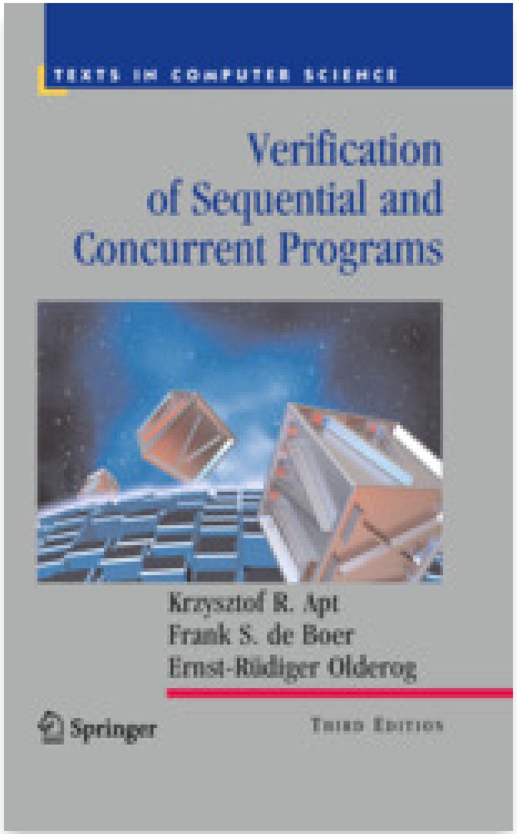
Literature Recommendation


Apt · Olderog

Programmverifikation

Sequentielle, parallele und verteilte Programme

Springer-Lehrbuch


TEXTS IN COMPUTER SCIENCE

Verification of Sequential and Concurrent Programs

Krzysztof R. Apt
Frank S. de Boer
Ernst-Rüdiger Olderog

Springer

THIRD EDITION

Content

- **Formal Program Verification**
  - **Deterministic Programs**
    - **Syntax**
    - **Semantics**
    - Termination, Divergence
  - **Correctness** of deterministic programs
    - **partial** correctness,
    - **total** correctness.
  - **Proof System PD**
- **The Verifier for Concurrent C**

Assertions

Assertions

- Extend the **syntax** of **deterministic programs** by

$$S ::= \cdots \mid \mathbf{assert}(B)$$

- and the **semantics** by rule

$$\langle \mathbf{assert}(B),\, \sigma \rangle \to \langle E,\, \sigma \rangle \text{ if } \sigma \models B.$$

(If the asserted boolean expression B does not hold in state σ, the empty program is not reached; otherwise the assertion remains in the first component: **abnormal** program termination).

Extend PD by axiom:

$$\text{(A7) } \{p\}\ \mathbf{assert}(p)\ \{p\}$$

- That is, if p holds **before** the assertion, then we can **continue** with the derivation in PD.

  If p does not hold, we **“get stuck”** (and cannot complete the derivation).

- So we **cannot** derive $\{\mathit{true}\}\ x := 0;\ \texttt{assert}(x = 27)\ \{\mathit{true}\}$ in PD.

Modular Reasoning

Modular Reasoning

We can add another rule for calls of functions $f : F$ (simplest case: only global variables):

$$\text{(R7)}\ \frac{\{p\}\ F\ \{q\}}{\{p\}\ f()\ \{q\}}$$

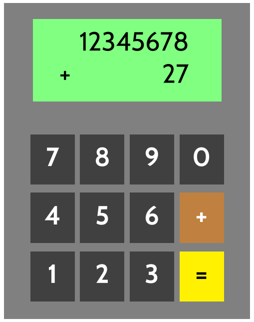
"If we have $\vdash \{p\}\ F\ \{q\}$ for the **implementation** of function f,
then if f is **called** in a state satisfying p, the state after return of f will satisfy q."

p is called **pre-condition** and q is called **post-condition** of f.

Example: if we have

- $\{\mathit{true}\}$ `read_number` $\{0 \leq \mathit{result} < 10^8\}$
- $\{0 \leq x \wedge 0 \leq y\}$ `add` $\{(\mathit{old}(x) + \mathit{old}(y) < 10^8 \wedge \mathit{result} = \mathit{old}(x) + \mathit{old}(y)) \vee \mathit{result} < 0\}$
- $\{\mathit{true}\}$ `display` $\{(0 \leq \mathit{old}(\mathit{sum}) < 10^8 \implies \text{"}\mathit{old}(\mathit{sum})\text{"}) \wedge (\mathit{old}(\mathit{sum}) < 0 \implies \text{"}\texttt{-E-}\text{"})\}$

we may be able to prove our pocket calculator correct.



```
int main() {

  while (true) {
    int x = read_number();
    int y = read_number();

    int sum = add( x, y );

    display(sum);
  }
}
```

Return Values and Old Values

- For **modular reasoning**, it's often useful to refer in the post-condition to
  - the **return value** as $result$,
  - the **values** of variable x **at calling time** as $old(x)$.

- Can be defined using **auxiliary variables**:
  - Transform function

$$T\ f()\ \{\ldots;\ \mathbf{return}\ expr;\}$$

(over variables $V = \{v_1, \ldots, v_n\}$; where $result, v_i^{old} \notin V$) into

```
$T\ f()\ \{$
    $v_1^{old} := v_1; \ldots; v_n^{old} := v_n;$
    $\ldots;$
    $result := expr;$
    $\mathbf{return}\ result;$
$\}$
```

over $V' = V \cup \{v^{old} \mid v \in V\} \cup \{result\}$.

- Then $old(x)$ is just an abbreviation for x^{old}.

The Verifier for Concurrent C

VCC

- The **Verifier for Concurrent C** (VCC) basically implements Hoare-style reasoning.
- **Special syntax**:
  - `#include <vcc.h>`
  - `_(requires` p`)` — **pre-condition**, p is (basically) a C expression
  - `_(ensures` q`)` — **post-condition**, q is (basically) a C expression
  - `_(invariant` $expr$`)` — **loop invariant**, $expr$ is (basically) a C expression
  - `_(assert` p`)` — **intermediate invariant**, p is (basically) a C expression
  - `_(writes &v)` — VCC considers **concurrent** C programs; we need to declare for each procedure which global variables it is allowed to write to (also checked by VCC)
  - **Special expressions**:
    - `\thread_local(&v)` — no other thread writes to variable v (in pre-conditions)
    - `\old(v)` — the value of v when procedure was called (useful for post-conditions)
    - `\result` — return value of procedure (useful for post-conditions)

VCC Syntax Example

```c
#include <vcc.h>

int a, b;

void div( int x, int y )
  _(requires x >= 0 && y >= 0)
  _(ensures a * y + b == x && b < y)
  _(writes &a)
  _(writes &b)
{
  a = 0;
  b = x;
  while (b >= y)
  _(invariant a * y + b == x && b >= 0)
  {
    b = b - y;
    a = a + 1;
  }
}
```

$$DIV \equiv a := 0;\ b := x;\ \textbf{while}\ b \geq y\ \textbf{do}\ b := b - y;\ a := a + 1\ \textbf{od}$$

$$\{x \geq 0 \wedge y \geq 0\}\ DIV\ \{x \geq 0 \wedge y \geq 0\}$$

VCC Web-Interface

VCC

Does this C program always work?

```
#include <vcc.h>

int a, b;

void div( int x, int y )
_(requires x >= 0 && y >= 0)
_(ensures a * y + b == x && b < y)
_(writes &a)
_(writes &b)
{
  a = 0;
  b = x;
  while (b >= y)
  _(invariant a * y + b == x && b >= 0)
  {
    b = b - y;
    a = a + 1;
  }
}
```

samples
hello
lsearch
safestring
bozosort
spinlock

about Vcc - A Verifier for Concurrent C

VCC is a tool that proves correctness of annotated concurrent C programs or finds problems in them. VCC extends C with design by contract features, like pre- and postcondition as well as type invariants. Annotated programs are translated to logical formulas using the Boogie tool, which passes them to an automated SMT solver Z3 to check their validity.

tools developer about

rise4fun © 2016 Microsoft Corporation - terms of use - privacy & cookies - code of conduct

Example program DIV: `http://rise4fun.com/Vcc/4Kqe`

Interpretation of Results

- VCC result: "**verification succeeded**"
  - We can **only** conclude that the tool
    — under its interpretation of the C-standard, under its platform assumptions (32-bit), etc. —
    claims that there is a proof for $\models \{p\}\ DIV\ \{q\}$.
  - May be due to an error in the tool! (That's a **false negative** then.)

    Yet we can ask **for a printout of the proof** and check it manually
    (hardly possible in practice) or with other tools like interactive theorem provers.
  - **Note**: $\models \{\mathit{false}\}\ f\ \{q\}$ **always** holds.

    That is, a **mistake** in writing down the pre-condition can make errors in the program go undetected!

- VCC result: "**verification failed**"
  - May be a **false positive** (wrt. the goal of finding errors).

    The tool **does not provide counter-examples** in the form of a computation path,
    it (only) gives **hints on input values** satisfying p and causing a violation of q.
  - $\rightarrow$ try to construct a (true) counter-example from the hints.

    or: make loop-invariant(s) (or pre-condition p) stronger, and try again.

- Other case: "**timeout**" etc. — completely **inconclusive** outcome.

VCC Features

- For the exercises, we use VCC only for **sequential, single-thread programs**.
- VCC checks a number of **implicit assertions**:
  - **no arithmetic overflow** in expressions (according to C-standard),
  - **array-out-of-bounds access**,
  - **NULL-pointer dereference**,
  - and many more.
- Verification **does not always succeed**:
  - The backend SMT-solver may not be able to discharge proof-obligations (in particular non-linear multiplication and division are challenging);
  - In many cases, we need to provide **loop invariants** manually.
- VCC also supports:
  - **concurrency**: different threads may write to shared global variables; VCC can check whether concurrent access to shared variables is properly managed;
  - **data structure invariants**: we may declare invariants that have to hold for, e.g., records (e.g. the length field l is always equal to the length of the string field str); those invariants may **temporarily** be violated when updating the data structure.
  - and much more.

Tell Them What You've Told Them. . .

Formal Verification:

- **Program verification** is **another approach** to software quality assurance.
- **Proof System PD** can be used
  - to **prove**
  - that a given program is
  - **correct** wrt. its specification.

  This approach considers **all inputs** inside the specification!
- Tools like **VCC** implement this approach.

References

References

Hoare, C. A. R. (1969). An axiomatic basis for computer programming. *Commun. ACM*, 12(10):576–580.

Ludewig, J. and Lichter, H. (2013). *Software Engineering*. dpunkt.verlag, 3. edition.