

# *Software Design, Modelling and Analysis in UML*

## *Lecture 04: OCL Cont'd, Object Diagrams*

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## Contents & Goals

### Last Lecture:

- OCL Syntax

### This Lecture:

- **Educational Objectives:** Capabilities for following tasks/questions.
  - What is an object diagram? What are object diagrams good for?
  - When is an object diagram called partial? What are partial ones good for?
  - When is an object diagram an object diagram (wrt. what)?
  - Is this an object diagram wrt. to that other thing?
  - How are system states and object diagrams related?
  - What does it mean that an OCL expression is satisfiable?
  - When is a set of OCL constraints said to be consistent?
  - Can you think of an object diagram which violates this OCL constraint?
- **Content:**
  - OCL Semantics
  - Object Diagrams
  - Example: Object Diagrams for Documentation
  - OCL : consistency, satisfiability

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## OCL Semantics [OMG, 2006]

### The Task

OCL Syntax 1/4: Expressions

$expr ::=$		
$w$	$: \tau(w)$	
$  expr_1 = expr_2$	$: \tau \times \tau \rightarrow Bool$	
$  oclIsUndefined_r(expr_1)$	$: \tau \rightarrow Bool$	
$  \{expr_1, \dots, expr_n\}$	$: \tau \times \dots \times \tau \rightarrow Set(\tau)$	
$  isEmpty(expr_1)$	$: Set(\tau) \rightarrow Bool$	
$  size(expr_1)$	$: Set(\tau) \rightarrow Int$	
$  allInstances_C$	$: Set(\tau_C)$	
$  v(expr_1)$	$: \tau_C \rightarrow \tau(v)$	
$  r_1(expr_1)$	$: \tau_C \rightarrow \tau_D$	
$  r_2(expr_1)$	$: \tau_C \rightarrow Set(\tau_D)$	

Where, given  $\mathcal{S} = (\mathcal{F}, \mathcal{C}, V, atr)$ ,

- $W \supseteq \{self\}$  is a set of typed logical variables,  $w$  has type  $\tau(w)$
- $\tau$  is any type from  $\mathcal{F} \cup T_B \cup T_{\mathcal{C}}$   $\cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{C}}\}$
- $T_B$  is a set of basic types, in the following we use  $T_B = \{Bool, Int, String\}$
- $T_{\mathcal{C}} = \{\tau_C \mid C \in \mathcal{C}\}$  is the set of object types,
- $Set(\tau_0)$  denotes the set-of- $\tau_0$  type for  $\tau_0 \in T_B \cup T_{\mathcal{C}}$  (sufficient because of "flattening" (cf. standard))
- $v : \tau(v) \in atr(C), \tau(v) \in \mathcal{S}$ ,
- $r_1 : D_{0,1} \in atr(C)$ ,
- $r_2 : D_* \in atr(C)$ ,
- $C, D \in \mathcal{C}$ .

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- Given an OCL expression  $expr$ , a system state  $\sigma \in \Sigma_{\mathcal{S}}$ , and a valuation of logical variables  $\beta$ , define

$$I[\![\cdot]\!](\cdot, \cdot) : OCLExpressions(\mathcal{S}) \times \Sigma_{\mathcal{S}} \times (W \rightarrow I(\mathcal{F} \cup T_B \cup T_{\mathcal{C}})) \rightarrow I(Bool)$$

such that

$$I[\![expr]\!](\sigma, \beta) \in \{true, false, \perp_{Bool}\}.$$

## Basically business as usual...

- (i) Equip each OCL (!) **basic type** with a reasonable **domain**, i.e. define function

$$I \text{ with } \text{dom}(I) = T_B$$

- (ii) Equip each **object type**  $\tau_C$  with a reasonable **domain**, i.e. define function

$$I \text{ with } \text{dom}(I) = \tau_C$$

(most reasonable:  $\mathcal{D}(C)$  determined by structure  $\mathcal{D}$  of  $\mathcal{S}$ ).

- (iii) Equip each **set type**  $\text{Set}(\tau_0)$  with reasonable **domain**, i.e. define function

$$I \text{ with } \text{dom}(I) = \{\text{Set}(\tau_0) \mid \tau_0 \in T_B \cup T_{\mathcal{E}}\}$$

- (iv) Equip each **arithmetical operation** with a reasonable **interpretation** (that is, with a **function** operating on the corresponding **domains**).

$$I \text{ with } \text{dom}(I) = \{+, -, \leq, \dots\}, \text{ e.g., } I(+) \in I(\text{Int}) \times I(\text{Int}) \rightarrow I(\text{Int})$$

- (v) **Set operations** similar:  $I \text{ with } \text{dom}(I) = \{\text{isEmpty}, \dots\}$

- (vi) Equip each **expression** with a reasonable **interpretation**, i.e. define function

$$I : \text{Expr} \times \Sigma_{\mathcal{S}}^{\mathcal{D}} \times (W \rightarrow I(\mathcal{T} \cup T_B \cup T_{\mathcal{E}})) \rightarrow I(\text{Bool})$$

...except for OCL being a **three-valued logic**, and the "iterate" expression.

## (i) Domains of Basic Types

### Recall:

- $T_B = \{\text{Bool}, \text{Int}, \text{String}\}$

### We set:

- $I(\text{Bool}) := \{\text{true}, \text{false}\} \cup \{\perp_{\text{Bool}}\}$
- $I(\text{Int}) := \mathbb{Z} \cup \{\perp_{\text{Int}}\}$
- $I(\text{String}) := \dots \cup \{\perp_{\text{String}}\}$

\*undefined\*

We may omit index  $\tau$  of  $\perp_{\tau}$  if it is clear from context.

## (ii) Domains of Object and (iii) Set Types

- Now we need a structure  $\mathcal{D}$  of our signature  $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$ .
- **Recall:**  $\mathcal{D}$  assigns an (infinite) domain  $\mathcal{D}(C)$  to each class  $C \in \mathcal{C}$ .
- Let  $\tau_C$  be an (OCL) **object type** for a class  $C \in \mathcal{C}$ .
- We set

$$I(\tau_C) := \mathcal{D}(C) \cup \{\perp_{\tau_C}\} \quad \begin{array}{l} \text{disjoint union, i.e. assume} \\ \perp_{\tau_C} \notin \mathcal{D}(C) \\ \text{otherwise rename in } \mathcal{D}(C) \end{array}$$

- Let  $\tau$  be a type from  $T_B \cup T_{\mathcal{C}}$ .
- We set

$$I(\text{Set}(\tau)) := \mathcal{P}^{I(\tau)} \cup \{\perp_{\text{Set}(\tau)}\} \quad \begin{array}{l} \text{powerset of } I(\tau), \text{ i.e. set of} \\ \text{subsets of } I(\tau) \end{array}$$

**Note:** in the OCL standard, only **finite** subsets of  $I(\tau)$ .  
But infinity doesn't scare **us**, so we simply allow it.

## (iv) Interpretation of Arithmetic Operations

- **Literals** map to fixed values:

$$I(\text{true}) := \text{true}, \quad I(\text{false}) := \text{false}, \quad I(0) := 0, \quad I(1) := 1, \dots$$

$$I(\text{OclUndefined}_{\tau}) := \perp_{\tau} \quad \text{equality relation}$$

- **Boolean operations** (defined point-wise for  $x_1, x_2 \in I(\tau)$ ):

*symbol in OCL syntax*

$$I(=_{\tau})(x_1, x_2) := \begin{cases} \text{true} & \text{, if } x_1 \neq \perp_{\tau} \neq x_2 \text{ and } x_1 = x_2 \\ \text{false} & \text{, if } x_1 \neq \perp_{\tau} \neq x_2 \text{ and } x_1 \neq x_2 \\ \perp_{\text{Bool}} & \text{, otherwise} \end{cases}$$

- **Integer operations** (defined point-wise for  $x_1, x_2 \in I(\text{Int})$ ):

$$I(+)(x_1, x_2) := \begin{cases} x_1 + x_2 & \text{, if } x_1 \neq \perp \neq x_2 \\ \perp & \text{, otherwise} \end{cases}$$

**Note:** There is a **common principle**.

Namely, the **interpretation** of an operation  $\omega : \tau_1 \times \dots \times \tau_n \rightarrow \tau$

is a function  $I(\omega) : I(\tau_1) \times \dots \times I(\tau_n) \rightarrow I(\tau)$  on corresponding semantical domain(s).

### (iv) Interpretation of OclIsUndefined

- The **is-undefined** predicate (defined point-wise for  $x \in I(\tau)$ ):

$$I(\text{oclIsUndefined}_\tau)(x) := \begin{cases} true & , \text{ if } x = \perp_\tau \\ false & , \text{ otherwise} \end{cases}$$

### (v) Interpretation of Set Operations

Basically the same principle as with arithmetic operations...

Let  $\tau \in T_B \cup T_\emptyset$ .

- **Set comprehension** ( $x_1, \dots, x_n \in I(\tau)$ ):

$$I(\{\}_n^\tau)(x_1, \dots, x_n) := \{x_1, \dots, x_n\}$$

for all  $n \in \mathbb{N}_0$

- **Empty-ness check** ( $x \in I(\text{Set}(\tau))$ ):

$$I(\text{isEmpty}^\tau)(x) := \begin{cases} true & , \text{ if } x = \emptyset \\ \perp_{Bool} & , \text{ if } x = \perp_{\text{Set}(\tau)} \\ false & , \text{ otherwise} \end{cases}$$

- **Counting** ( $x \in I(\text{Set}(\tau))$ ):

*cardinality*

$$I(\text{size}^\tau)(x) := |x| \text{ if } x \neq \perp_{\text{Set}(\tau)} \text{ and } \perp_{Int} \text{ otherwise}$$

## (vi) Putting It All Together

### OCL Syntax 1/4: Expressions

```

expr ::=
| w : τ(w)
| expr1 = expr2 : τ × τ → Bool
| oclUndefinedτ(expr1) : τ → Bool
| {expr1, ..., exprn} : τ × ... × τ → Set(τ)
| isEmpty(expr1) : Set(τ) → Bool
| size(expr1) : Set(τ) → Int
| allInstancesC : Set(τC)
| v(expr1) : τC → τ(v)
| r1(expr1) : τC → τD
| r2(expr1) : τC → Set(τD)
    
```

Where, given  $\mathcal{S} = (\mathcal{T}, \mathcal{C})$ ,

- $W \supseteq \{self\}$  is a set of logical variables,  $w$  has
- $\tau$  is any type from  $\mathcal{T} \cup \{Set(\tau_0) \mid \tau_0 \in T_B \cup T_\emptyset\}$
- $T_B$  is a set of basic types, the following we use  $T_B = \{Bool, Int, St\}$
- $T_\emptyset = \{\tau_C \mid C \in \mathcal{C}\}$  set of object types,
- $Set(\tau_0)$  denotes the set-of- $\tau_0$  type for  $\tau_0 \in T_B \cup T_\emptyset$  (sufficient because of "flattening" (cf. star
- $v : \tau(v) \in atr(C), \tau(v)$
- $r_1 : D_{0,1} \in atr(C),$
- $r_2 : D_* \in atr(C),$
- $C, D \in \mathcal{C}.$

### OCL Syntax 2/4: Constants, Arithmetical Operat

For example:

```

expr ::= ...
| true, false : Bool
| expr1 {and, or, implies} expr2 : Bool × Bool → Bool
| not expr1 : Bool → Bool
| 0, -1, 1, -2, 2, ... : Int
| OclUndefined : τ
| expr1 {+, ...,} expr2 : Int × Int → I
| expr1 {<, ≤, ...} expr2 : Int × Int → I
    
```

Generalised notation:

```

expr ::= ω(expr1, ..., exprn) : τ1 × ... × τn -
with ω ∈ {+, -, ...}
    
```

### OCL Syntax 3/4: Iterate

```

expr ::= ... | expr1 -> iterate(w1 : τ1; w2 : τ2 = expr2 | expr3)
or, with a little renaming,
expr ::= ... | expr1 -> iterate( iter : τ1; result : τ2 = expr2 | expr3)
    
```

### OCL Syntax 4/4: Context

```

context ::= context w1 : τ1, ..., wn : τn inv : expr
where w ∈ W and τi ∈ T∅, 1 ≤ i ≤ n, n ≥ 0.
    
```

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## Valuations of Logical Variables

- Recall:** we have typed logical variables ( $w \in W$ ),  $\tau(w)$  is the type of  $w$ .
- By  $\beta$ , we denote a valuation of the logical variables, i.e. for each  $w \in W$ ,

$$\beta(w) \in I(\tau(w)).$$

$$\beta: W \longrightarrow \bigcup_{w \in W} I(\tau(w))$$

e.g. if  $w: \tau_c$  then

$$\beta(w) \in I(\tau_c) = \mathcal{D}(C) \cup \{\perp\}$$

(vi) Putting It All Together...

$$\text{expr} ::= w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)$$

- $I[[w]](\sigma, \beta) := \beta(w)$
- $I[[\omega(\text{expr}_1, \dots, \text{expr}_n)]](\sigma, \beta) := I(\omega) \left( I[[\text{expr}_1]](\sigma, \beta), \dots, I[[\text{expr}_n]](\sigma, \beta) \right)$
- $I[[\text{allInstances}_C]](\sigma, \beta) := \text{dom}(\sigma) \cap \mathcal{D}(C)$

**Note:** in the OCL standard,  $\text{dom}(\sigma)$  is assumed to be **finite**.  
Again: doesn't scare us.

(vi) Putting It All Together...

$$\text{expr} ::= w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)$$

Assume  $\text{expr}_1 : \tau_C$  for some  $C \in \mathcal{C}$ . Set  $u_1 := I[[\text{expr}_1]](\sigma, \beta) \in \overset{I}{\mathcal{D}}(\tau_C) = \mathcal{D}(C) \cup \{\perp\}$

- $I[[v(\text{expr}_1)]](\sigma, \beta) := \begin{cases} \sigma(u_1)(v) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp & , \text{ otherwise } \end{cases}$
- $I[[r_1(\text{expr}_1)]](\sigma, \beta) := \begin{cases} u & , \text{ if } u_1 \in \text{dom}(\sigma) \text{ and } \sigma(u_1)(r_1) = \{u\} \\ \perp & , \text{ otherwise } \end{cases}$
- $I[[r_2(\text{expr}_1)]](\sigma, \beta) := \begin{cases} \sigma(u_1)(r_2) & , \text{ if } u_1 \in \text{dom}(\sigma) \\ \perp & , \text{ otherwise } \end{cases}$

(Recall:  $\sigma$  evaluates  $r_2$  of type  $C_*$  to a set)

## (vi) Putting It All Together...

$$\text{expr} ::= w \mid \omega(\text{expr}_1, \dots, \text{expr}_n) \mid \text{allInstances}_C \mid v(\text{expr}_1) \mid r_1(\text{expr}_1) \mid r_2(\text{expr}_1) \mid \text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)$$

*assign to hlp the set denoted by expr<sub>1</sub>*
*assign to v<sub>2</sub> the initial expression*

$I[\text{expr}_1 \rightarrow \text{iterate}(v_1 : \tau_1 ; v_2 : \tau_2 = \text{expr}_2 \mid \text{expr}_3)](\sigma, \beta)$

$$:= \begin{cases} I[\text{expr}_2](\sigma, \beta) & , \text{ if } I[\text{expr}_1](\sigma, \beta) = \emptyset \\ \text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta') & , \text{ otherwise} \end{cases}$$

*modification of  $\beta$  at hlp and*

where  $\beta' = \beta[\text{hlp} \mapsto I[\text{expr}_1](\sigma, \beta), v_2 \mapsto I[\text{expr}_2](\sigma, \beta)]$  and

$$\text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta')$$

$$:= \begin{cases} I[\text{expr}_3](\sigma, \beta'[v_1 \mapsto x]) & , \text{ if } \beta'(\text{hlp}) = \{x\} \\ I[\text{expr}_3](\sigma, \beta'') & , \text{ if } \beta'(\text{hlp}) = X \cup \{x\} \text{ and } X \neq \emptyset \end{cases}$$

where  $\beta'' = \beta'[v_1 \mapsto x, v_2 \mapsto \text{iterate}(\text{hlp}, v_1, v_2, \text{expr}_3, \sigma, \beta'[\text{hlp} \mapsto X])]$

**Quiz:** Is (our)  $I$  a function?

### Example

$\sigma = \{ \tau_{1h} \mapsto \{ \text{name} = \text{"Schulz"}; \text{age} = 27, \text{meetings} = \{ \tau_{1h} \} \}, \tau_{3M} \mapsto \{ \text{title} = \text{"Briefing"}, \text{numPart} = 2, \text{start} = 1.5.23, \text{duration} = 120, \text{participants} = \{ \tau_{1h}, \tau_{8h} \}, \text{location} = \{ \tau_{2l} \} \}, \tau_{2l} \mapsto \{ \text{name} = \text{"Hall"}, \text{meeting} = \{ \tau_{3M} \} \}, \tau_{8h} \mapsto \{ \text{name} = \text{"Boss"}, \text{age} = 57, \text{meetings} = \{ \tau_{3M} \} \} \}$

$\beta = \{ \text{self} \mapsto \tau_{1h} \}$

$I[\text{self}](\sigma, \beta) = \beta(\text{self}) = \tau_{1h}$

$I[\text{age}(\text{self})](\sigma, \beta) = \sigma(\tau_{1h})(\text{age}) = 27$

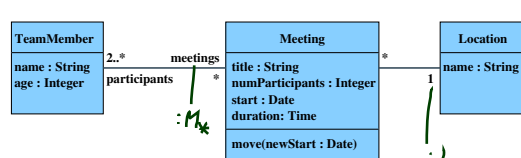
$I[\geq(\text{age}(\text{self}), 18)](\sigma, \beta) = I[\geq](I[\text{age}(\text{self})](\sigma, \beta), I[18]) = \geq(27, 18) = \text{true} \in I[\text{bool}]$

$I[\text{allInstances}_{\tau_{1h}}](\sigma, \beta) = \text{dom}(\sigma) \cap \mathcal{D}(\tau_{1h}) = \{ \tau_{1h}, \tau_{2l}, \tau_{8h}, \tau_{3M} \} \cap \{ \tau_{1h}, \tau_{2m}, \dots \} = \{ \tau_{1h}, \tau_{8h} \}$

$I[\text{meetings}(\text{self})](\sigma, \beta) = \{ \tau_{3M} \}$

$I[\text{location}(x)](\sigma, \{ x \mapsto \tau_{3M} \}) = \tau_{2l}$

• something will  $\perp$ : exercises/tutorial



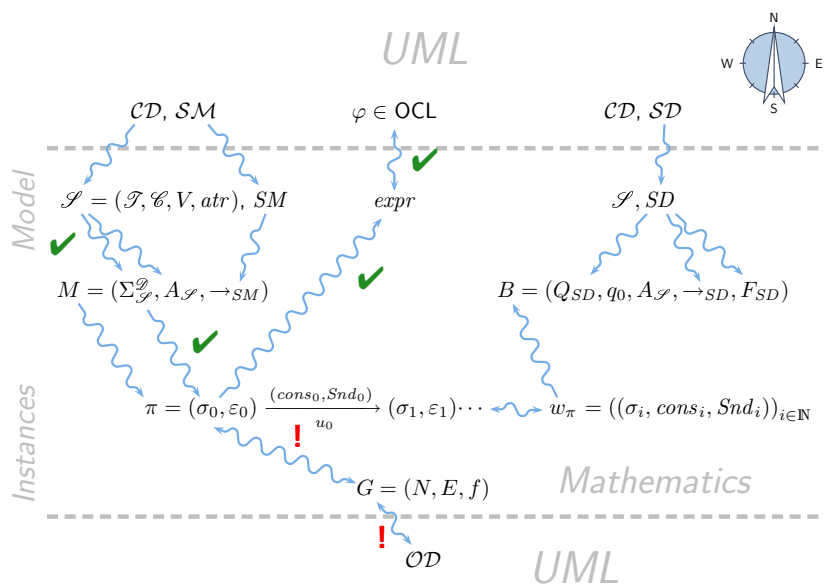
- context TeamMember inv: age  $\Rightarrow$  18
- context Meeting inv: duration  $>$  0

$\text{allInstances}_{\tau_{1h}} \rightarrow \text{iterate}(\text{self: TeamMember}; \text{res: Bool} = \text{true} \mid \text{res and } \geq(\text{age}(\text{self}), 18))$



# Where Are We?

## You Are Here.



## Object Diagrams

### Graph

**Definition.** A node labelled **graph** is a triple

$$G = (N, E, f)$$

consisting of

- vertexes  $N$ ,
- edges  $E$ ,
- node labeling  $f : N \rightarrow X$ , where  $X$  is some label domain,

# Object Diagrams

**Definition.** Let  $\mathcal{D}$  be a structure of signature  $\mathcal{S} = (\mathcal{T}, \mathcal{C}, V, atr)$  and  $\sigma \in \Sigma_{\mathcal{D}}$  a system state.

Then any graph  $G = (N, E, f)$  with

- nodes are identities (not necessarily alive), i.e.

- edges correspond to "links" of objects, i.e.

$$E \subseteq N \times \{v : \tau \in V \mid \tau \in \{C_{0,1}, C_* \mid C \in \mathcal{C}\}\} \times N,$$

$\forall (u_1, r, u_2) \in E : u_1 \in \text{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r)$

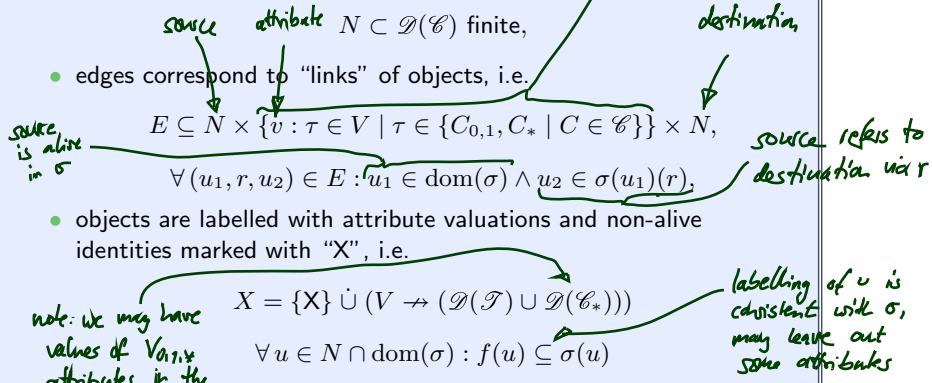
- objects are labelled with attribute valuations and non-alive identities marked with "X", i.e.

$$X = \{X\} \dot{\cup} (V \rightarrow (\mathcal{D}(\mathcal{T}) \cup \mathcal{D}(\mathcal{C}_*)))$$

$$\forall u \in N \cap \text{dom}(\sigma) : f(u) \subseteq \sigma(u)$$

$$\forall u \in N \setminus \text{dom}(\sigma) : f(u) = \{X\}$$

is called **object diagram** of  $\sigma$ . (redundant with edges)



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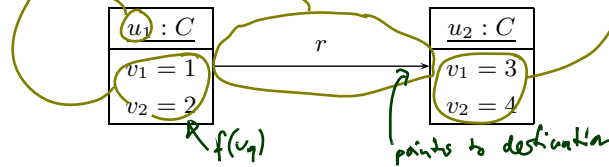
$\sigma = \{$   
 $1_{TH} \mapsto \{ \text{name} = \text{"Schulze"}, \text{age} = 2, \text{meetings} = \{3_M\} \},$   
 $3_M \mapsto \{ \text{title} = \text{"Briefing"}, \text{NumPart} = 2, \text{start} = 13:23, \text{duration} = 120, \text{participants} = \{1_{TH}, 8_{TH}\} \cup \{5_{TH}\}, \text{location} = \{7_L\} \},$   
 $7_L \mapsto \{ \text{name} = \text{"Hall"}, \text{meeting} = \{3_M\} \}$   
 $8_{TH} \mapsto \{ \text{name} = \text{"Boss"}, \text{age} = 57, \text{meetings} = \{3_M\} \}$

$(N, E, f)$   
 $N = \{1_{TH}, 3_M, 5_{TH}\}$   
 $E = \{ (3_M, \text{participants}, 1_{TH}), (3_M, \text{participants}, 5_{TH}) \}$   
 $f = \{ 1_{TH} \mapsto \{ \text{age} = 27 \}, 3_M \mapsto \{ \text{participants} = \{1_{TH}, 8_{TH}, 5_{TH}\} \}, 5_{TH} \mapsto X \}$

## Graphical Representation of Object Diagrams

$N \subset \mathcal{D}(\mathcal{C})$  finite,  $E \subset N \times V_{0,1,*} \times N$ ,  $X = \{X\} \dot{\cup} (V \rightarrow (\mathcal{D}(\mathcal{T}) \cup \mathcal{D}(\mathcal{C}_*)))$   
 $u_1 \in \text{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r)$ ,  $f(u) \subseteq \sigma(u)$  or  $f(u) = \{X\}$

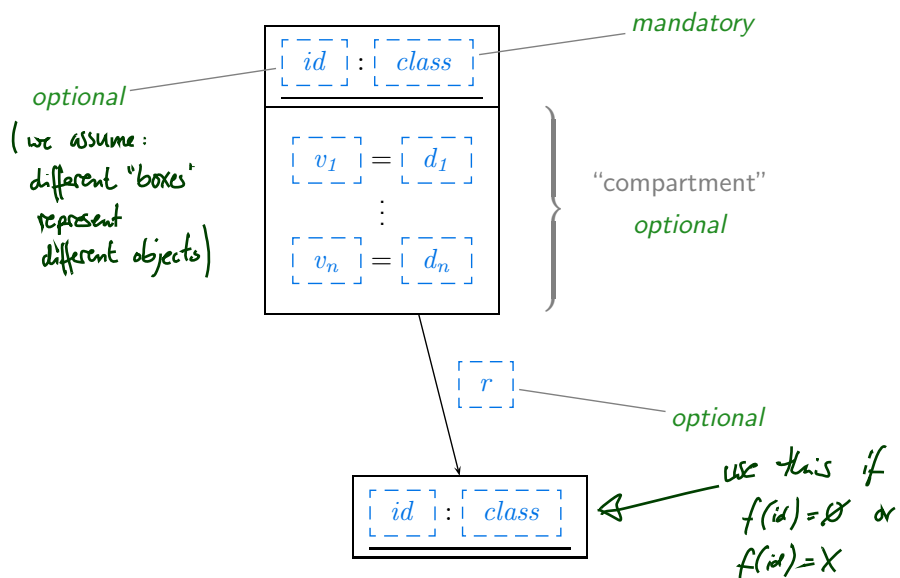
- Assume  $\mathcal{S} = (\{Int\}, \{C\}, \{v_1 : Int, v_2 : Int, r : C_*\}, \{C \mapsto \{v_1, v_2, r\}\})$ .
- Consider  $\sigma = \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2, r \mapsto \{u_2\}\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4, r \mapsto \emptyset\}\}$
- Then  $G = (N, E, f)$   
 $= (\{u_1, u_2\}, \{(u_1, r, u_2)\}, \{u_1 \mapsto \{v_1 \mapsto 1, v_2 \mapsto 2\}, u_2 \mapsto \{v_1 \mapsto 3, v_2 \mapsto 4\}\},$   
 is an object diagram of  $\sigma$  wrt.  $\mathcal{S}$  and any  $\mathcal{D}$  with  $\mathcal{D}(Int) \supseteq \{1, 2, 3, 4\}$ .
- We may equivalently (!) **represent**  $G$  graphically as follows:



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## UML Notation for Object Diagrams



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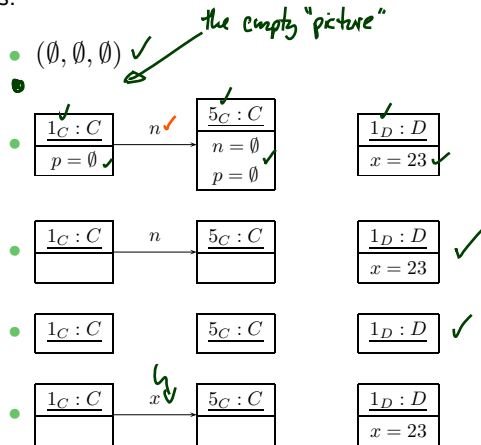
## Object Diagrams: More Examples

$$N \subset \mathcal{D}(\mathcal{C}) \text{ finite, } E \subset N \times V_{0,1,*} \times N, \quad X = \{X\} \dot{\cup} (V \rightarrow (\mathcal{D}(\mathcal{T}) \cup \mathcal{D}(\mathcal{C}_*)))$$

$$u_1 \in \text{dom}(\sigma) \wedge u_2 \in \sigma(u_1)(r), \quad f(u) \subseteq \sigma(u) \text{ or } f(u) = \{X\}$$

$$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$$

vs.



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## Complete vs. Partial Object Diagram

**Definition.** Let  $G = (N, E, f)$  be an object diagram of system state  $\sigma \in \Sigma_{\mathcal{G}}$ .

We call  $G$  **complete** wrt.  $\sigma$  if and only if

- $G$  is **object complete**, i.e.
  - $G$  consists of all alive objects, i.e.  $N = \text{dom}(\sigma)$ ,
- $G$  is **attribute complete**, i.e.
  - $G$  comprises all "links" between alive objects, i.e. if  $u_2 \in \sigma(u_1)(r)$  for some  $u_1, u_2 \in \text{dom}(\sigma)$  and  $r \in V$ , then  $(u_1, r, u_2) \in E$ , and
  - each node is labelled with the values of all  $\mathcal{T}$ -typed attributes, i.e. for each  $u \in \text{dom}(\sigma)$ ,
 
$$f(u) \supseteq \sigma(u)|_{V_{\mathcal{T}} \cup \{r \mapsto (\sigma(u)(r) \setminus N) \mid r \in V : \sigma(u)(r) \setminus N \neq \emptyset\}}$$
 where  $V_{\mathcal{T}} := \{v : \tau \in V \mid \tau \in \mathcal{T}\}$ .

Otherwise we call  $G$  **partial**.

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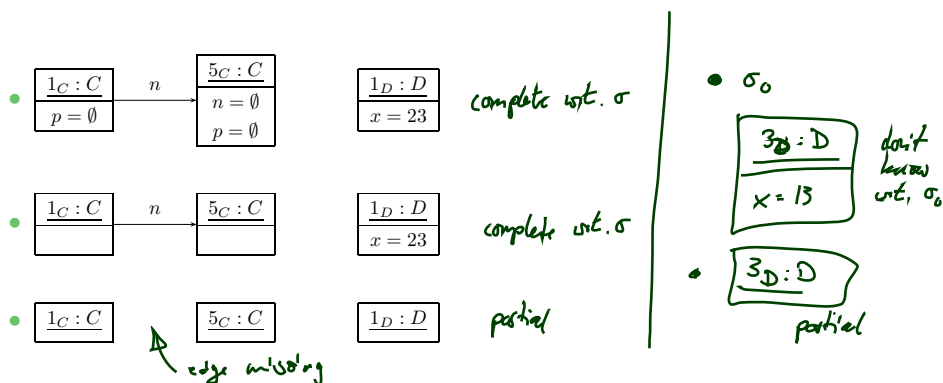
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## Complete vs. Partial Examples

- $N = \text{dom}(\sigma)$ , if  $u_2 \in \sigma(u_1)(r)$ , then  $(u_1, r, u_2) \in E$ ,
- $f(u) = \sigma(u)|_{V_{\mathcal{G}}} \cup \{r \mapsto (\sigma(u)(r) \setminus N) \mid \sigma(u)(r) \setminus N\}$

Complete or partial?

$$\sigma = \{1_C \mapsto \{p \mapsto \emptyset, n \mapsto \{5_C\}\}, 5_C \mapsto \{p \mapsto \emptyset, n \mapsto \emptyset\}, 1_D \mapsto \{x \mapsto 23\}\}$$



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## Complete/Partial is Relative

- Claim:
  - Each finite system state has **exactly one complete** object diagram.
  - A finite system state can have **many partial** object diagrams.

- Each object diagram  $G$  represents a set of system states, namely

$$G^{-1} := \{\sigma \in \Sigma_{\mathcal{G}} \mid G \text{ is an object diagram of } \sigma\}$$

- **Observation:** If somebody **tells us**, that a given (consistent) object diagram  $G$  is **complete**, we can uniquely reconstruct the corresponding system state.

In other words:  $G^{-1}$  is then a singleton.

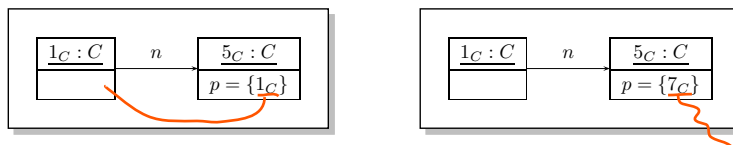
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## Corner Cases

### Closed Object Diagrams vs. Dangling References

Find the 10 differences! (Both diagrams shall be complete.)



**Definition.** Let  $\sigma$  be a system state. We say attribute  $v \in V_{0,1,*}$  has a **dangling reference** in object  $u \in \text{dom}(\sigma)$  if and only if the attribute's value comprises an object which is not alive in  $\sigma$ , i.e. if

$$\sigma(u)(v) \notin \text{dom}(\sigma).$$

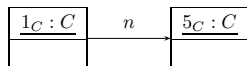
We call  $\sigma$  **closed** if and only if no attribute has a dangling reference in any object alive in  $\sigma$ .

**Observation:** Let  $G$  be the (!) complete object diagram of a **closed** system state  $\sigma$ . Then the nodes in  $G$  are labelled with  $\mathcal{T}$ -typed attribute/value pairs only.

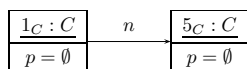
## Special Notation

- $\mathcal{S} = (\{Int\}, \{C\}, \{n, p : C_*\}, \{C \mapsto \{n, p\}\})$ .

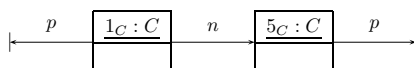
- Instead of



we want to write



or

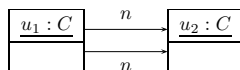


to **explicitly** indicate that attribute  $p : C_*$  has value  $\emptyset$  (also for  $p : C_{0,1}$ ).

## Aftermath

We slightly deviate from the standard (for reasons):

- In the course,  $C_{0,1}$  and  $C_*$ -typed attributes **only** have **sets as values**. UML also considers multisets, that is, they can have



(This is not an object diagram in the sense of our definition because of the requirement on the edges  $E$ . Extension is straightforward but tedious.)

- We **allow** to give the valuation of  $C_{0,1}$ - or  $C_*$ -typed attributes in the **values compartment**.
  - Allows us to indicate that a certain  $r$  is not referring to another object.
  - Allows us to represent “dangling references”, i.e. references to objects which are not alive in the current system state.
- We introduce a graphical representation of  $\emptyset$  values.



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